

## AN APPROACH BASED ON NEURAL NETWORKS TO DETERMINE PARAMETERS OF FREE FORM SURFACES

José Carlos de Lima Júnior, [limajrcarlos@hotmail.com](mailto:limajrcarlos@hotmail.com)

João Bosco de Aquino Silva, [jbdeaquino@uol.com.br](mailto:jbdeaquino@uol.com.br)

Department of Mechanical Engineering, Federal University of Paraiba

**Abstract.** The Coordinate Measuring Machine (CMM) is recognized as a powerful tool and is frequently used in the inspection of industrialized products. Inspection results of a part may be affected by various error sources, such as: geometric errors, thermal variations, probing system, and the CMM's software. Currently, CMM software is based on algorithms which use the least squares method or the minimum zone method in the determination of substitute geometries. This article presents a new methodology based on artificial neural networks to determine substitute geometry parameters in measurements by CMMs, such as: involute profile of spur gear as well as free form surface..

**Keywords:** Artificial Neural Networks, Substitute Geometries, Coordinate Measuring Machine.

### 1. INTRODUCTION

The concept of design, manufacture and inspection of the modern industrialized products have been changing with the advent of computer numeric control machine tools and technologies such as computer aided design (CAD) and computer aided manufacturing (CAM). The dimensional control of manufactured parts is indispensable for the guarantee of the fulfillment of tolerances specified in the design (Silva and Burdekim, 2002). Thus, an inspection system capable of attending to current manufacture requirements must answer to the following requisites: velocity compatible with production speed, capacity to control complex geometries, measurement uncertainty compatible with the tolerances of the part, be flexible in order to control a great diversity of geometries, and have a high degree of automatization and informatization (Bosch, 1995). Conventional dimensional inspection techniques have not been capable of answering these requisites, required by new manufacture technologies, which require devices which will perform dimensional control at high speed and with high accuracy (Lima Jr, 2007). Coordinate measurement machines (CMMs) have been showing that they are capable of fulfilling all of these requisites. These machines operate according to the coordinate metrology principle. Based on this principle, the evaluation process of the geometric entities is carried out in an indirect way, having the measurement of Cartesian coordinates as a basis, related to a referential of points on the surface of the object to be measured, followed by the treatment of these data, executed with the goal of obtaining all the information needed to determine the desired geometric characteristics. CMMs have the following main characteristics: high-speed inspection, high accuracy and flexible results. Therefore, these measuring machines are able to measure parts with formats which are considered complex (Curran and Phelan, 2004). CMMs are frequently used to check whether the dimensions of the part conform to design specifications, taking into account the dimensional tolerances (Webera et al, 2002).

Despite having the aforementioned characteristics, the CMM is a measurement tool and, as such, may present measurement errors, which may come from various sources, as shown in Figure 1 (Weckenmann, 2001). This figure shows an error in the software which may also influence the measured results. Majority of CMM softwares incorporate algorithms based on Euclidean geometry and use the least squares or minimum zone methods to obtain substitute geometry parameters. With an intention to contribute to the application of CMMs in measurement of complex geometries, this article presents an approach based on artificial neural networks (ANNs) technique. points  $(X_i, Y_i, Z_i)$ .

In the first part of this research (Lima Jr and Silva, 2009), an approach based on ANNs it was developed to determine parameters of substitute geometries such as: circle, ellipse and sphere. Therefore, the aim of this present work is to extend the application of ANNs to determine parameters of spur gears and evaluation of complex and free form surfaces.

### 2. ARTIFICIAL NEURAL NETWORKS

The main component of an artificial neural network (ANN) is the artificial neuron. By the combination of various artificial neurons, an artificial neural network is formed. An ANN is defined as a parallel distributed processor of simple processing units which have a natural inclination to store experimental knowledge and make it available for use (Haykin, 1998). ANNs differ in the learning method used, that is, the way in which the synaptic weights learn the existing relation between the network incoming and outgoing data. Another difference is in the composition of the network topology, where the determination of the number of layers of which the network is comprised is done empirically, as well as the number of neurons in each ANN layer. The artificial neural network has the capacity to learn from its environment and improve its performance through a learning process also known as training of the network. The neural network learns about its environment through an interactive process of adjustments applied to its synaptic

weights which store, at the end of the process, the knowledge that the network has acquired from the environment in which it is operating. According to Haykin (1998), the learning process may be defined as the means by which the parameters (synaptic weights) are adjusted through a continued stimulation by the environment in which the network is operating. The kind of learning is determined by the way in which the modification of the parameters (synaptic weights) occur (Haykin, 1998).

The majority of ANN models have some training rule, where the weights of its connections are adjusted according to the presented standards. In other words, they learn through examples. The neural network architecture is typically organized in layers. The neural network goes through a training process starting from known real and numeric cases, acquiring, from then on, the necessary systematic to adequately execute the desired processing of the supplied data. Thus, the neural network is able to extract basic rules from real data, differing from programmed computation, where a set of rigid pre-set rules and algorithms are needed (Haykin, 1998)

There are, nowadays, many algorithms used to train ANNs. Back-propagation is a supervised algorithm that uses pairs (input, desired output) to, by means of learning through error correction, adjust the synaptic weights. The goal of this learning is to adjust the parameters (synaptic weights) of the network to find a relation between the input and output data supplied to the network.

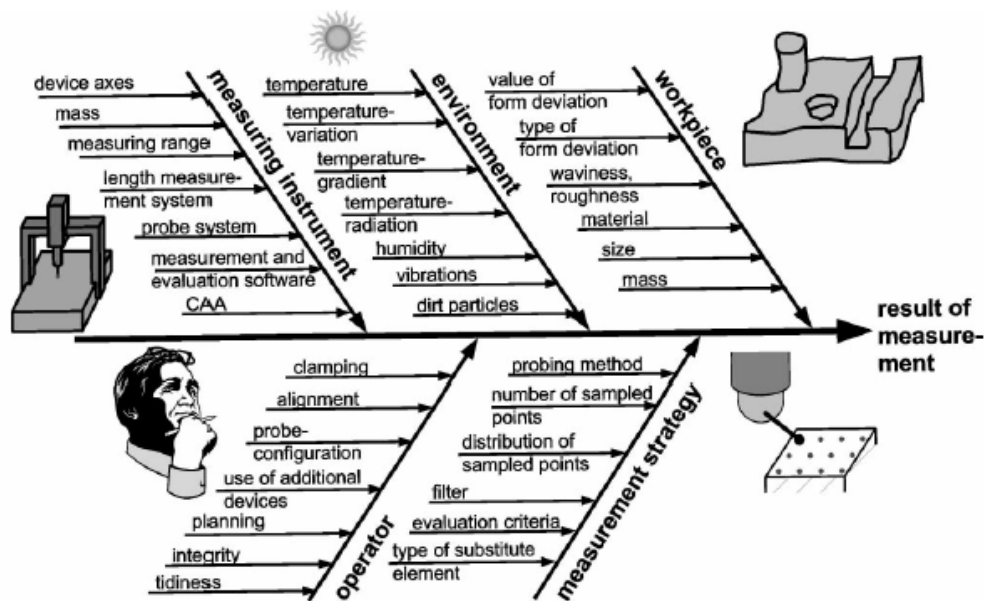


Figure 1. Some factors that influence the CMM results (Weckenmann, 2001)

### 3. DEVELOPMENT OF ARTIFICIAL NEURAL NETWORKS FOR DETERMINATION OF GEOMETRIC PARAMETERS

In this section it will be described the development of artificial neural networks (ANNs) for determination of the geometric parameters of standard geometric entities. For each of the geometric entities input and output data were generated to be used in the network training. It should be stressed that the ANNs presented here are based on the back-propagation training algorithm, and the networks topologies vary according to the entity in question.

#### 2.1. Application of artificial neural networks to analyze the involute profile of spur gear

On a practical level, a gear production system aims to produce gears with geometric parameters defined in the design. One method for checking whether the gear geometric parameters meet those defined in the design consists of measuring the gear after its manufacturing process. For example, if the measured geometric parameter is the involute profile, then the nominal or design profile is compared with the actual profile measured on the part. Coordinate measuring machine (CMM) may be used for measuring the involute profile of the gear tooth, however, this machine normally does not have, incorporated to its software, a module dedicated to measuring gears. Therefore, it is necessary to implement a specific module in the CMM to measure gears. It is worth to note that is not an easy task. In order to analyze the involute profile from coordinated points  $P_i(X_i, Y_i, 0)$ , an Artificial Neural Network(ANN) was developed to determine the differences or errors, if any, existent between the nominal and actual involute profiles after the manufacturing process of a spur gear. For this ANN, it was considered as nominal data a spur gear defined by the following geometric parameters: pressure angle,  $\phi=20^\circ$ ; module,  $m = 5$  mm and number of teeth,  $Z = 20$ . From these

parameters it was possible to obtain other gear parameters, such as diameters: external,  $d_e$ , internal,  $d_i$ , primitive,  $d_p$ , and base,  $d_b$ , (Shigley, 2003). Figure 2 shows a representation of the nominal gear with 20 teeth.

The involute profiles of the teeth shown in Figure 2 were obtained from the Eqs. (1) to (4), which depend on the evolving angle  $\beta$  and this depends on the pressure angle  $\varphi$  and radius,  $r$ , measured from the base diameter.

$$X = r(\cos \beta + \beta \text{sen} \beta) \tag{1}$$

$$Y = r(\text{sen} \beta - \cos \beta) \tag{2}$$

$$\beta = \tan(\varphi) - \varphi \tag{3}$$

$$r = \frac{r_b}{\cos(\beta)} = \frac{r_p \cos(\varphi)}{\cos(\beta)} = \frac{(Z * m / 2) \cos(\varphi)}{\cos(\beta)} \tag{4}$$

With the use of Eq. (1) to (4), it was possible to obtain nominal curve of the gear involute profile shown in Figure 2. Therefore, the generated profile has no errors. However, it is known that, in practice, a manufactured gear may not be in conformity with the dimensions specified in the design. In this work, it was developed an Artificial Neural Network (ANN) which is capable of checking whether the involute profile conforms to the design specifications.

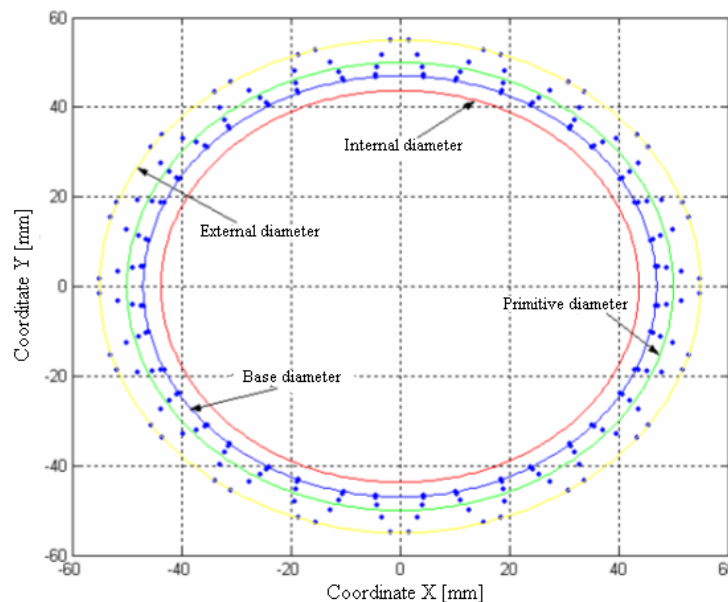


Figure 2. Gear representation for:  $\varphi = 20^\circ$ , module:  $m = 5$  mm and  $Z = 20$ .

The nominal involute profile, which is shown in Figure 2 was obtained using the Eq. (1) to (4) and by considering the following data: pressure angle,  $\varphi = 20^\circ$ ; module,  $m = 5$  mm and number of teeth,  $Z = 20$ . These data were the same used for obtaining Figure 3. Once defined the nominal involute, it is necessary to generate, by simulation, an involute profile with errors. In order to simulate these errors, a modification was made in one of the gear parameters, in this case the pressure angle  $\varphi$ . When this angle changes, the geometric shape of the tooth changes, as seen in Figure 3.

This kind of situation may happen in practice in the following situation: one desire to produce a gear with a defined pressure angle, but because of errors from the manufacturing process, the pressure angle is different of that specified in the design. Figure 3 shows this situation, where the gear defined in the design has a pressure angle equal to  $\varphi = 20^\circ$  (blue curve) and after the manufacturing process the gear has a pressure angle equal to  $\varphi = 14,5^\circ$  (red curve).

The ANN developed in this research has the aim to determine the existing difference, if any, between the nominal involute profile and the actual profile. This network has the following topology:

- 4 neurons with linear activation function at the input layer, where 2 neurons represent the coordinates  $X_{nominal}$  and  $Y_{nominal}$  of a point which is on the nominal involute profile and the other 2 neurons represent

the coordinates  $X_{actual}$  and  $Y_{actual}$  of a point which is on the actual involute profile. These coordinated points have in common the coordinate X as the aim is to evaluate the difference, if any, between the coordinates Y of the nominal and actual involute profiles. This comparison is done only for the region where the nominal and actual involute profile have the same interval of the coordinate X, as shown in Figure 3. In this figure, the blue and red curves represent the actual and nominal involute profiles, respectively.

- 20 neurons in the intermediary layer with hyperbolic tangent activation function;
- 5 neurons with linear activation function in the output layer, where 4 neurons are the same as in the input layer, described above, and the other one represents  $\Delta(Y)$ , that is the difference between the coordinate Y of the points which are on the nominal and actual involute profile, respectively.

In order to get simulation data for training the artificial neural network (ANN), different involute profiles were generated by considering the following pressure angles,  $(\varphi)$ : 14,5°; 17°; 20° and 25°. For each involute profile were collected 50 coordinate points  $P_i (X_i, Y_i)$ . It is important to note that the comparisons between the involute profiles took place by considering that the nominal involute profile had a pressure angle,  $\varphi = 20^\circ$ . Table 1 shows an example of the points used for training the ANN, which are on the nominal and actual involutes shown in Figure 3.

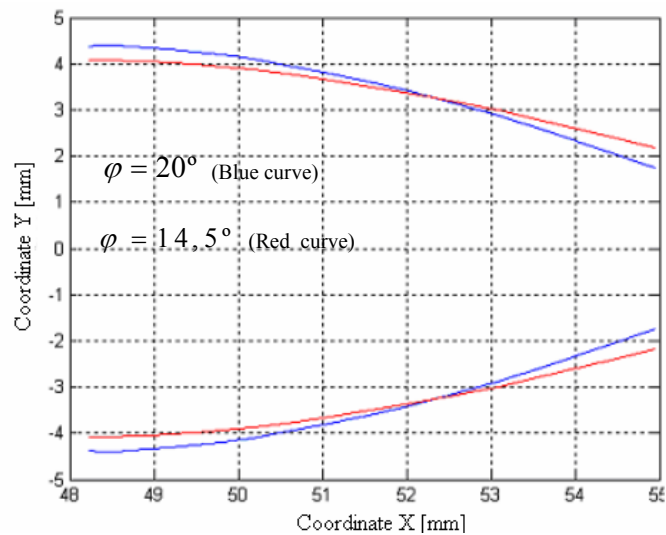


Figure 3. Nominal and actual involute profile for ANN training

Table 1. Example of points used in the ANN training

Input data	Point 1	Point 2	Point 3	Point 4
$X_{20^\circ}$	48,2363	49,8075	53,0828	54,3048
$Y_{20^\circ}$	-4,3840	-4,1920	-2,8787	-2,1637
$X_{14,5^\circ}$	48,2363	49,8075	53,0828	54,3048
$Y_{14,5^\circ}$	-4,0455	-3,9296	-2,9876	-2,4723

Output data	Point 1	Point 2	Point 3	Point 4
$X_{20^\circ}$	48,2363	49,8075	53,0828	54,3048
$Y_{20^\circ}$	-4,3840	-4,1920	-2,8787	-2,1637
$X_{14,5^\circ}$	48,2363	49,8075	53,0828	54,3048
$Y_{14,5^\circ}$	-4,0455	-3,9296	-2,9876	-2,4723
$\Delta(Y)$	-0,3385	-0,2624	0,1089	0,3086

where:  $\Delta(Y) = Y_{nominal} - Y_{actual} = Y_{20^\circ} - Y_{14,5^\circ}$

The table data are presented in mm.

After determining input and output data the ANN was trained by using the backpropagation algorithm. Figure 4 shows that the ANN error was found to be, after 250 iterations,  $2,14 \cdot 10^{-13}$ . This error is the difference between the expected and actual output of the ANN. Therefore, it can be concluded that the network was successfully trained.

In order to verify the ANN performance various simulations were performed after its training process. In one of the simulations, it was considered that one of the gears should have its pressure angle defined as  $\varphi = 20^\circ$ , however, in practice, as consequence of manufacturing process errors that pressure angle was equal to  $\varphi = 17^\circ$ . It should be stressed that in each of the simulations were used points that were not in the set of points used for training the neural network. Table 2 shows points used in this simulation, where the difference between the involute profiles, in analysis, is known beforehand. These points were collected as shown in Figure 5. Table 3 shows the results obtained with the ANN for this set of points.

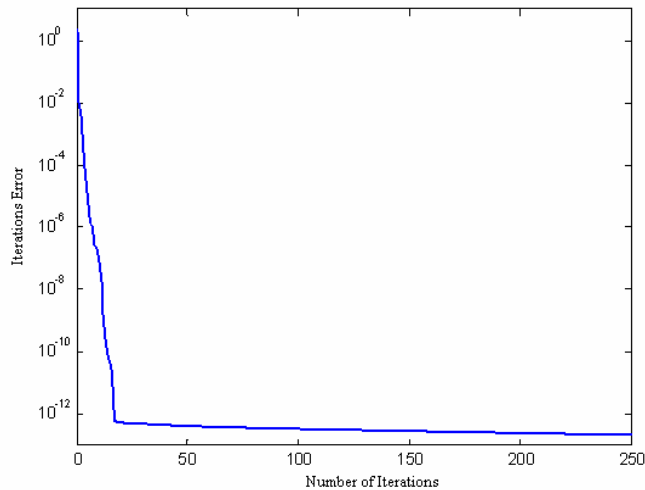


Figure 4. Network error decrease in the evolvent ANN's training

After the simulation, the results show that the ANN developed in this research is capable of checking whether the gear involute profile meets the specifications of design.

Tabela 2. Pontos da simulação das evolventes:  $\varphi = 20^\circ$  e  $\varphi = 17^\circ$

Collects Points	$X_{20^\circ}$	$Y_{20^\circ}$	$X_{17^\circ}$	$Y_{17^\circ}$	$\Delta(Y)$
Point 1	47,6320	-4,3841	47,6320	-4,1816	-0,2025
Point 2	47,7273	-4,3892	47,7273	-4,1859	-0,2033
Point 3	48,0133	-4,3855	48,0133	-4,1828	-0,2026
Point 4	48,4888	-4,3437	48,4888	-4,1483	-0,1954
Point 5	49,1502	-4,2353	49,1502	-4,0585	-0,1767
Point 6	49,9916	-4,0318	49,9916	-3,8899	-0,1420
Point 7	51,0050	-3,7061	51,0050	-3,6195	-0,0866
Point 8	52,1801	-3,2315	52,1801	-3,2251	-0,0064
Point 9	53,5047	-2,5830	53,5047	-2,6856	0,1026
Point 10	54,9643	-1,7369	54,9643	-1,9807	0,2438

Note: the table data are presented in mm.

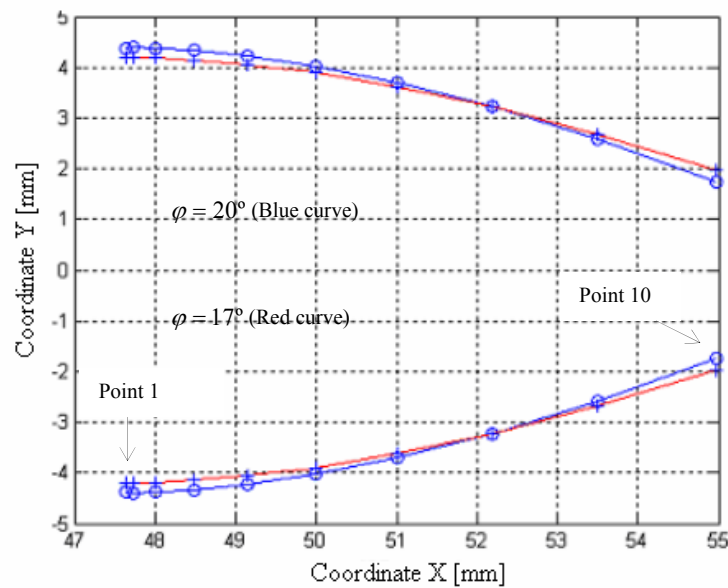


Figure 5. Points collected on the involute profiles for verifying the ANN performance

Table 3. Results obtained with the ANN for the data in Table 2

Collect Points	Delta(Y) (mm)		Error (µm)
	Obtained by Simulation	Obatinbed by ANN	
Point 1	-0,2025	-0,2025	0,0
Point 2	-0,2033	-0,2033	0,0
Point 3	-0,2026	-0,2026	0,0
Point 4	-0,1954	-0,1954	0,0
Point 5	-0,1767	-0,1767	0,0
Point 6	-0,1420	-0,1420	0,0
Point 7	-0,0866	-0,0866	0,0
Point 8	-0,0064	-0,0064	0,0
Point 9	0,1026	0,1026	0,0
Point 10	0,2438	0,2438	0,0

where: Error = Delta(Y)by simulation - Delta(Y)by ANN

## 2.2. Application of artificial neural network for evaluation of 3D surfaces

The following situation will be considered in this section: a part must be manufactured and its profile, a 3D surface, must be that shown in Figure 6. This profile is mathematically defined by Eq. (5).

$$Z = \frac{X^2}{A^2} - \frac{Y^2}{B^2} \tag{5}$$

where:  $A^2 = 10$  e  $B^2 = 5$

$X$ ,  $Y$  and  $Z$  : coordinates of a point on the surface

In practice, when a part is manufactured it is possible that the final dimensions, after the manufacturing process, to be different from those defined in the design. This difference may be attributed to errors from the manufacturing process, caused by geometric and dynamic errors of the machines tool. In order to determine the existing difference between the nominal and actual surfaces, from coordinate points  $P_i (X_i, Y_i, Z_i)$ , an artificial neural network (ANN) was

developed as part of this research work. The nominal surface was defined by Eq. (5) using the values of the parameters  $A^2 = 10$  and  $B^2 = 5$ . The actual surface was, also, defined by equation (5), but with parameters  $A^2$  and  $B^2$  defined by the equations (6) and (7), respectively.

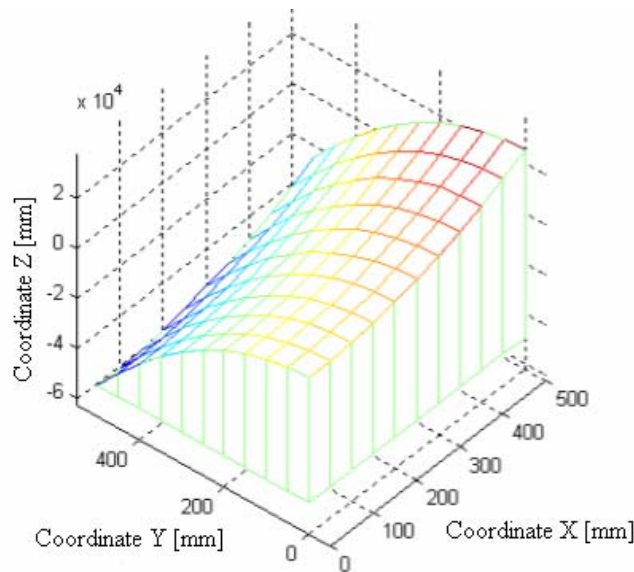


Figure 6. Nominal 3D surface of a part to be manufactured

$$A^2 = 10 \cos(\psi) \quad (6)$$

$$B^2 = 5 [\sin(\psi) + 1] \quad (7)$$

The value of the angle ( $\psi$ ), in the simulation process, represents the influence of eventual errors from the manufacturing process. This was done in order to make it easier to generate surfaces which are different from the nominal surface. Therefore, it is possible to develop and train an ANN capable of determining the difference between the nominal and actual surfaces. If the angle ( $\psi$ ) is null, the manufactured surface will have the same dimensions and form as specified in design. Figure 7 shows the nominal and actual surfaces, by considering that ( $\psi$ ) = 10 degree. To determine the difference between the nominal and actual surfaces the following procedure was applied: first, the actual surface is measured by scanning or digitalizing process in order to get a set of points  $P_{ia}(X_{ia}, Y_{ia}, Z_{ia})$  on the actual surface. Second, the mathematical model that defines the nominal surface is used for each pairs ( $X_{ia}, Y_{ia}$ ) to determine the set of nominal points  $P_{in}(X_{in}, Y_{in}, Z_{in})$ . Finally, it is possible to determine the error,  $\Delta Z_{i}$ , between the coordinates  $Z_{ia}$  and  $Z_{in}$  of the actual and nominal surfaces, respectively.

The artificial neural network (ANN) used to check the difference between the nominal and actual surfaces has the following topology:

- Input layer: 6 neurons with linear activation function where 3 neurons represent the coordinates of a point ( $X_{nominal}, Y_{nominal}, Z_{nominal}$ ) which belongs to the nominal surface and the other 3 neurons are the coordinates of a point ( $X_{actual}, Y_{actual}, Z_{actual}$ ) which belongs to the actual surface. Remembering that the points have in common the coordinates X and Y.
- Intermediary layers: 15 neurons with hyperbolic tangent activation function in the first layer and 10 neurons with hyperbolic tangent activation function in the second intermediary layer.

Output layer: 7 neurons with linear activation function where 3 neurons represent the coordinates of a point ( $X_{nominal}, Y_{nominal}, Z_{nominal}$ ) which belongs to the nominal surface and the other 3 neurons are the coordinates of a point ( $X_{actual}, Y_{actual}, Z_{actual}$ ) which belongs to the actual surface and 1 neuron represents the error,  $\Delta Z_{i}$ , between the coordinates  $Z_{ia}$  and  $Z_{in}$  of the actual and nominal surfaces, respectively.

Various surfaces were generated by using different values of  $\psi$  such as: 5°, 10° 20° and 30°. After the neural network was defined, it was then trained using the Back-propagation algorithm. Some of the used data are presented in Table 4, and they are the network desired input and output data. In this table the actual surface is defined with  $\psi = 10^\circ$ .

After the neural network was defined, it was then trained, and afterwards various simulations were performed to check the ANN performance. In these simulations were considered points which were not in the ANN training data.

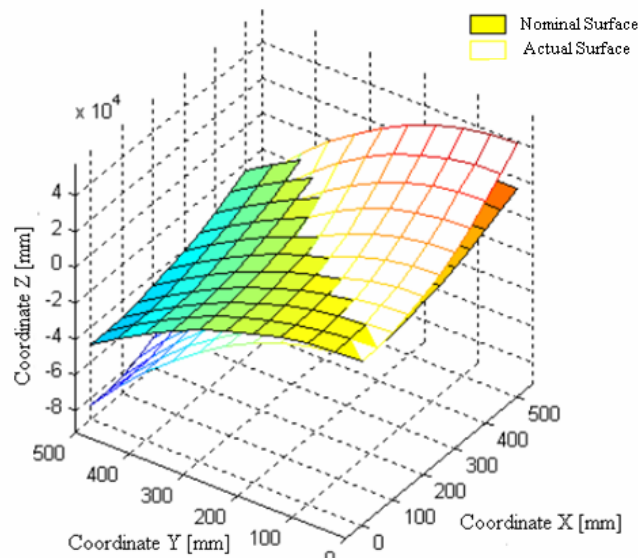


Figure 7. Comparison between the part's ideal and real surfaces

Table 4. Some of the data used in training the ANN

Input data of the ANN (mm)					
		Point 1	Point 2	Point 3	Point 4
$X_{ideal}$	0,0000	100,0000	200,0000	400,0000	500,0000
$Y_{ideal}$	300,0000	0,0000	400,0000	500,0000	500,0000
$Z_{ideal}$	-18000,000	1000,0000	-28800,00000	-34000,0000	-25000,0000
$X_{real}$	0,0000	100,0000	200,0000	400,0000	500,0000
$Y_{real}$	300,0000	0,0000	400,0000	500,0000	500,0000
$Z_{real}$	-30.207,5867	2000,0000	-45702,3763	-51909,9631	-33909,9631
Target of the ANN (mm)					
$X_{ideal}$	0,0000	100,0000	200,0000	400,0000	500,0000
$Y_{ideal}$	300,0000	0,0000	400,0000	500,0000	500,0000
$Z_{ideal}$	-18000,000	1000,0000	-28800,00000	-34000,0000	-25000,0000
$X_{real}$	0,0000	100,0000	200,0000	400,0000	500,0000
$Y_{real}$	300,0000	0,0000	400,0000	500,0000	500,0000
$Z_{real}$	-30.207,5867	2000,0000	-45702,3763	-51909,9631	-33909,9631
$\Delta(Z)$	12207,5867	-1000,0000	17702,3763	17909,9631	8909,9631

where:  $\Delta(Z) = (Z)_{nominal} - (Z)_{actual}$

Table 5 shows some of the points used in the simulation, where the actual surface has  $\psi = 25,6^\circ$ . Coordinates X and Y vary from 125 to 125 mm and 25 points were collected on the actual surface. Figure 8 shows the actual and nominal surfaces for the situation in question. Also, in Table 5 are shown the results obtained by the ANN. The maximum error was found to be 0.2  $\mu\text{m}$ , which proves that the ANN has a good performance.



Table 5. Data used for checking the ANN performance

<b>Data of simulation (mm)</b>					
	<b>Point 1</b>	<b>Point 2</b>	<b>Point 3</b>	<b>Point 4</b>	<b>Point 5</b>
$X_{ideal}$	0,0000	125,0000	250,0000	375,0000	500,0000
$Y_{ideal}$	375,0000	500,0000	375,0000	375,0000	250,0000
$Z_{ideal}$	-28125,0000	-48437,0000	-21875,0000	-	12500,0000
				14062,5000	
$X_{real}$	0,0000	125,0000	250,0000	375,0000	500,0000
$Y_{real}$	375,0000	500,0000	375,0000	375,0000	250,0000
$Z_{real}$	-35422,5154	-59848,3608	-22922,5154	-7297,5154	34256,6597
$\Delta(Z)$	7297,5154	11410,8608	1047,5154	-6764,9845	-21756,6597
<b>Result from the ANN (mm)</b>					
$X_{ideal}$	0,0000	124,9999	250,0000	375,0000	500,0000
$Y_{ideal}$	374,9999	500,0000	375,0000	375,0000	250,0000
$Z_{ideal}$	-28125,0000	-48437,0000	-21874,9999	-	12500,0000
				14062,4999	
$X_{real}$	0,0000	124,9999	250,0000	375,0000	500,0000
$Y_{real}$	375,0000	500,0000	374,9999	375,0000	250,0000
$Z_{real}$	354225154	-59848,3608	-22922,5154	-7297,5154	34256,6597
$\Delta(Z)$	7292,5156	11410,8607	1047,5154	-6754,9845	-21756,6598
<b>Difference between data obtained from simulation and ANN (<math>\mu m</math>)</b>					
$X_{ideal}$	0,0	0,1	0,0	0,0	0,0
$Y_{ideal}$	0,1	0,0	0,0	0,0	0,0
$Z_{ideal}$	0,0	0,0	0,1	0,1	0,0
$X_{real}$	0,0	0,1	0,0	0,0	0,0
$Y_{real}$	0,0	0,0	0,1	0,0	0,0
$Z_{real}$	0,0	0,0	0,0	0,0	0,0
$\Delta(Z)$	-0,2	0,1	0,0	0,0	-0,1

Other simulations were carried out and the results are shown in Table 10. This table shows the obtained results for various simulations, in which the number of points collected on the surface varies, as well as angle  $\psi = 25,6^\circ$ , since when this angle varies, we obtain a surface that is different from the nominal one.

Tabela 6. Results of further simulations

	<b>Number of points</b>	$\psi$	<b>Maximum Error (<math>\mu m</math>)</b>
<b>Simulation 1</b>	36	$8^\circ$	0,1
<b>Simulation 2</b>	49	$13^\circ$	0,1
<b>Simulation 3</b>	100	$24^\circ$	0,1
<b>Simulation 4</b>	625	$40^\circ$	0,7
<b>Simulation 5</b>	900	$0^\circ$	0,6

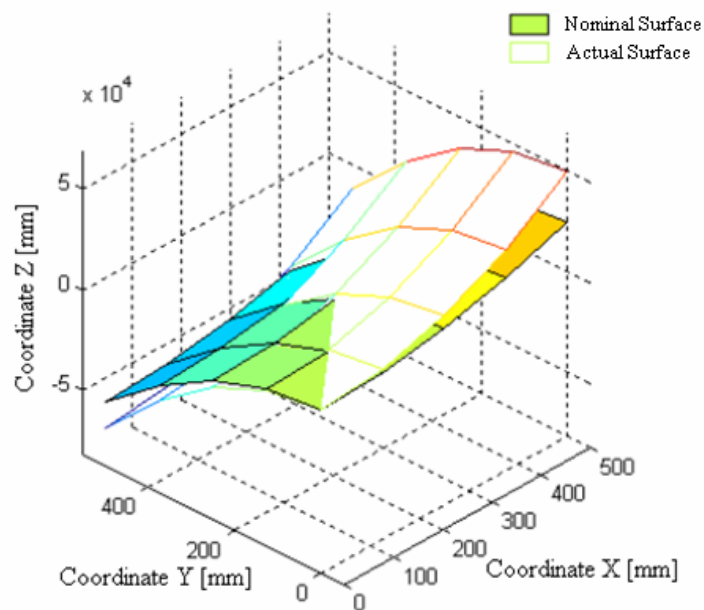


Figure 8. Distribution of simulation points for  $\psi = 25,6^\circ$  and 25 collected points

#### 4. CONCLUSIONS

This work presented an alternative to determine parameters of substitute geometries in coordinate measuring machines (CMMs). Differently from the current computational systems of the most CMMs, which are based on Euclidean geometry and numeric methods such as least squares and minimum zones, the methodology proposed in this work uses artificial neural networks (ANNs). The ANNs were applied to complex geometries such as: involute profile of the tooth of a spur gear and free form surface. In both cases, the results obtained by using the ANNs were considered effective. The results obtained, in this work, show that the artificial neural networks are an important and powerful tool to be applied in coordinate measuring machines as the maximum error of the ANNs was below  $1 \mu\text{m}$ . Further research work is to be developed in order to apply this proposed approach in reverse engineering applications.

#### 5. REFERENCES

- Bosch, J. A. 1995, "Coordinate Measuring Machines and Systems". New York, Marcel Dekker, Inc.
- Curran E. and Phelan, P. 2004, "Quick Check Error Verification of Coordinate Measuring Machines", *Journal of Materials Processing Technology*, pp. 1207–1213.
- Haykin, S., 1998, "Neural Networks: A Comprehensive Foundation", 2nd edition, MacMillan.
- Lima Jr, J and Silva, J. B. A., 2009, "Determining Geometries Through Artificial Neural Networks and Comparing to Least Square Methods". *Máquinas e Metais*, Vol. 518, pp. 148-163.
- Shigley, J. E., et. al. 2003, "Mechanical Engineering Design", McGraw-Hill ScienceArtmed, 7th ed.
- Silva, J. B. A. and Burdekim, M., 2002, "A Modular Space Frame for Assessing the Performance of Co-ordinate Measuring Machines (CMMs)", *Precision Engineering*, 26, pp. 37 – 48.
- Webera, T., Motavalli, S., Fallahi, B., and Cheraghi, S. H., 2002, "A Unified Approach to Form Error Evaluation", *Precision Engineering*, vol. 26, pp. 269–278.
- Weckenmann, A., Knauer, M. and Killmaier, T., 2001, "Uncertainty of Coordinate Measurements on Sheet-Metal Parts in the Automotive Industry", *Journal of Materials Processing Technology*, Vol.115, pp 9-13.

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