DYNAMIC MODEL AND CONTROL OF HUMAN POSTURE

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Abstract. *The biomechanic model of a human musculoskeletal system and the simulation of behavior in movement can be applyed in several areas, such as sports, engineering and medicine. The purpose of this work is obtain a dynamic model that represents a musculoskeletal system of a leg. The description of kinematic and dynamic links movements is based on Newton-Euler and Euler-Lagrange formulation. The resulting movements and forces are produced by sets of actuators muscle-tendinideous. These dynamic models are non-linear with multiples input and output and many degrees of freedom. This results in many difficulties in the determination of parameters like forces and moments. The model must be generic enough for accept several muscle models and control techniques. In this paper, it was used a control based on optimal theory control. A geometric model for simulations of postural control is obtained with Matlab/Simulink software.*

Keywords*:* dynamic model, musculoskeletal system, optimal control.

1. INTRODUCTION

Researchers on robotic enthusiastically dreamed with smart machines that realize movements and tasks that a human can realize, with many expectation about modern control. Therefore, progresses in robotic control did not correspond to the expectation and the biggest difficulties were about the understanding of human motion in day-tasks. Human beings can manipulate objects and realize movements and complex tasks with facility and ability, through a biological evolution and trainee. A lot of researchers from several areas are involving in dynamic of human body. While a comprehensive theory of human movement is still far away, it is true that great progress has been made in the last few decades, with important contributions coming from researchers engaged in robotics. Mathematical models of biological systems are a field with the biggest increase in scientific development in the present-day. Although all techniques in simulation and mathematical models, the generalized application in musculoskeletal system is very complex (Thelen, 2006). The biomechanic model of musculoskeletal system and the simulation of the system behavior in motion can contribute to understanding the relationship among musculoskeletal properties and movement and articulation forces with application on medicine, diseases (Anderson *et al*. 2001, Pandy,1995) and sports (Thelen, 2005).

In this work, we use a simpler biomechanical planar models with dynamic torque actuators of posture. The availability this model may lead to new applications of such devices. One of them is the reproduction of human posture, previously acquired in study laboratories, since a correspondence between the real postural problem and the mathematical model can be established. Not only normal postural equilibrium should be reproduced, but also pathological patterns. This can increase the comprehension of the phenomenon by the physician and the chance of success of the therapy.

In the context of optimal control theory applied to the class of dynamical systems we are dealing with, the biomechanical model defines a topology where the solution is a set of control signals, is valid at the same time that an optimization procedure is performed. The advantage of this approach is the possibility of finding open-loop muscle excitation control signals, as well as muscle forces and kinematical trajectories, without previous measurement of the motion (Kuo, 1995, Menegaldo *et al*. 2003).

2. EQUATION OF MOTION

In order to surpass the numerical difficulties associated with the optimal control solution, simpler biomechanical planar models with dynamic torque actuators of posture were adopted (Gruber, 1998). The human system include four rigid segments representing the foot, the leg, the thigh and the upper part of the body, which are linked by three articulations, ankle, knee and hip joints modeled as frictionless hinges as shown in Figure 1.

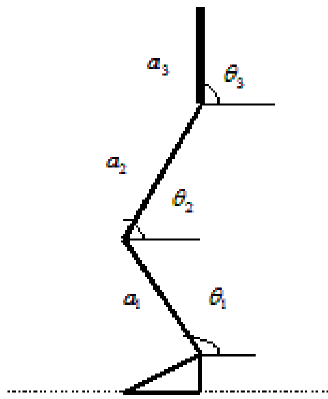

Figure 1. Simpler biomechanical planar model.

In Figure 1, the vector of generalized variables is $q = [\theta_1, \theta_2, \theta_3]$ that represents articulations angle and a_1, a_2 and a_3 the leg, thigh and pelvis length, respectively.

2.1. Position and Orientation of a Rigid Body

An human link, in this case, a leg, can be seen, in a mechanic point of view, like a kinematic open chain, formed by rigid bodies connected by rotation joints (Yang, 1990). An end is connected to the base and the other one to the terminal element. The structure movement is realized by a composition of elementary movements for each link, with respect to the preceding. In order to simulate a movement like walking or pedaling it is necessary a description of position and orientation of joints and links. It is also necessary a derivation of kinematic equations of leg, describing the position and orientation of the terminal element, as function of joint variables with respect to a reference coordinates system. These equations can be obtained through Denavit-Hartenberg convention (Siciliano and Valavanis, 1998). We express the transformation of coordinates that relate the system $O_i(x_i, y_i)$ with the system $O_{i-1}(x_{i-1}, y_{i-1})$, through the following steps:

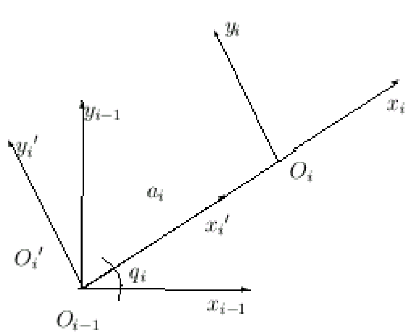

Figure 2. Different coordinated systems.

1. It starts from coordinated system O_{i-1}.

2. It does the rotation θ_i around of z_i axis. This operation take to the system O'_{i-1}, described by homogeneous rotation transformation matrix.

$$A^{\mathrm{i}}_{\mathrm{i-1}} = \begin{bmatrix} \cos\theta_{\mathrm{i}} & -\sin\theta_{\mathrm{i}} & 0 & 0 \\ \sin\theta_{\mathrm{i}} & \cos\theta_{\mathrm{i}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

3. It dislocates the coordinated system O'_{i-1} in a_i, through x'_{i-1} axis. This operation take to the system O_i, described by homogeneous dislocation transformation matrix.

$$D^{\mathrm{i}}_{\mathrm{i-1}} = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

Finally, it is obtained the transformation of coordinates that relate the system $O_i(x_i, y_i)$ with the system $O_{i-1}(x_{i-1}, y_{i-1})$

$$Q^i_{i-1} = A^i_{i-1} D^i_{i-1} \tag{3}$$

For successions of several transformations, as a musculoskeletal system, the position and the orientation of terminal element is represented by total matrix transformation

$$T^3_0 = Q^1_0 Q^2_1 Q^3_2 \tag{4}$$

2.2. Geometric Jacobian

Once knew the direct kinematic equations, we obtain the relationship among velocity of joints and linear and angular velocities of links, through the geometric jacobian (Bottega, 2005). These relations are necessaries for the derivation of movement equation of the musculoskeletal model as a whole.

The linear and angular velocity of a point p of the terminal element are expressed, like free vector in function of velocity of the joints $\dot{q} = \dot{\theta}$, with relations as

$$\begin{aligned} \dot{p} &= J_p(q)\dot{q}, \\ w &= J_0(q)\dot{q} \end{aligned} \tag{5}$$

which can be written in the following form

$$v = \begin{bmatrix} J_p \\ J_0 \end{bmatrix} \dot{q} = J(q)\dot{q}, \tag{6}$$

where the transformation matrix $J_{6\times n}$ is called geometric jacobian. The Equation (4) can be written in vectors

$$J = \begin{bmatrix} J_{p_1} & \cdots & J_{p_n} \\ J_{O_1} & \cdots & J_{O_n} \end{bmatrix}, \tag{7}$$

where $J_{pi}(q_i)\dot{q}_i$ represents the contribution of joint i to the linear velocity of the terminal link, while $J_{0i}(q_i)\dot{q}_i$ represents the contribution of this joint to the angular velocity of the terminal link.

2.3. Lagrange's Formulation

In order to obtain a set of differential equations of motion to adequately describe the dynamics of the musculoskeletal system, the Lagrange's approach can be used (Cannon 1982). A system with n generalized coordinates $q_i = \theta_i$ must satisfy n differential equations of the form

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \xi_{f_i}, \qquad i=1,...,n, \tag{8}$$

where ξ_{f_i} are the generalized forces with respect to the generalized coordinates q_i. L is the so called Lagrangian which is given by

$$L = \mathrm{T} - U, \tag{9}$$

where T represents the kinetic energy of the system and U the potential energy.

The Equation 8 define the relations among the generalized forces applied on the system and the joints velocity and acceleration.

2.4. Kinetic Energy

The kinetic energy of link i can be expressed as

$$T = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}(q)\dot{q}_i\dot{q}_j = \frac{1}{2}\dot{q}^T B(q)\dot{q} \tag{10}$$

where

$$B(q) = \sum_{i=1}^{n}\left(m_{l_i} J_p^{l_i T} J_p^{l_i} + J_0^{(l_i)T} I_{l_i} J_0^{(l_i)}\right) \tag{11}$$

is the generalized inertia matrix.

2.5. Potential Energy

The potential energy is given by

$$U = -\sum\left(m_{l_i} g_0^T p_{l_i}\right) \tag{12}$$

where g_0 is the gravity vector expressed in the base frame.

2.6. Equations of motion

By taking Equation and Equation , into account, the Lagrangian of Eq. (6) can be written as

$$\sum_{j=1}^{n} b_{ij}(q)\ddot{q}_j + \sum_{j=1}^{n}\sum_{k=1}^{n} h_{ijk}(q)\dot{q}_k\dot{q}_j + g_i(q) = \xi_{f_i}, \text{i=1,...,n}, \tag{13}$$

where $h(q)$ represents a vector with centripetal, Coriolis and gravitational forces given by

$$h_{ijk} = \frac{\partial b_{ij}}{\partial q_k} - \frac{\partial b_{jk}}{\partial q_i}. \tag{14}$$

Finally, the equations of motion Eq. (13) for our system, which are modeled as a set of coupled rigid bodies, can be written in matrix form

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u \quad , \tag{15}$$

where $B(q)$ is the mass matrix, $g(q)$ is the gravity vector, u is the $n \times 1$ vector of applied joint torques and $C(q,\dot{q})\dot{q}$ is the Coriolis matrix.

With an appropriate coordinated system, the Jacobian of linear velocity in Eq. (7) is given by

$$J_p^{(l_1)} = \begin{bmatrix} -l_1 s_1 & 0 & 0 \\ l_1 c_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad J_p^{(l_2)} = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} & 0 \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{16}$$

$$J_p^{(l_3)} = \begin{bmatrix} -l_3 s_{123} - l_2 s_{12} - l_1 s_1 & -l_3 s_{123} - l_2 s_{12} & -l_3 s_{123} \\ l_3 c_{123} + l_2 c_{12} + l_1 c_1 & l_3 c_{123} + l_2 c_{12} & l_3 c_{123} \\ 0 & 0 & 0 \end{bmatrix} \quad , \tag{17}$$

where $c_{(12...n)}$ and $s_{(12...n)}$ indicate, respectively, cossino and sino of ($\theta_1 + \theta_2 + ... + \theta_n$).

The Jacobians of angular velocity of baricenters in Eq. (7) are

$$J_0^{(l_1)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \; J_0^{(l_2)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \; J_0^{(l_3)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}. \tag{18}$$

3. CONTROL MODEL

Although the theory of optimal control is pretty well worked out in literature, there seems to be less information about implementing optimal controller on systems with many degrees of freedom and many uncertain parameters (Kuo 1995).

The dynamic system defined by Eq. (15) can be parameterized in first order equations and written in the state-dependent coefficient (SDC) form

$$\begin{aligned} \dot{x} &= A(x)x + B(x)u \\ y &= S(x)x \end{aligned} \quad , \tag{19}$$

where $x = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{bmatrix}^T$ is a state time dependent, $\dot{x} \in \Re^6$ is the vector of the first order time derivates of the states, $u \in U \in \Re^3$ is the control vector, *U* is the control constraint set and *S(x)* is the output matrix. This system represents the constrains from the nonlinear regulator problem, together with $x(t_0) = x_0$, $x(\infty) = 0$, respectively the initial and final conditions.

The coefficient dependent matrices are given by

$$\begin{aligned} &A(x) = \begin{bmatrix} 0 & \mathrm{I} \\ -M^{-1} & -M^{-1}(C+g) \end{bmatrix}, \quad B(x) = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}, \\ &Q(x) = S^T(x)S(x), \quad S(x) = diag\left\{\sqrt{q_i}\right\}_{i=1,\dots,6} \quad , \end{aligned} \tag{20}$$

where $A \in \Re^{6x6}$ and $B \in \Re^{6x3}$. The state *x* and the control *u* is given by $f(x) = A(x)x$, $b(x) = B(x)$ and $d(x) = S(x)x$ (Mracek and Cloutier, 1998). It is assumed that *f(0)=0*, which imply that the origin is an equilibrium point.

A state feedback rather that output feedback is adopted to enhance the control performance. The non-quadratic cost function for the regulator problem is given by

$$J = \frac{1}{2}\int_{t_0}^{\infty}\left[x^T Q(x)x + u^T R(x)u\right]dt \tag{21}$$

where $Q(x)$ is semi-positive-definite matrix and $R(x)$ positive definite. There are weighting matrices on the outputs and control inputs, respectively. For a pointwise linear fashion there matrices are assumed with constant coefficients.

Assuming full state feedback, the control law is given by

$$u = -R^{-1}(x)B^T(x)P(x)x \tag{22}$$

The estate-dependent Riccati equation to obtain $P(x)$, is given by

$$A^T(x)P(x) + P(x)A(x) - P(x)B(x)R^{-1}(x)B^T(x)P(x) + Q(x) = 0\ . \tag{23}$$

It is show in Mracek and Cloutier (1998) that

1) In the neighborhood Ω about the origin the SDRE method guarantees a closed-loop solution, local asymptotic stability.
2) In the scalar case, the SDRE method reaches the optimal solution of the feedback regulator problem performance index (20), even when Q and R are functions of x.
3) In general multivariable case, the SDRE nonlinear feedback controller satisfy the first necessary condition for optimality, $\mathbf{H}_{\mathbf{u}} = \mathbf{0}$ (**H** is the Hamiltonian from the problem (19)-(21), while the second necessary condition for optimality, $\dot{\boldsymbol{\lambda}} = -\mathbf{H}_{\mathbf{x}}$, is asymptotically satisfied at a quadratic rate as **x** goes to zero.
4) The system (19) is pointwise controllable and observable, for a region in neighborhood Ω about the origin. For controllability this mean $\left[B \vdots A^n B\right]_{n=1,\dots,5}$ from the static problem $\dot{x} = Ax + Bu$, in this neighborhood. SDRE method considers a solution for this static pointwise problem, for small time interval.

The SDRE technique to obtain a suboptimal solution for this problem has the following procedure (Mracek and Cloutier, 1998).

Step 1. Define the space-state model of the manipulator with the state-dependent coefficient form as in eq. (20).
Step 2. Measure the state of the system $x(t)$, i.e. define $x(0)=x_0$, and choose the coefficients of weight matrices Q and R.
Step 3. Solve the Riccati equation (23) for the state $\mathbf{x}(t)$, considering pointwise static solutions, i.e. solve $A^T P + PA - PBR^{-1}B^T P + Q = 0$ for each step.
Step 4. Calculate the input signal from eq. (22)
Step 5. Integer the system eq.(19) and update the state of the system $x(t)$ with this results. Go to step 3.

4. DYNAMIC MODEL

For the purpose of design, simulation, and control the dynamic equations of the musculoskeletal system can be represented in the state-space form. A state vector is defined as $z = [z, \dot{z}]$ defined as the difference between the regulated θ_i output and the value of the set-point θ_{id}.

$$z = \begin{bmatrix} \theta_1 - \theta_{1d} \\ \theta_2 - \theta_{2d} \\ \theta_3 - \theta_{3d} \end{bmatrix}, \qquad \dot{z} = \begin{bmatrix} \dot{\theta}_1 - \dot{\theta}_{1d} \\ \dot{\theta}_2 - \dot{\theta}_{2d} \\ \dot{\theta}_3 - \dot{\theta}_{3d} \end{bmatrix} \tag{26}$$

Therefore, the model in the Eq. (15) can be written as

$$\begin{aligned} \dot{Z} &= A(t)Z + B_e(t)u, \\ y &= C_e(t)Z, \\ Z(0) &= Z_0, \\ Z(\infty) &= 0. \end{aligned} \tag{27}$$

After inverting matrix B in Equation 15 and performing some algebraic manipulations. The state-dependent system matrices are

$$A = \begin{bmatrix} 0 & I \\ -B^{-1}K_e & -B^{-1}[C+g] \end{bmatrix}, \; B_e = \begin{bmatrix} 0 \\ B^{-1} \end{bmatrix}, \; C_e = diag(\sqrt{q_i}). \tag{28}$$

Solve the state-dependent Riccati equation, using the LQR function in MATLAB, to obtain *P(t)* and construct the non-linear feedback controller

$$\mathrm{u} = -\mathrm{R}^{-1}\mathrm{B}^{\mathrm{T}}\mathrm{P(t)Z}\,, \tag{29}$$

minimizing the functional

$$J_z = \frac{1}{2}\int_0^\infty Z^T QZ + u^T Rudt \tag{30}$$

where the matrices Q and R are chosen as $Q = diag(50, ..., 50)$ and $R = diag(1, ..., 1)$.

5. RESULTS

We consider a simplified model presented in Figure 1 that uses torque as variable of input control, where the mathematical model is given by Eq. (25). Let m_1, m_2 and m_3 be the masses, l_1, l_2 and l_3 the baricenter length, I_1, I_2 and I_3 are the inertial moment respectively and g is the gravity acceleration..

This model uses the following anthropometric and geometric parameters of musculoskeletal system (Menegaldo, 2003),

$l_1 = 0.24$, $l_2 = 0.27$, $l_3 = 0.1$,
$a_1 = 0.4$, $a_2 = 0.4$ $a_3 = 0.2$,
$m_1 = 14.0$, $m_2 = 6.7$, $m_3 = 0.5$,
$I_1 = 0.2640$, $I_2 = 0.1295$, $I_3 = 0.063$,
$g = 9.8$.

The movement of musculoskeletal system was simulated on PC, using MatLab/Simulink with time period $\Delta t = 1\ ms$, numerical method for differential equation solution by Runge Kutta, with 1 second period. In this case, we obtained the following results.

In order to check the performance of the controllers presented, using a path that represents an initial condition of the human body crouching down to erect condition to simulate a control of human posture (Pandy, 2001), where the initial conditions represent angles of approximately 57 degrees to the hip joint θ_3, approximately -57 degrees to the knee joint θ_2 and about 40 degrees to the ankle joint θ_1. In Figure 3 is shown by the trajectory angle of the joints, there is a convergence to zero of the angle of joints, representing the upright position of the model.

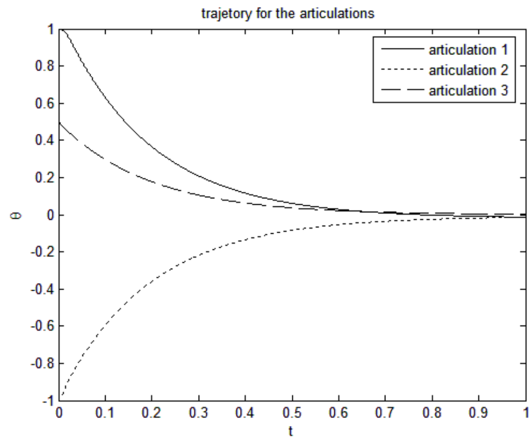

Figure 3. Tracking trajectory for angle of articulations to musculoskeletal system.

In Figure 4 there is a convergence to zero of the speed trajectories. The torques applied to joints, limited by the gain matrix of control, are shown in Figure 5.

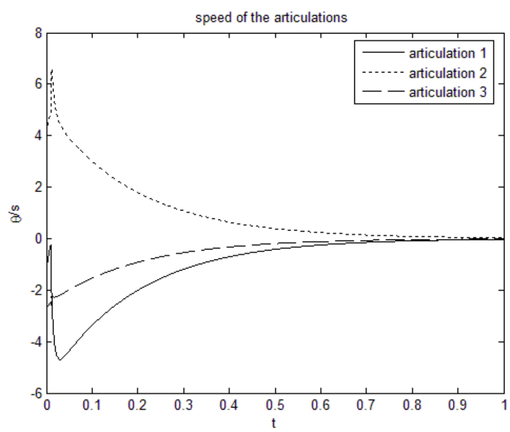

Figure 4. Tracking trajectory for the speed angle of articulations to musculoskeletal system.

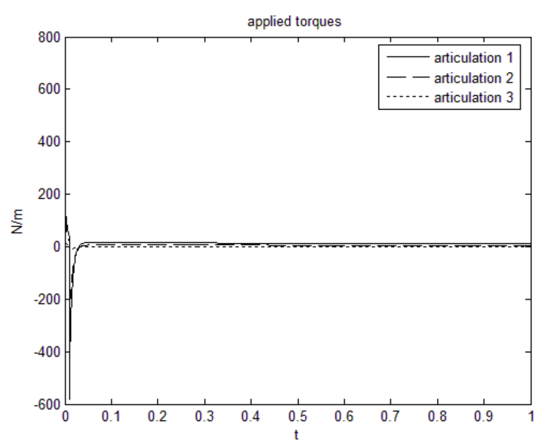

Figure 5. The torques applied to joints.

This simulation shows the good performance of the adaptive control system presented in both the stationary and the transient state.

6. ACKNOWLEDGEMENTS

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7. CONCLUSIONS

The biomechanic model of a human musculoskeletal system and the simulation of his behavior in movement can be applied in several areas, such as sports, engineering and medicine. The objective of this work was obtain a dynamic model that represents the musculoskeletal system of a leg. The description of kinematic and dynamic links movements was based on Newton-Euler and Euler-Lagrange formulation. In this paper, it was used a control based on optimal theory control. A geometric model for simulations was obtained with Matlab/Simulink software. We presented the desired and the tracking trajectories for angle of articulations to musculoskeletal system. The trajectory error remained next to zero. The simulation showed the good performance of adaptative control system presented here.

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