Effects of flow transpiration on pressure losses in duct flow

Mario G. D. Cruz
Fernanda B. C. C. Sousa
Atila P. Silva Freire
Mechanical Engineering Program (COPPE/UFRJ), C.P. 68503, 21945-970, Rio de Janeiro, Brazil

Daniel A. Rodrigues
Juliana B. R. Loureiro
Scientific Division, Inmetro, 22.050-050, Rio de Janeiro, Brazil

Abstract. *This work introduces a resistance law for transpired flows in smooth and rough pipes at large Reynolds numbers. The law is derived directly from the first principles through perturbation techniques and a simple turbulence modelling. In addition to the classical logarithmic term, a bilogarithmic dependence of the friction coefficient on the transpiration rate, the effective roughness and the local Reynolds number is disclosed. The results are compared with the experiments of other authors.*

Keywords: *horizontal wells, resistance law, transpiration, roughness.*

1. INTRODUCTION

Horizontal drilling technology in oil exploration, development, and production operations has become a standard procedure over the last 20 years. The great advances in technology have meant that horizontal wells can be found with more than 2000 meters. The objective of horizontal drilling is to expose very large areas of reservoir rock to the wellbore surface. To fill the space between the screen casing and the bore hole, a high conductivity hydraulic sand or gravel packed "wall" is normally installed. Upon the action of the reservoir large pressures, fluid is then transpired from the rock reservoir into the wellbore. Over short lengths, the effects of transpiration on the flow properties including the pressure drop can be neglected. However, this is not the case for the existing very long horizontal wells.

The purpose of the present work is to show how some previously derived results for transpired turbulent boundary layers can be used to develop a resistance law for smooth and rough pipes at large Reynolds numbers. In previous studies of the problem, the application of asymptotic techniques to the Reynolds averaged Navier-Stokes equations (RANS) has allowed analytical solutions to be determined. In fact, an application of the matched asymptotic expansions method to RANS together with the mixing length concept results in local equations of motion which upon two successive integrations give a bi-logarithmic law for the mean velocity profile. Such asymptotic bi-logarithmic law naturally incorporates the effects of local Reynolds number and transpiration rate.

Here, we show how the wall roughness effects can be further incorporated in the existing theories so that a universal resistance law for transpired flow in rough pipes at very large Reynolds numbers can be proposed. This approach is quite different from other authors (see, e.g., the review paper of Clemo (2006)), who instead have preferred to describe frictional losses in perforated pipes through a decomposition of effects: wall friction, perforation friction and mixing effects. In most cases, perforation friction is associated with an increase in roughness. The mixing effects, on the other hand, are compared to the problem of multiple interacting jets in a cross flow. This solution strategy throws authors into such an complex analysis so that only external empirical evidence can be used to determine the correct behaviour of the friction term.

The present solution is validated against the experimental data of Olson and Eckert (1996) and Su and Gudmunsson (1993, 1998).

2. SHORT LITERATURE REVIEW

A major contribution to the description of transpired turbulent flows in pipes was given by Olson and Eckert (1966). Interested in studying the beneficial effects of flow transpiration on the thermal protection of walls, these authors analyzed the turbulent flow of air in a porous tube with circular cross section. The tube had a 50 percent porosity and was wrapped with several layers of rayon cloth to ensure a uniform injection. The data reduction was based on a balance of the mass and momentum equations. Results were presented for the local mean velocity profiles, static pressure, mass flow rate and wall shear stress.

Regarding external flow applications, authors have approached the transpired turbulent boundary layer problem differently. Differential analyses based on the Reynolds averaged Navier-Stokes equations (RANS) are normally preferred. In fact, for external flows, results obtained through the momentum integral equation tend to be very inaccurate. Dahm and Kendall (1968), in particular, show that an uncertainty of one percent in the momentum thickness variation and in the blowing rate result in an uncertainty of 32% in the predictions of the friction coefficient. In many works, thus, the solution strategy resorts to asymptotic arguments and the mixing-lentgh hypothesis for turbulence closure. Then, by treating the small parameters independently, a local solution can be found directly from a double integration of the RANS for the mean velocity profile that involves logarithmic and bilogarithmic terms (Stevenson(1963), Simpson (1970), Silva Freire (1988)).

Su and Gudmundsson (1998) split the total pressure drop in horizontal wellbores into four distinct effects: flow acceleration, wall friction, perforation roughness and fluid mixing. Their report suggests that the pressure drop due to perforation roughness is eliminated by the inflow when the transpiration rate reaches a certain limit. Experiments were conducted in a pipe with 0.022 mm internal diameter and 0.6 m length. The perforation was provided by drilling 158 holes of 3 mm in the pipe. The resulting porosity was 0.027. The measured pressure drop due to wall friction, perforation roughness and mixing effects was obtained by subtracting the pressure drop due to flow acceleration from the total pressure drop. The ordinary wall frictional pressure drop was calculated through the Darcy-Weisbach equation (White, 1986).

Schulkes and Utvik (1998) conducted experiments in perforated pipes much in the some fashion as Su and Gudmunsson (1998). Some differences were present in pipe geometry and flow conditions. However, the basic conclusions were the same. Authors found that when the transpiration rate is small, lubrication of the pipe flow occurs. For high transpiration rates, on the other hand, a sudden jump in pressure drop was observed.

The work of Yalniz and Ozkan (2001) simulated flow injection in a horizontal well with just two discrete perforations. Thus, the dynamics of the flow is more associated to a description of the mixing effects of jets in cross flow than to the effects of a homogeneous flow transpiration at the wall. The investigation is reported to use theoretical and experimental models for the derivation of an expression for the friction factor. The so-called theoretical models, however, do not appear to the first principles. Instead they are based on simple dimensional analysis and strong empirical input. With no influx, the wall perforations are said to produce a reduction in pressures gradients proportional to their area.

Clemo (2006) performed a comparison of some existing theories for pressure loss prediction with the data of Olson and Eckert (1966), Siwon (1987) and Su and Gudmunsson (1998). The models of Siwon (1987) and Yuan et al. (1997) recognize the flow acceleration effects reported in the integral analysis advanced by Olson and Eckert (1966) and use experimental correlations to propose a wall shear stress equation. The analysis concludes that (i) the perforations provoke an increase in pressure losses irrespective of the transpiration rate and (ii) fluid transpiration provokes larger pressure losses than would occur without inflow, but not as large as would be expected considering only the momentum increase induced by increasing velocities.

3. THEORY

3.1 Resistance law for smooth pipes

In the following discussion we shall consider a fluid cylinder of length L and radial coordinate y_c.

In a fully developed turbulent flow the balance between shearing and pressure forces gives

$$\tau = \frac{p_1 - p_2}{L}\frac{y_c}{2}, \tag{1}$$

so that the friction coefficient λ can be defined through

$$\frac{p_1 - p_2}{L} = \frac{\lambda}{D}\frac{\rho}{2}\overline{U}^2, \tag{2}$$

where D denotes the pipe diameter and $\overline{U}$ the mean flow velocity.

For very large Reynolds numbers, asymptotic arguments can be used together with dimensional analysis or the mixing length theory of Prandtl to show that the velocity distribution in the near wall fully turbulent region is given by

$$\frac{u}{u_*} = \frac{1}{\varkappa}\ln\left(\frac{y u_*}{\nu}\right) + A, \tag{3}$$

where y is the wall distance, u_* is the friction velocity ($=\sqrt{\tau_w/\rho}$), $\varkappa$ (= 0.4) is von Karman's constant and A = 5.5.

Of course, the above equation is only valid in regions where laminar viscous effects are not important. However, if this is taken aside for the moment, Eq. (3) can be integrated over the cross-sectional area of a pipe to give

$$\frac{\overline{U} - U}{u_*} = -3.75 \tag{4}$$

An algebraic manipulation of the previous equations yields

$$\frac{1}{\sqrt{\lambda}} = 2.035 \log\left(R_e\sqrt{\lambda}\right) - 0.91 \tag{5}$$

with $R_e = \overline{U}D/\nu$.

Equation (5) provides a simple way to find predictions of λ for given values of R_e. Actually, a rigorous comparison of Eq. (5) experiment shows that the constant 0.91 should be replaced by 0.8, a value commonly found in literature.

3.2 Resistance law for smooth pipes with wall transpiration

For flow subject to wall transpiration, Eq. (5) is clearly not valid anymore. The result of the injection or suction of fluid into an oncoming flow is to modify the velocity distribution throughout the boundary layer so that drag is either reduced or increased. Any expression advanced with the purpose of determining the friction coefficient should therefore reflect this. Furthermore, an explicit dependence of Eq. (5) on the transpiration rate should be expected.

In Silva Freire (1988) the matched asymptotic expansions method was applied to the equations of motion to find a law of the wall in which the additive parameter A varied with transpiration. The resulting expression is

$$u_+ = \frac{1}{\varkappa}\ln\left(y^+\right) + A + \frac{\Pi}{\varkappa}W\left(\frac{y}{\delta}\right) + v_w^+\left(\frac{1}{2\varkappa}\ln\left(\frac{yu_*}{\nu}\right) + \frac{A}{2}\right)^2 + \frac{\tilde{\Pi}}{\varkappa}W\left(\frac{y}{\delta}\right) \tag{6}$$

with $u^+ = u/u_*$, $y^+ = yu_*/\nu$, $v_w^+ = v_w/u_*$, v_w = normal velocity at the wall and A is given by:

$$A = 5 - 512\frac{v_w}{U} \tag{7}$$

and parameters Π and $\tilde{\Pi}$ and function W are related to the universal function of Coles (1956).

Much in the same way as Eq. (3), Eq. (6) can be integrated over the cross-sectional area of a pipe, so that

$$\overline{U} = U - 3.75u_* - v_w(1.86A + 2.34\ln {Re^+}^2 - 5.47) \tag{8}$$

with $Re^+ = Ru_*/\nu$.

Some further algebraic manipulations with

$$\frac{\overline{U}}{u_*} = \frac{2\sqrt{2}}{\sqrt{\lambda}}, \tag{9}$$

imply that

$$1 = \frac{\sqrt{\lambda}}{2\sqrt{2}}(2,5\ln(Re^+) + A - 3,75) + v_w^+(1,56\ln^2(Re^+) + (1,25A - 4,68)\ln(Re^+) + \frac{A^2}{4} + 1,86A + 5,47) \tag{10}$$

where

$$v_w^+ = \frac{v_w}{\overline{U}} \qquad \text{and} \qquad Re^+ = \frac{\overline{U}D}{\nu}\frac{\sqrt{\lambda}}{4\sqrt{2}}. \tag{11}$$

The transcedental equation, Eq. (10), gives λ for given Re^+ and v_w^+.

3.3 Resistance law for rough pipes

The pipes that are used in industrial applications are not smooth. The effect of roughness of high Reynolds number flows is to increase significantly the loses in pressure. Hence, both flow resistance equations previously presented must be modified.

One fundamental difficulty in dealing with rough surfaces lies on the description of its geometric form. The frequently large number of parameters that are necessary to characterize a rough wall seems to conspire against any rational attempt at developing simple, workable rules. However, some early experiment has made it clear that basically two types of roughness must be considered: (i) those corresponding to relatively coarse and tightly spaced roughness elements and (ii) those corresponding to protrusions that are more gentle and distributed over a larger area. In the first case, the resistance coefficient is independent of the Reynolds number R_e and can be expressed through a single roughness parameter k_s/R, where k_s is a characteristic length of the roughness and R the radius of the cross section. In the second case, both a R_e and k_s/R dependence is apparent.

The first systematic study of flow in rough pipes was carried out by Nikuradse (1933) who used circular pipes with maximum density sand glued to the wall. The changing the size of grain, Nikuradse was able to change k_s/R from 1/5000 to 1/15. As it turned out, three different flow regimes were identified (Schlichting, 1979, pg 617): (i) a regime said hydraulically smooth in which "the size of the roughness is so small that all protrusions are contained within the laminar sub-layer" ($0 \leq k_s^+ \leq 5$, $k_s^+ = (k_s u_*)/\nu$, k_s = sand grain roughness), (ii) a transitional regime where "protrusions extend partly outside the laminar sub-layer and the additional resistance, as compared with a smooth pipe, is mainly due to the form drag experienced by the protrusions in the boundary layer" ($5 \leq k_s^+ \leq 70$); and (iii) a fully rough regime where "all protrusions reach outside the laminar sub-layer and by far the largest part of the resistance to flow is due to the form drag which acts on them" ($k_s^+ \geq 70$). As further pointed out by Schlichting (1979), in practical applications, the typical roughness density is much smaller than tightly glued sand, so that the effective length of a roughness must be correlated with Nikuradse's sand roughness.

The data of Nikuradse have also shown that the mean velocity profile for flow over rough walls can be expressed in terms of a logarithmic law, accordingly with

$$\frac{u}{u_*} = \frac{1}{\varkappa} \ln\left(\frac{y}{k_s}\right) + B, \tag{12}$$

where B = 8.5 (completely rough regime).

In fact, B is a function of Re_k (= $k_s u_*/\nu$). The behaviour of B for the three types of flow regime discussed previously has been studied by several authors. For example, Ligrani e Moffat (1986) suggest the following functional dependence

$$B = 8.5\sigma + \frac{1-\sigma}{\varkappa} \ln\left(Re_k\right) + (1-\sigma)C, \tag{13}$$

where $Re_k = k_s u_*/\nu$, C = 5.1 and $\sigma = \sin((1/2)\pi g)$ with

$$g = \frac{\ln\left(\frac{Re_k}{Re_{k,s}}\right)}{\ln\left(\frac{Re_{k,r}}{Re_{k,s}}\right)}, \tag{14}$$

$Re_{k,s}$ = 5, $Re_{k,r}$ = 70 and this approximation is valid in $5 \leq Re_k \leq 70$.

The resistance formula for flow in a rough pipe can be obtained by integrating Eq. (12) over the cross-sectional area of a pipe. The result is

$$\lambda = [0.88 \ln(R/k_s) + 0.35B - 1.33]^{-2} \tag{15}$$

A comparison of Eq. (15) with the experiment of Nikuradse shows that for a fully rough regime the ln additive term should be replaced by 1.74.

3.4 Resistance law for rough pipes with wall transpiration

A law of resistance for rough pipes with wall transpiration can now be deduced provided the results of the previous sections are taken into account. Let us define $Re^+ = R/k_s$ and $A_k = B$ - 512 v_w/U. It follows immediately from Eqs. (10) and (15) that

$$\begin{aligned} 1 = {} & \frac{\sqrt{\lambda}}{2\sqrt{2}}(2.5\ln(R/k_s) + A_k - 3.75) + v_w^+(1.56\ln^2(R/k_s) \\ & + (1.25A_k - 4.68)\ln(R/k_s) + \frac{A_k^2}{4} + 1.86A_k + 5.47). \end{aligned} \tag{16}$$

4. Experimental validation

The predictions of Eq. (16) are validated against the data of Olson and Eckert (1966) and of Su and Gudmunsson (1998). The flow conditions of both works are quite different. In particular, Olson and Eckert (1966) worked with a high porosity wall that was “made by winding two layers of alternated 0.010-in. solid and stranded (7-strand, 0.050-in. wire per strand) stainless-steel wire on a cylindrical mandrel, wrapping a stainless-steel perforated sheet 0.010 in. thick with 50 percent porosity around these layers, then wrapping two similar layers of alternate solid-stranded wires over the perforated sheet”. Su and Gudmunsson (1998), on the other hand, worked with a low porosity pipe that was made by “drilling perforation holes through the pipe wall that were geometrically similar to a 7-inch perforated casing with 0.83 inch perforation diameter, 12 SPF perforation density, and 60^o phasing. The perforations were covered with a 25 μm pore size filter on top of a water resistant glue pad.”

From the above description, it is clear that the homogeneous wall transpiration condition preconceived by Eq. (16) is much more likely to be satisfied by the data of Olson and Eckert (1966). The very low porosity of 2.7 percent of Su and Gudmunsson (1998), in principle, does not characterize a uniform wall condition. In view of the present availability of experimental data and the previous comparison of models and experiments performed by Clemo (2006), however, we have decided to assess the data of Su and Gudmunsson (1998) against Eq. (16).

Figure 1 compares Eq. (16) with the data of Olson and Eckert (1966) for several transpiration rates and measuring positions. The measurements of Olson and Eckert (1966) show a decrease in friction coefficient with an increase in the transpiration rate as expected. However, their data also show an increase in λ with an increase in the local Reynolds number. This has not been observed in the classical experiments of Nikuradse, for example. The predictions of Eq. (16), however, are coherent with the expected trends.

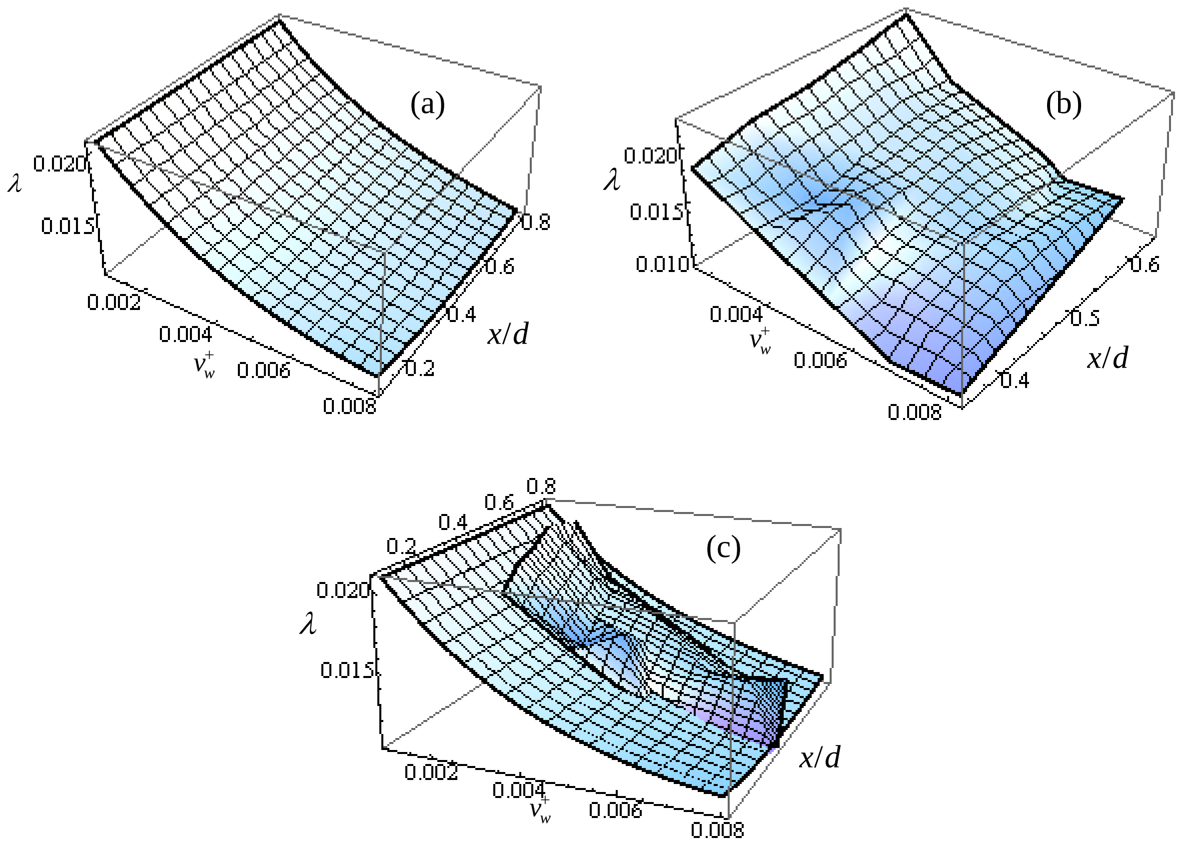

Figure 1. Friction coefficient for transpired flow over rough walls. a: present theory, b: data of Olson and Eckert (1966), c: comparison of (a) and (b).

The pressure drop due to frictional losses for the experimental conditions of Su and Gudmunsson (1998) are shown in Table 1. Note that results are presented for three distinct total flow rate ratio and Reynolds number. To all considered R_e, the theoretical predictions are very good for the low σ = 0.02 and 0.05. In fact, in these ranges, the maximum prediction error is about 5%. For the highest σ (= 0.1), The error increases to 18%. Thus, the overall agreement of the present theory with the data of Su and Gudmunsson (1998) is surprisingly good. We have previously commented that the wall boundary condition for their experimental arrangement is not likely to approach a uniform condition

Table 1. Predictions given by Eq. (16) as compared with the data of Su e Gudmunsson (1998). σ = total flow rate ratio. Δp (Pa) = frictional pressure loss.

R_e	σ	Δp *(Pa) Experiment*	Δp *(Pa) Theory*
40,000	0.02	1000	958
40,000	0.05	950	890
40,000	0.1	900	788
65,000	0.02	2450	2323
65,000	0.05	2350	2131
65,000	0.01	2250	1845
90,000	0.02	4900	4653
90,000	0.05	4700	4250
90,000	0.1	4500	3638

5. CONCLUSION

The present work has proposed an expression for the calculation of the pressure drop in transpired flows over smooth and rough surfaces. The results are compared with the data of other authors showing good agreement. The present theory is particularly interesting because it has been derived directly form the first principles through perturbation techniques and some very simple turbulence modelling. The necessary additional modelling of an integration constant was obtained from some external boundary layer data. No particular model fitting was made to the experiments to which the present theory was compared.

A new experiment is currently under way so that further assessment of Eq. (16) can be made.

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