

CONTROL OF A PARALLEL PROPELLER SYSTEM

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Abstract. *The application of multivariable control techniques with the aid of computing tools has been largely used in projects of complex control systems that are based on the concept of state-variables. The objective of this paper is to show the development of a tracking-system with state-feedback for a prototype of a parallel propeller system presenting nonlinear dynamics with three degrees-of-freedom but having only two controlled. This system is basically composed by a beam articulated at its center with two spindles fixed at one end and a balance weight on the other side. Each of these spindles is driven by a DC motor with a propeller attached to it used to generate the thrust necessary to accomplish the pitch and yaw motions. The weight can be slid along the beam in order to keep the system unbalanced in the propeller thrust range. This maintains the system working within specified displacement and velocity ranges, helping to maintain the system flying. This kind of system does not need a tail rotor. Also, the system has three encoders that supply the real time position and motion speed. Two mathematical models are used: a linear model and a nonlinear model. The linear representation is used to design the gain matrices of the linear control system. The nonlinear model is obtained through the virtual prototyping environment system ADAMS[®], using modeling techniques of multi-body systems to get the dynamic motion equations. Then, the nonlinear model is exported to Simulink[®] (Matlab[®]) in order to be simulated. The simulation is used to verify if the applied linear control techniques are able to control the nonlinear model and to analyze the system behavior (control system plus nonlinear plant). The design of the control system is done using the entire eigenstructure assignment technique. The results show the performance and the efficiency of the proposed methodology in terms of displacement and angular velocity responses.*

Keywords: *Virtual prototyping, tracking systems, dynamic simulation, parallel propeller system*

1. INTRODUCTION

The development of didactic systems that make possible the study of control techniques can result in great benefits for the presentation of current themes in an introductory control course (Campo, 2007).

The modern tendency of the engineering systems is to increase the complexity mainly due to the need of accomplishing high precision tasks. In order to support crescent and rigorous exigency of control systems performance, increasing systems complexity and the easy access to computers, the theory of modern control, which is an algebraic approach for the analysis and project of complicate control systems, is based on the state space concept.

So, the main purpose of this paper is to present a tracking system with states feedback for a prototype of a parallel propeller system where the dynamics is nonlinear and has three degrees of freedom with two of them controlled.

2. SYSTEM DESCRIPTION

The tracking system strategy developed in this paper addresses a nonlinear plant with two parallel propellers, controlling two degrees of freedom.

This system is basically composed by a beam articulated at its center with two spindles fixed at one end and a balance weight on the other side as shown in Figure 2. Each of these spindles is driven by a DC motor with a propeller attached to it used to generate the thrust necessary to accomplish the pitch and yaw motions. Figure 1 shows the magnitude of the thrust force obtained against the motor input voltage given in terms of pulse width modulation (PWM) duty cycle. The weight can be slid along the beam in order to keep the system unbalanced in the propeller thrust range and maintaining it working within specified displacement and velocity ranges and flying. This kind of system does not need a tail rotor.

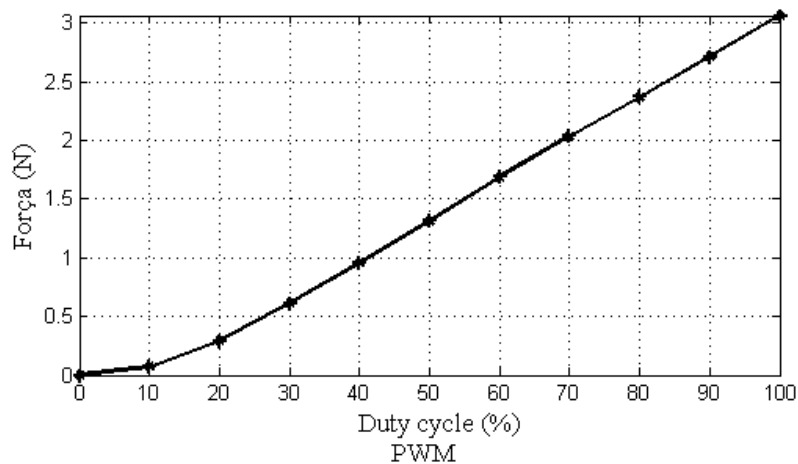


Figure 1. Experimental curve of the relation between duty cycle and thrust generated by the propeller

The mathematical model is obtained using the virtual prototyping environment system *ADAMS*®, shown in Fig. 2, that uses modeling techniques of multi-body systems to obtain the dynamic motion equations. This software has a graphic interface to aid the creation of the mathematical model and for the simulation results visualization where the wanted parameters are supplied in a simple and fast way. Soon afterwards the model is exported to the simulations environment *MatLab*®.

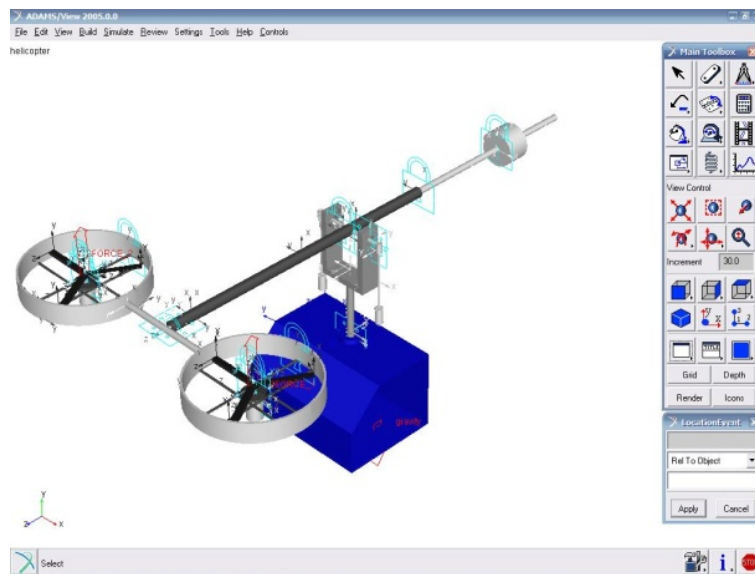


Figure 2. Parallel propeller system modeled in ADAMS

2.1 Control System

A controllable open-loop system is represented by the n th-order state and p th-order output equations of the form

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx = \begin{bmatrix} E \\ F \end{bmatrix} x \tag{2}$$

where y is an $p \times 1$ vector and $w = Ex$ is a $m \times 1$ vector representing the outputs which are required to follow a $m \times 1$ input vector r .

The feedback controller is required to cause the output vector w to track the command input r in the sense that the steady-state response

$$\lim_{t \rightarrow \infty} w(t) = r(t) \tag{3}$$

when r consists of piecewise-constant command input. The design method consists of the addition of a vector comparator and an integrator which satisfies the equation

$$\dot{z} = r - w = r - Ex \tag{4}$$

The composed open-loop system is therefore governed by the augmented state and output equations formed from Eqs. (1) to (4)

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -E & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ I \end{bmatrix} r = \bar{A}x' + \bar{B}u + \bar{B}'r \tag{5}$$

$$y = [C \ 0] \begin{bmatrix} x \\ z \end{bmatrix} = \bar{C}x' \tag{6}$$

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ -E & 0 \end{bmatrix}; \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}; \bar{B}' = \begin{bmatrix} 0 \\ I \end{bmatrix}; \bar{C} = [C \ 0]. \tag{7}$$

D'Azzo and Houpis, (1995) show the state-feedback control law to be used here which is

$$u = K_1x + K_2z = [K_1 \ K_2] \begin{bmatrix} x \\ z \end{bmatrix} \tag{8}$$

$$\bar{K} = [K_1 \ K_2] \tag{9}$$

A block diagram representing the feedback control system, consisting of the plant state and output equations given by equations (1) and (2) and the control law given by equations (8), is shown in Fig. 3. This control law assigns the desired closed-loop eigenvalue spectrum if and only if the augmented plant and control matrix pair (\bar{A}, \bar{B}) is controllable. It has been shown that this conditions is satisfied if (A, B) is a controllable pair and

$$\text{Rank} \begin{bmatrix} B & A \\ 0 & -E \end{bmatrix} = n + m \tag{10}$$

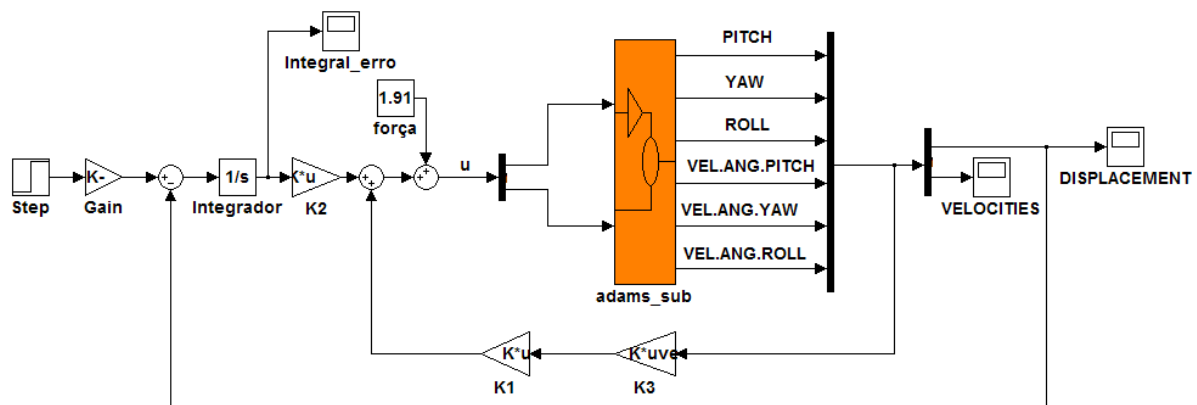


Figure 3 Tracking System for the nonlinear plant

This condition is much easier to use than the determination of the rank of the controllability matrix for (\bar{A}, \bar{B}) . Equation (10) can be satisfied only if the number of outputs p which are required to track the input r is less than or equal to the number of control inputs m . Satisfaction of the condition of Eq. (10) guarantees that a control law of the form of Eq. (8) can be synthesized such that the closed-loop output tracks the command input. In that case the closed-loop state equation is

$$\dot{x}' = \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A + BK_1 & BK_2 \\ -E & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} r = A'_{cl}x' + B'r \quad (11)$$

The feedback matrix of Eq. (9) can be selected so that the eigenvalues are in the left-half plane for the closed-loop plant matrix of Eq. (11).

The purpose in applying state feedback is to assign a closed-loop eigenvalue spectrum

$$\sigma(\bar{A} + \bar{B}\bar{K}) = \{\lambda_1, \lambda_2, \dots, \lambda_{n+m}\} \quad (12)$$

and an associated set of eigenvectors

$$v(\bar{A} + \bar{B}\bar{K}) = \{v_1, v_2, \dots, v_{n+m}\} \quad (13)$$

which are selected in order to achieve the desired time-response characteristics. The closed-loop eigenvalues and eigenvectors are related by the equation

$$(\bar{A} + \bar{B}\bar{K})v_i = \lambda_i v_i \quad (14)$$

This equation can be put in the form

$$[\bar{A} - \lambda_i I \ \bar{B}] \begin{bmatrix} v_i \\ g_i \end{bmatrix} = 0 \quad \text{for } i = 1, 2, \dots, n + m \quad (15)$$

where v_i is an eigenvector and

$$g_i = \bar{K}v_i \quad (16)$$

In order to satisfy Eq. (15), the vector $[v_i^T \ g_i^T]$ must lie in the kernel or null space of the matrix

$$\bar{S}(\lambda_i) = [\bar{A} - \lambda_i I \ \bar{B}] \quad \text{for } i = 1, 2, \dots, n + m \quad (17)$$

The notation $\ker \bar{S}(\lambda_i)$ is used to define the null space which contains of all vectors $[v_i^T \ g_i^T]$ for which Eq. (15) is satisfied. Equation (16) can be used to form the matrix equality

$$[g_1 \ g_2 \ \dots \ g_{n+m}] = [\bar{K}v_1 \ \bar{K}v_2 \ \dots \ \bar{K}v_{n+m}] \quad (18)$$

Since \bar{K} can be factored from the right-hand matrix of Eq. (18)

$$\bar{K} = [g_1 \ g_2 \ \dots \ g_{n+m}][v_1 \ v_2 \ \dots \ v_{n+m}]^{-1} = QV^{-1} \quad (19)$$

Therefore, if the desired eigenvalue spectrum of Eq. (12) is specified and the associated eigenvectors are selected to satisfy Eq. (15), then Eq. (19) specifies the required states-feedback matrix \bar{K} .

The $\ker \bar{S}(\lambda_i)$ imposes constraints on the eigenvector v_i that may be associated with the assigned eigenvalue λ_i . The $\ker \bar{S}(\lambda_i)$ identifies a specific subspace, and the selected eigenvectors v_i must be located within this subspace. In addition, the selected eigenvectors must be linearly independent so that the inverse matrix V^{-1} in Eq. (19) exists.

3. RESULTS

The linearized model in terms of state-variables, according to Fig. 4, is used to accomplish the control around an operation position. It is represented by four matrices: A , B , C and D . The matrix A has dimensions $n \times n$ where n is the number of states which is twice the number of degrees of freedom of the dynamic system. Since the parallel propeller system has three degrees of freedom, n is equal to six. The matrix B has dimensions $n \times m$ where m is equal to two, the number of inputs of the system. The matrix C is $p \times n$ where p is the number of outputs of the system and the matrix D is $p \times m$ which has all their elements null in the case. Also the linear model is used to calculate the gains of the control system.

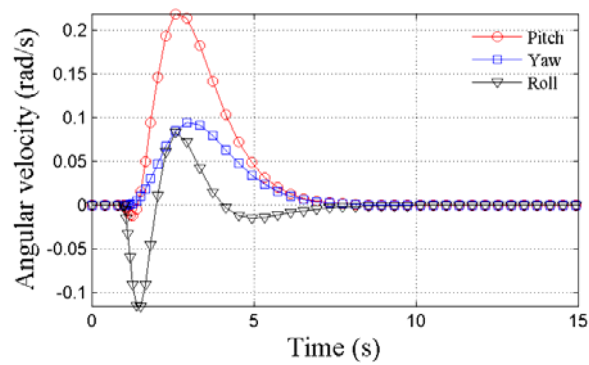


Figure 6 Angular velocities in the linear plant

The nonlinear model is used to simulate the behavior of the real parallel propeller system, and to test the efficiency of the control, that is, to verify if the control obtained for the linear model is able to control the nonlinear model in an appropriate way about the linearization point. This can be observed in Figure 7 and 8.

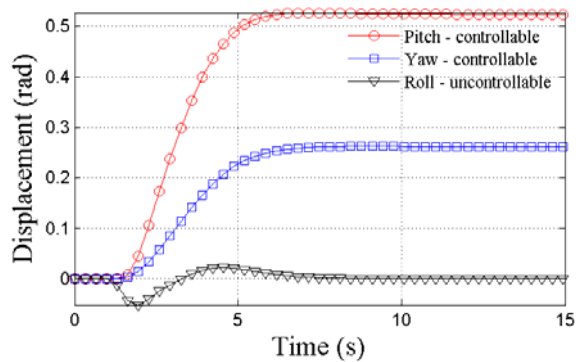


Figure 7 Displacements obtained in a nonlinear plant

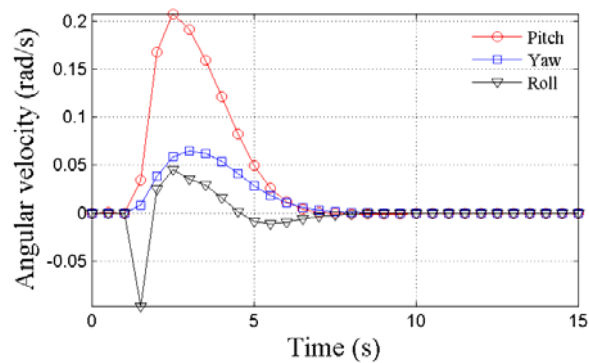


Figure 8 Angular velocities obtained in a nonlinear plant

A white noise was added to the control action, simulating an external disturbance, as a way to verify if the tracking system would still be able to track the command inputs. Figure 9 and 10 show the results of this case.

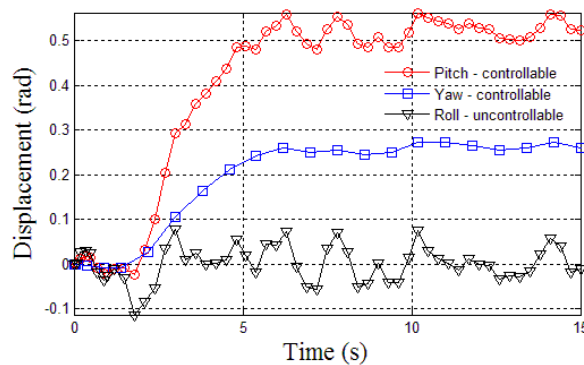


Figure 9 Displacements obtained in a nonlinear plant with noise

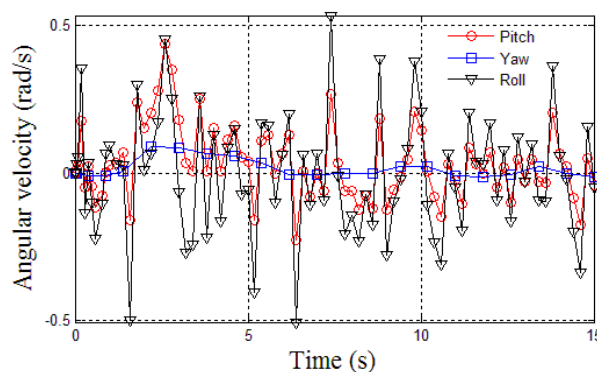


Figure 10 Angular Velocities obtained in a nonlinear plant with white noise

The pitch and yaw angles are used as command input r that should be tracked by the output w which is the corresponding controllable part of the control system outputs.

One can observe in the displacement graph that the controller has adequate action, because the output converged to the wanted values, requested by the command input.

4. CONCLUSIONS

The study presented in this paper demonstrates the possibility of using tracking systems in a multivariable plant, and the importance of the use of the dynamic modeling package ADAMS®, to give flexibility and speed in obtaining the dynamic model. The tracking system used to control the parallel propeller system is efficient, since the feedback controller forced the output to track the command input in terms of steady-state response.

In future works, this control law will be implemented in an experimental nonlinear plant, so that one can compare the experimental results with the simulated ones presented in this paper and to validate the control system.

5. ACKNOWLEDGEMENTS

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