

## INVERSE ANALYSIS APPLIED TO ILLUMINATION DESIGN: OPTIMUM LOCATIONS AND POWERS OF THE LIGHT SOURCES

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**Abstract.** *This work presents a methodology to find a prescribed uniform luminous flux on the working area (the design surface) of a three-dimensional enclosure. The major contribution of the present work is to show how to determine both the luminous powers and locations of the light sources to attain the uniform illumination on the design surface. The illumination design is inherently an inverse problem, in which the design surface is subjected to two conditions – luminous flux and null luminous power – while the light sources locations and luminous powers are left unconstrained. The inverse problem is formulated as an optimization problem, which is solved through the generalized extremal optimization (GEO) algorithm. This method is especially advantageous to be applied in complex problems where there are different kinds of design variables involved (in this case, the locations and the power inputs of the light sources). Two objective functions are considered, based on the minimization of the least-square deviation and on the minimization of the maximum deviation. The results show that the inverse analysis can lead to a luminous flux on the design surface with a maximum local deviation of less than 3% of the prescribed illumination.*

**Keywords:** *Inverse analysis, Illumination design, Radiation exchanges, GEO Optimization, Luminous flux*

### 1. INTRODUCTION

The analytical design of artificial lighting of environments started in the first quarter of the 20th century, based on the knowledge that the luminous flux on a given working area was not only dependent on the power of the light sources, but also on the absorbing and reflecting effect of the remaining surfaces. Among the first works to deal with illumination design, Harrison and Anderson (1916 and 1920) proposed an experimental procedure, the lumen method, in which the luminous flux on a working plane was determined from a combination of a series of proposed assembling of punctual and continuous light sources. The lumen method (OSRAM, 2005) is probably the most widely employed for the design of illumination, for its algebraic relations provide a rapid, simple procedure to determine the power of the lamps, although the method lacks on precision. Many studies have been carried out to provide recommending lighting for several possible applications. In general, not only the intensity of light (luminous flux intensity) is specified, but also it is required uniformity of the lighting. The major goal of the illumination designer is to determine the positions and powers of the light sources to provide the prescribed luminous flux on the working area.

Relying on computational simulation, Garrocho and Amorim (2004) focused on energy saving issues by using daylight to optimize the illumination. Computational simulation of daylight was also presented by Papst et al. (1998) and Tavares (2007). The first work, using the commercial software Lumen Micro, considered the influence of daylight through different hours of the day and year, developing an analytical methodology concerning the quantity and distribution of light in environments. The second work focused on the popularization of the computational tools, simulating with the software ECOTECH and Lumen Designer both the daylight and artificial illumination in buildings and indicating the best options for each situation. Using both a computational method and the lumens method, Souza et al. (2004) developed a code based on MS-DOS interface to calculate the distributions of light sources in the environment, counting with a database of some lamps and luminaires that could be chosen for the design. A more elaborate solution can be achieved with the WinElux code (EEE, 2002), which contains a database of different types of lamps. In spite of their widespread use, both the lumen method and the WinElux code are in general not capable of providing solutions that can satisfy uniformity of luminous flux on the design surface.

Trying to resolve the problems identified in empirical methods, a new approach was proposed in the works of Smith Schneider and França (2004), Seewald et al. (2006), and Mossi et al. (2007), in which the illumination design was treated as an inverse problem. Starting from the radiation exchange relations within an enclosure, those works proposed a methodology based on fundamental luminous exchange relations, obtaining a luminous flux on the design that satisfied the uniformity and the required intensity. The first and the second works employed the truncated singular value decomposition (TSVD) regularization (Hansen, 1990); the latter applied the generalized extremal optimization (GEO) algorithm (de Sousa et al., 2003). Costa et al., 2000, approached the design of lighting based on the inverse analysis method, which has clear advantages over the empirical methods. The proposed algorithm is based on the Simulate Anneling (SA) method, and can consider optical models that simulate real environments, thereby enabling the use of

lighting objects (representing tables, chairs, computers and walls) as input data to obtain results that were closer to reality.

This paper considers the illumination design of the three-dimensional rectangular enclosure that was studied in Smith Schneider and França (2004) and Mossi et al. (2007). While the locations of the light sources were fixed in those two works, and the objective of the design was to find their luminous powers, so as to contribute to the optimization of the illumination design, the present work aims at determining the optimum locations of the lamps. Although the search for the location of the lamps was also presented in Cassol et al. (2008), the present work extends the solution to consider that the powers of the lamps were not the same, leading to a problem with a higher number of unknowns. In addition, two objective functions are employed to solve the design problem: one based on the minimization of the least-square of the deviations between the imposed and the actual illumination heat flux on the design surface; and the other based on the minimization of the maximum deviation. All the surfaces that form the enclosure are assumed diffuse and having spectral hemispherical emissivities that are wavelength independent in the visible region of the spectrum.

## 2. PHYSICAL AND MATHEMATICAL MODELING

### 2.1. Luminous flux and thermal radiation

Visible light is contained in the spectrum of thermal radiation, corresponding to wavelengths ranging from 0.4 to 0.7  $\mu\text{m}$ . The luminous flux, in units of lumens/m<sup>2</sup> or lux, can be related to the thermal radiation flux, in units of W/m<sup>2</sup>, by means of the following relation:

$$dq^{(l)} = CV_{\lambda} dq^{(w)} \quad (1)$$

where  $dq^{(l)}$  and  $dq^{(w)}$  correspond respectively to the luminous flux and to the thermal radiation flux for a specific wavelength  $\lambda$ , within an interval  $d\lambda$ ,  $C$  is a conversion factor constant, equal to 683 lumens/W, and  $V_{\lambda}$  is the photopic spectral luminous efficacy of the human eye, which takes into account the human eye sensitivity to the thermal radiation comprehended in the visible region of the spectrum. As shown in Smith Schneider and França (2004) and Mossi et al. (2007),  $V_{\lambda}$  peaks with a value of 1.0 for a thermal radiation in the wavelength of 0.555  $\mu\text{m}$ , and then decay monotonically to zero as the lower and upper limits of the visible region, 0.4  $\mu\text{m}$  and 0.7  $\mu\text{m}$ , are approached.

In general, a source of light is composed of radiation covering the entire range of the visible region. In such a case, Eq. (1) must be applied to each infinitesimal amount of the spectral energy, and then be integrated in the visible spectrum. Details of this procedure can be found in Smith Schneider and França (2004).

### 2.2. Problem definition

A schematic view of a three-dimensional enclosure is shown in Fig. 1, which is formed by surfaces that are diffuse and have spectral hemispherical emissivities that are wavelength independent in the visible region of the spectrum. The design surface, where a luminous flux is to be specified, is located on the bottom of the enclosure; the light sources are located on the top surface. The remaining of the enclosure is formed by walls that do not emit but reflect incident light. The length, width and height of the enclosure are designated by  $L$ ,  $W$  and  $H$ , respectively.

Figure 2 shows the division of the enclosure into finite-sized square elements,  $\Delta x = \Delta y = \Delta z$ , in which the luminous energy balance can be applied. In this analysis, it is considered that an uniform luminous flux (in lumens/m<sup>2</sup> or lux), designated by  $q_{\text{specified}}^{(l)}$ , is specified on the design surface. The problem consists of finding the position of each light source element, and its luminous powers, imposed to be the same for all the light sources.

As shown in Smith Schneider and França (2004), the light energy balance applied to a surface element  $j$  can be expressed in different but complementing forms, as seen below:

$$q_{o,j}^{(l)} = \varepsilon_j e_{b,j}^{(l)} + (1 - \varepsilon_j) \sum_{j^*=1}^J F_{j-j^*} q_{o,j^*}^{(l)} \quad (2)$$

$$q_{o,j}^{(l)} = q_{r,j}^{(l)} + \sum_{j^*=1}^J F_{j-j^*} q_{o,j^*}^{(l)} \quad (3)$$

$$q_{i,j}^{(l)} = \sum_{j^*=1}^J F_{j-j^*} q_{o,j^*}^{(l)} \quad (4)$$

where  $q_o^{(l)}$  is the outgoing luminous flux, in lumens/m<sup>2</sup> or lux, which takes into account both emission and reflection;  $q_r^{(l)}$  is the net luminous flux, in lumens/m<sup>2</sup>, which takes into account emission minus absorption;  $q_i^{(l)}$  is the incident luminous flux, in lumens/m<sup>2</sup>;  $e_b^{(l)}$  is the blackbody luminous power, in lumens/m<sup>2</sup>, which is solely dependent on the temperature;  $F_{j-j^*}$  is the view factor between surface elements  $j$  and  $j^*$ ;  $\epsilon_j$  is the hemispherical emissivity of the surface in the visible range of the spectrum; finally,  $J$  is the total of elements on the enclosure. In the derivation of Eqs. (2) and (3), it was assumed that the spectral emissivity was independent of the wavelength in the visible region of the spectrum. Since the objective of this work is mainly the presentation of a methodology for the determination of the optimum location of the light sources, the gray surfaces assumption is adopted for simplicity, but extension to non-gray surfaces is immediate.

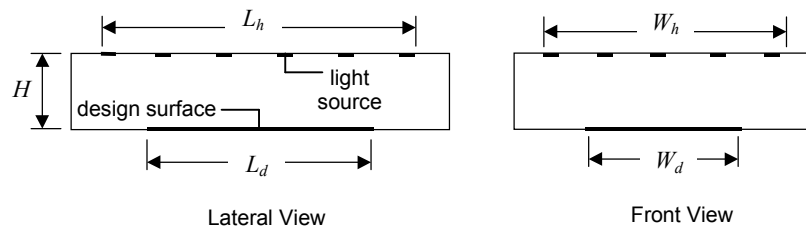


Figure 1. Three-dimensional rectangular enclosure.

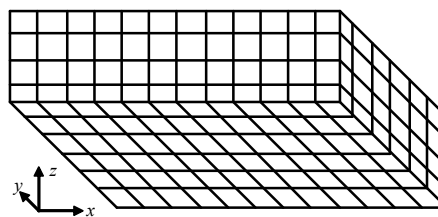


Figure 2. Division of the bottom and two side surfaces of the enclosure into finite size elements.

In the illumination design, no condition is known for the light source elements, but they need to be found to satisfy the specifications on the design surface. For an element  $jw$  on the side walls, the luminous power is null,  $e_{b,jw}^{(l)} = 0$ , since they do not emit light. For a design surface element  $jd$ , two conditions are specified: the luminous power is also null,  $e_{b,jd}^{(l)} = 0$ , and the luminous flux is equal to  $q_{specified}^{(l)}$ . Depending on the problem, the luminous flux can correspond to either the net or to the incident luminous fluxes,  $q_{r,jd}^{(l)}$  and  $q_{i,jd}^{(l)}$ , respectively. In fact, the combination of Eqs. (2) to (4), with  $e_{b,jd}^{(l)} = 0$ , show that these two quantities are related by  $q_{i,jd}^{(l)} = -q_{r,jd}^{(l)} / \epsilon_{jd}$ , so prescribing one condition is equivalent to prescribing the other. In this work, it is considered that the prescribed luminous flux is related to the net luminous flux, that is,  $q_r^{(l)} = -q_{specified}^{(l)}$ . Note that the negative signal arises from the adopted convention that the net luminous flux corresponds to emission minus absorption of light. For a surface that is illuminated, it should be negative.

One possible treatment for this problem is to specify the positions as well as the net luminous fluxes,  $q_{r,jl}^{(l)}$ , of the light source elements  $jl$  (alternatively, it could be the blackbody luminous power,  $e_{b,jl}^{(l)}$ , instead of  $q_{r,jl}^{(l)}$ ), and to impose the condition of null luminous power to the elements on the design surface and walls,  $e_{b,jd}^{(l)} = e_{b,jw}^{(l)} = 0$ . Equation (2) is written for each element on the design surface and on the wall, and Eq. (3) is written for each light source element, forming a system of  $J$  linear equations on the  $J$  unknown luminous radiosities of each surface  $j$ ,  $q_{o,j}^{(l)}$ . This system is in general well-conditioned and can be solved by any standard matrix inversion technique, such as Gaussian elimination, or by iterative techniques, such as the Gauss-Seidel. Once the system is solved for the outgoing luminous flux, Eq. (3) is written for each design surface element to determine the net luminous power,  $q_{r,jd}^{(l)}$ , which can be compared to the prescribed luminous power,  $q_{specified}^{(l)}$ . The process is repeated with the placing of the light sources in different positions and specifying a different value for  $q_{r,jl}^{(l)}$ , repeating the process until the conditions on the design surface is attained

within a maximum error. Searching through all possible solutions is not a feasible task, unless an efficient searching technique is devised. In this work, this will be done with the aid of the GEO algorithm.

### 3. THE GENERALIZED EXTREMAL OPTIMIZATION ALGORITHM

The generalized extremal optimization (GEO) algorithm (De Sousa et al., 2003) is a new evolutionary algorithm devised to improve the Extremal Optimization method (Boettcher and Percus, 2001) so that it could be easily applicable to virtually any kind of optimization problem. Both algorithms were inspired by the evolutionary model of Bak and Sneppen (1993). Following the Bak and Sneppen (1993) model, in GEO  $L$  species are aligned and for each species it is assigned a fitness value that will determine the species that are more prone to mutate. One can think of these species as bits that can assume the values of 0 or 1. Hence, the entire population would consist of a single binary string. The design variables of the optimization problem are encoded in this string that is similar to a chromosome in a genetic algorithm (GA) with binary representation (see Fig. 2).

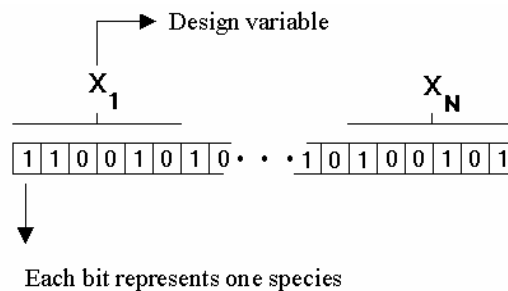


Figure 3. Design variables encoded in a binary string.

To each species (bit) it is assigned a fitness number that is proportional to the gain (or loss) the objective function value has in mutating (flipping) the bit. All bits are then ranked from rank 1, for the least adapted bit, to  $N$  for the best adapted. A bit is then mutated (flipped) according to the probability distribution of the  $k$  ranks, given by:

$$P(k) = k^{-\tau} \tag{5}$$

in which ( $1 \leq k \leq N$ ). In Eq. (5),  $\tau$  is a positive adjustable parameter (for  $\tau \rightarrow 0$ , the algorithm becomes a random walk, whereas for  $\tau \rightarrow \infty$ , we have a deterministic search). This process is repeated until a given stopping criteria is reached and the best configuration of bits found through the process is returned.

In a practical application of the GEO algorithm, the first decision to be made is on the definition of the number of bits that will represent each design variable. This can be done simply setting for each variable the number of bits necessary to assure a given desirable precision for each of them. For continuous variables the minimum number ( $m$ ) of bits necessary to achieve a certain precision is given by:

$$2^m \geq \left[ \frac{(x_j^u - x_j^l)}{p} + 1 \right] \tag{6}$$

where  $x_j^l$  and  $x_j^u$  are the lower and upper bounds, respectively, of the variable  $j$  (with  $j = 1, N$ ), and  $p$  is the desired precision. The physical value of each design variable is obtained through the equation:

$$x_j = x_j^l + (x_j^u - x_j^l) \frac{I_j}{(2^m - 1)} \tag{7}$$

in which  $I_j$  is the integer number obtained in the transformation of the variable  $j$  from its binary form to a decimal representation. The complete implementation of the GEO algorithm was presented in De Sousa et al. (2003).

#### 4. SOLUTION PROCEDURE

The optimization problem consists of minimizing an objective function  $F$  that can measure the difference between the specified luminous flux on the design surface,  $q_{specified}^{(l)}$ , and the luminous fluxes on the design surface that are obtained from a given choice of spatial configuration and luminous powers of the light sources,  $q_{r,jd}^{(l)}$ . In this work, two different objective functions are adopted, as shown below:

$$F_{lsq} = \sqrt{\sum_{jd} \left( q_{specified}^{(l)} - q_{r,jd}^{(l)} \right)^2} \quad (8)$$

and

$$F_{max} = \max_{jd} \left| q_{specified}^{(l)} - q_{r,jd}^{(l)} \right| \quad (9)$$

With the objective function defined by Eq. (8),  $F_{lsq}$ , the objective is to minimize the least-square of the deviations between the specified illumination and the illumination that is obtained from a given configuration of the light sources. On the other hand, the objective function  $F_{max}$  aims at minimizing the maximum deviation that can occur on the design surface. The optimization problem is subject to the following constraints:

$$i_{x\_low} \leq i_x \leq i_{x\_up} \quad (x \text{ direction restrictions}) \quad (10a)$$

$$i_{y\_low} \leq i_y \leq i_{y\_up} \quad (y \text{ direction restrictions}) \quad (10b)$$

$$q_{r,jl\_low} \leq q_{r,jl} \leq q_{r,jl\_up} \quad (\text{luminous flux restriction}) \quad (10c)$$

where subscripts *low* and *up* indicate the lower and upper limits of each variable, respectively. The variables  $i_x$  and  $i_y$  are indices that define the  $x$  and  $y$  positions of each light source; variable  $q_{r,jl}$  is the luminous power. These variables will be described below.

To minimize the above relation, the following procedure is followed:

1. Define the required precision,  $p$ , to find minimum number of bits,  $m$ , using Eq. (6);
2. Start with a given set of locations of the light sources and the value of the net luminous flux (according to the requirements of the design);
3. Solve the system of equations described in Section 2 to find net luminous fluxes on the design surface elements,  $q_{r,jd}^{(l)}$ , computing the error function from Eq. (8) or (9);
4. Choose a new set of luminous powers on the light sources according to the GEO algorithm;
5. Repeat from step 2 until satisfactory solutions for the locations of the light sources and of the value of the net luminous flux are found.

For each configuration (positions and luminous powers) of the light sources, it is possible to determine the luminous flux on the design surface,  $q_{r,jd}^{(l)}$ , which can be readily compared to the specified luminous flux,  $q_{specified}^{(l)}$ . The average error of the inverse solution is given by:

$$\gamma_{avg} = \frac{\sum_{jd=1}^{JD} \left| \frac{q_{r,jd}^{(l)} - q_{specified}^{(l)}}{q_{specified}^{(l)}} \right|}{JD} \times 100\% \quad (11)$$

where  $JD$  is the total number of elements on the design surface. The maximum error of the inverse solution is:

$$\gamma_{max} = \max_{jd} \left| \frac{q_{r,jd}^{(l)} - q_{specified}^{(l)}}{q_{specified}^{(l)}} \right| \times 100\% \quad (12)$$

## 5. RESULTS AND DISCUSSION

The case considered in this work consists of a three-dimensional enclosure as shown in the schematic representation in Fig. 1. The aspect ratio of the enclosure base is  $W/L = 0.8$ ; the dimensionless height is  $H/L = 0.2$ . The selection of the other dimensions of the enclosure will require a few considerations. First, the design surface ought not to cover the entire extension of the base, since the portions close to the corners would be mainly affected by the reflections from the side walls, not from the luminous radiation from the light source elements on the top surface. Therefore, the design surface dimensions are taken as  $L_d/L = 0.8$  and  $W_d/L = 0.6$ . The hemispherical emissivities in the visible light region of the design surface, of the light sources and of the walls are  $\varepsilon_d = 0.9$ ,  $\varepsilon_l = 0.9$  and  $\varepsilon_w = 0.5$ , respectively

The boundary conditions are: for the elements on the design surface and on the wall, the luminous emissive power is zero,  $e_{b,jd}^{(l)} = e_{b,jw}^{(l)} = 0$ ; the locations of the light sources as well their net luminous flux will be sought with the aid of the GEO algorithm to assure the expected dimensionless net luminous flux (defined as  $Q_{r,jd} = q_{r,jd}^{(l)} / q_{specified}^{(l)}$ ) is  $Q_{r,jd} = -1.0$ , within some acceptable error.

It is considered that a total of ten light sources are used, allowing a comparison with the work presented in Smith Schneider and França (2004), Mossi et al. (2007) and Cassol et al. (2008). The total of variables depends on the two integer indices  $i_x$  and  $i_y$  for each of the ten light sources, plus the dimensionless net luminous flux  $Q_{r,jh}$ , if the location and the power of the light sources are fixed or allowed to be different. In case 1, the positions of the light sources are fixed and the values of their net luminous fluxes can vary; in case 2, the positions and the luminous fluxes of the light sources are variables, but the luminous fluxes are imposed to be the same for all light sources; in case 3, the positions of the light sources and the luminous fluxes are all variables. In case 1, there are ten variables (the ten different luminous fluxes); in case 2, there are twenty-one variables (the twenty position indices and the unknown luminous flux); and in case 3, there are thirty variables (the twenty position indices and the ten different luminous fluxes).

Figure 4 shows the division of the bottom (or top) surface into fifteen and twelve elements in the  $x$  and  $y$  directions. The shaded area represents the design surface at the bottom surface. Due to the problem symmetry, indicated by the dashed lines, only one-quarter of the domain needs to be solved ( $0 \leq x/L \leq 0.5$ ,  $0 \leq y/L \leq 0.4$ ). It results that the position of each light source can be specified by varying integer indices  $i_x$  and  $i_y$  in the intervals  $[1, 15]$  and  $[1, 12]$ , respectively, in accordance with Eqs. (10a) and (10b). For the dimensionless net luminous flux of the light sources ( $Q_{r,jh} = q_{r,jh}^{(l)} / q_{specified}^{(l)}$ ), the chosen interval is  $[0, 50]$ , in accordance with Eq. (10c). The precision  $p$  for this problem was chosen to be 0.1. The objective functions were solved according Eqs. (8) and (9).

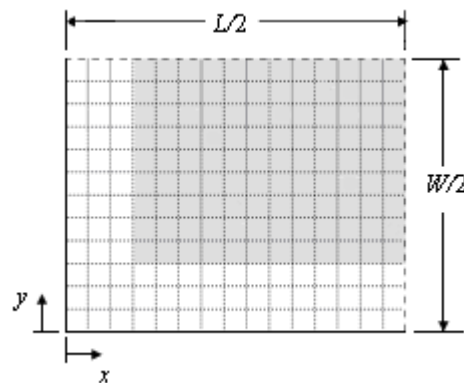


Figure 4. The design surface (shaded area) in one quarter of the bottom and top.

Since the performance of the GEO algorithm is dependent on the parameter  $\tau$ , which affects the probability of mutation (flip) of a bit according to Eq. (5), it was first performed a study to determine its optimum value for the present problem. With the aforementioned definitions, the GEO algorithm was run 100,000 times for the evaluation of the objective functions  $F_{lsq}$  and  $F_{max}$ , using different  $\tau$  values. Fifty independent runs were made for each algorithm. The results are shown in Fig. 5(a) and 5(b). From these results, considering  $F_{lsq}$ , it was selected the value of  $\tau = 1.25$  (for the case 1), and  $\tau = 1.00$  (for the cases 2 and 3). Considering the  $F_{max}$ , it was selected  $\tau = 1.00$  for all the cases.

For the next runs, the stopping criterion was modified, so that more evaluations were given to the algorithm for the search in the design space. The values of the error function towards the global minimum, concerning the minimization of  $F_{lsq}$  and  $F_{max}$ , can be seen in Figs. 6(a) and 6(b). As seen, and as expected, the objective function decreases continuously with the number of evaluations. However, although the decrease was abrupt for the first 100 thousand evaluations, further evaluations did not lead to considerable decrease in the objective function, establishing a clear point of stopping the evaluations.

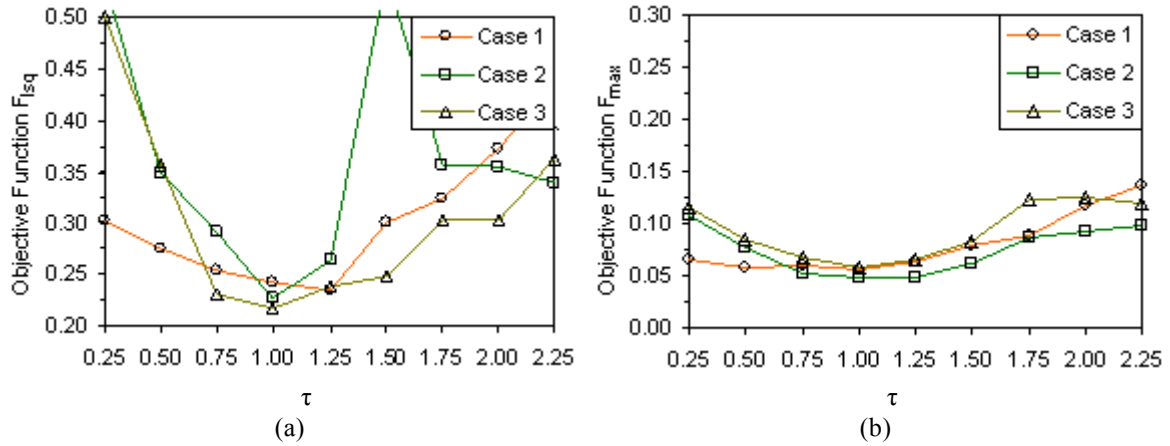


Figure 5. Average of best results for fifty independent runs of GEO for different  $\tau$ . Each run stopped after 100,000 evaluations of: (a) objective function  $F_{lsq}$ ; (b) the objective function  $F_{max}$ .

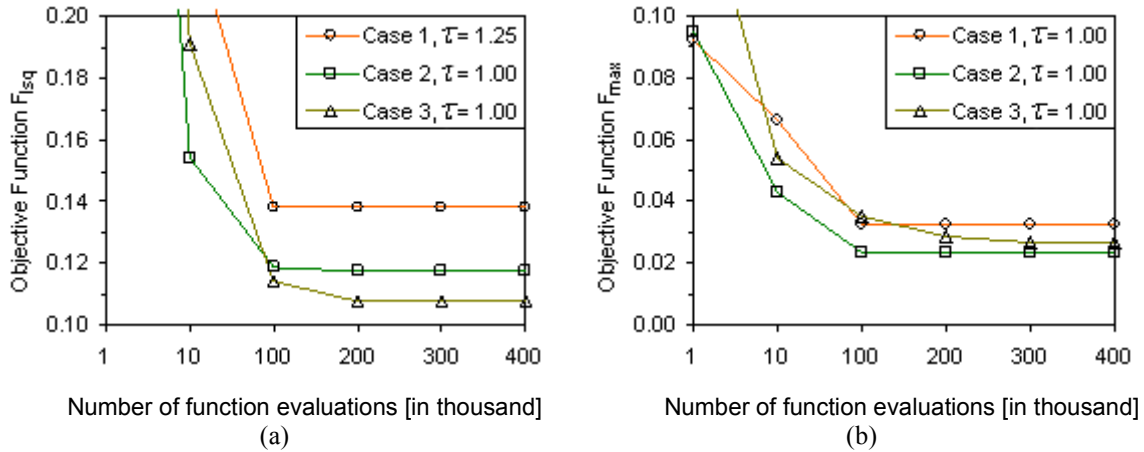


Figure 6. Lowest values of the objective functions for different numbers of evaluations: (a) minimization of the objective function  $F_{lsq}$ ; (b) minimization of the objective function  $F_{max}$ .

The positions and dimensionless powers of the light sources are shown in Tables 1 and 2 when considering the objective functions  $F_{lsq}$  and  $F_{max}$ , respectively. The case 1 corresponds to the light sources configuration recommended in Smith Schneider and França (2004), in which the positions were not found from optimization, but with the identification of the points of local maximum light powers when the entire top was covered with light sources, using the TSVD regularization. The cases 2 and 3 show the spatial configurations the light sources that were solved with the GEO method.

Table 1: Required dimensionless net luminous flux on the light source elements based on the minimization of objective function  $F_{lsq}$ .

Case 1 – Fixed Positions and different luminous flux				Case 2 – Different positions and fixed luminous flux			Case 3 – Different positions and luminous flux		
$jl$	$i_x$	$i_y$	$Q_{r,jl}$	$i_x$	$i_y$	$Q_{r,jl}$	$i_x$	$i_y$	$Q_{r,jl}$
1	2	2	25.50	1	1	24.70	1	3	41.70
2	2	8	11.20	2	6	24.70	2	9	30.30
3	4	4	25.50	2	10	24.70	3	4	16.20
4	4	10	38.40	5	3	24.70	6	11	23.70
5	8	2	33.20	6	10	24.70	7	2	18.60
6	8	8	19.20	8	5	24.70	8	6	24.60
7	12	4	25.60	11	1	24.70	10	8	10.80
8	12	10	32.00	12	11	24.70	12	2	38.80
9	14	2	16.30	13	1	24.70	13	10	28.30
10	14	8	8.00	13	7	24.70	15	5	11.10

Table 2: Required dimensionless net luminous flux on the light source elements based on the minimization of objective function  $F_{max}$ .

$jl$	Case 1 – Fixed Positions and different luminous flux			Case 2 – Different positions and fixed luminous flux			Case 3 – Different positions and luminous flux		
	$i_x$	$i_y$	$Q_{r,jl}$	$i_x$	$i_y$	$Q_{r,jl}$	$i_x$	$i_y$	$Q_{r,jl}$
1	2	2	26.20	1	3	24.50	1	9	49.60
2	2	8	0.20	1	10	24.50	2	3	38.30
3	4	4	38.90	2	5	24.50	5	1	3.50
4	4	10	39.20	5	10	24.50	7	9	32.10
5	8	2	21.70	6	3	24.50	8	1	37.00
6	8	8	25.70	8	2	24.50	8	2	14.70
7	12	4	25.00	9	9	24.50	10	10	11.60
8	12	10	27.30	12	5	24.50	13	3	25.80
9	14	2	18.20	13	10	24.50	13	7	23.30
10	14	8	11.80	14	2	24.50	13	11	14.80

Figures 7 and 8 present the resulting net luminous flux distribution on the design surface for the solutions obtained from the methodology presented in this work. All the solutions were capable of satisfying the net luminous on the design surface (specified as  $Q_{r,jd} = -1.0$ ) within an average error of 3.0% or less, which would be very difficult to obtain using a trial-and-error approach. This indicates the usefulness of the inverse analysis as a designing tool for illumination systems.

Table 3 presents the values of the objective function  $F_{lsq}$ , as defined by Eq. (8), as computed with the dimensionless net luminous fluxes, for the three net luminous flux distributions on the design surface that are shown in Fig. 7. The table also presents the average and maximum errors, as computed from Eq. (11) and (12). As seen in the table, case 3 led to the solution with the smallest average error, since both the positions and the luminous powers of the light sources were optimized in the solution. In addition, the objective function is based on the least-square deviation considering all elements in the design surface, so that the average error is also minimized.

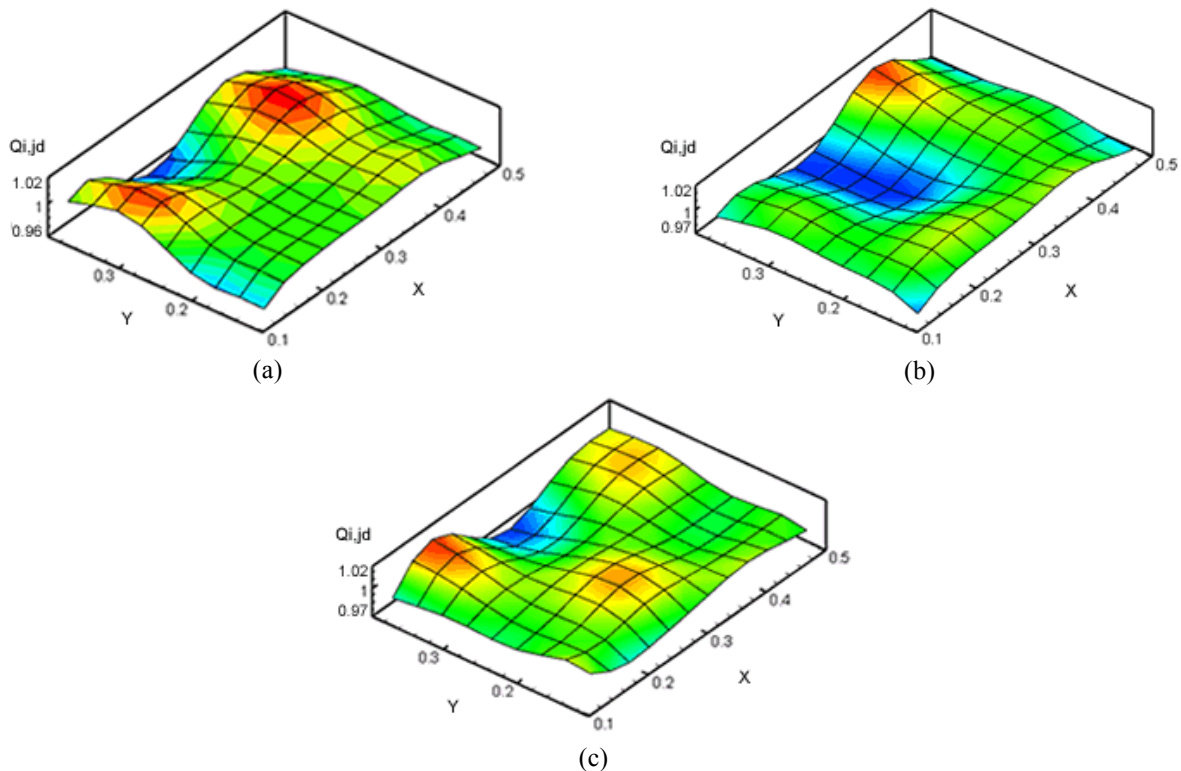


Figure 7. Dimensionless net luminous flux on the design surface. Minimization of objective function  $F_{lsq}$ . (a) Case 1; (b) Case 2; (c) Case 3.



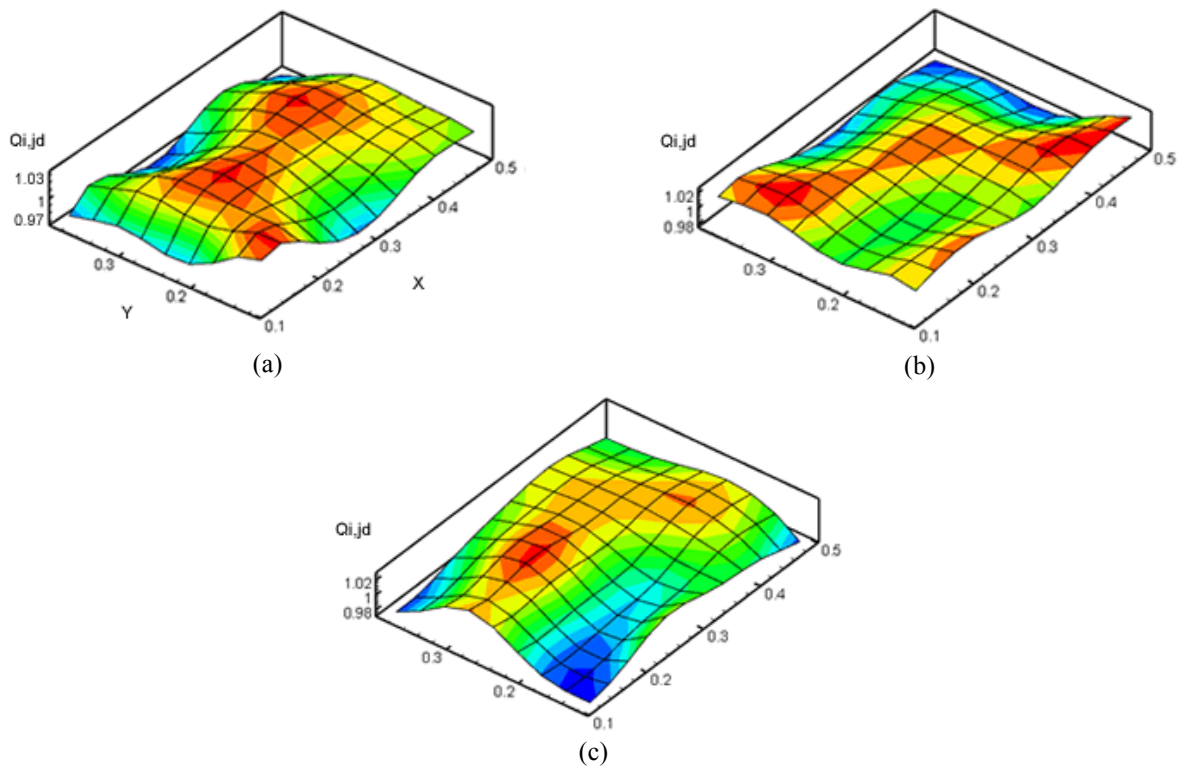


Figure 8. Dimensionless net luminous flux on the design surface. Minimization of objective function  $F_{max}$ . (a) Case 1; (b) Case 2; (c) Case 3.

Table 4 presents the values of the objective function  $F_{max}$ , as defined by Eq. (9). As seen, the maximum errors of the solutions were smaller than those that were obtained when the objective function was based on the least-squares of the deviations,  $F_{lsq}$ . This was expected due to the very nature of the objective function, which aimed at the minimization of the maximum average. On the other hand, the average errors proved to be larger.

Table 3: Minimized error function (in dimensionless form) for the minimization of  $F_{lsq}$ .

	$F_{lsq}$	Average error (%)	Maximum error (%)
Case 1	0.13820	1.01	3.78
Case 2	0.11768	0.88	2.88
Case 3	0.10788	0.80	3.31

Table 4: Minimized error function (in dimensionless form) for the minimization of  $F_{max}$ .

	$F_{max}$	Average error (%)	Maximum error (%)
Case 1	0.03215	1.66	3.21
Case 2	0.02361	1.13	2.36
Case 3	0.02686	1.28	2.68

## 6. CONCLUSIONS

This paper extended the inverse design of illumination systems, in the sense that the positions and powers of the light sources were all allowed to vary. The inverse problem was formulated as an optimization problem involving therefore two different kinds of variables simultaneously: position and luminous flux. Two different objective functions were applied, one based on the least-square and the other based on the maximum deviation between the specified illumination and the one obtained from each group of design settings. The generalized extremal optimization (GEO) algorithm was applied, and despite requiring a large computational effort, as typical of stochastic methods, it allowed finding a larger amount of satisfactory solutions. Another advantage is that the method can in general be more easily extended to non-linear problems than it would be possible with direct inversion and regularization. The problem showed different solutions depending of the objective function that was chosen. As possible next steps, the proposed inverse

design analysis can be applied using other objectives function, to consider the effect of external illumination and to consider the problem of finding the optimum number of the light sources.

## 7. ACKNOWLEDGEMENTS

FC thanks CNPq due to its financial support by means of a Masters Degree scholarship. FHRF and AJSN thank CNPq for research grants 304535/2007-9 and 300171/97-8, respectively. AJSN also thanks FAPERJ.

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