# STRUCTURAL DAMAGE ASSESSMENT USING THE ANT COLONY OPTIMIZATION TECHNIQUE

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Abstract. Structural systems in a variety of applications including aerospace vehicles, automobiles and engineering structures such as tall buildings, bridges and offshore platforms, accumulate damage during their service life. In several situations, such damage may not be visually observable. From the standpoint of both safety and performance, it is desirable to monitor the occurrence, location and extent of such damage. System identification methods, which may be classified in a general category of nondestructive evaluation techniques, can be employed for this purpose. Using experimental data, such as eigenmodes and static displacements, and an analytical structural model, parameters of the structures can be identified. The approach used in the present work is one where the structural properties of the analytical model are varied to minimize the difference between the analytically predicted and empirically measured response. This is an inverse problem where the structural parameters are identified. In this work a reduced number of static displacements and vibration modes were used as the measured response. For the damage assessment problem a close analytical model of the structural system is available and the model of the damaged structure will be identified. Damage will be represented by a reduction in the elastic stiffness properties of the structure. The problem described above was solved using global methods of optimization due to the fact that depending on the number of variables or the location of damage the resulting design space is nonconvex presenting several local minima. In the present work, the ant colony optimization (ACO) was used. It is a metaheuristic inspired by the ant's behavior developed initially for combinatorial optimization and extended for continuous domains. The approach was applied to simple truss structures with different levels of damage and presented promising results.

Keywords: structural damage, system identification techniques, inverse problems, ant colony optimization.

# 1. INTRODUCTION

In a typical load bearing structure, degradation of structural properties due to damage manifests itself as a change in the static and dynamic structural response. A correlation of the measured response with that obtained from an analytical model of the undamaged structure, allows for the possibility of determining a modified model that predicts the altered response. This process can be broadly categorized in the realm of system identification methods. The output error method of system identification, wherein the analytical model is refined to minimize the difference between the predicted and measured response of the structure was used in the present work (Stavroulakis, 2001). An iterative global optimization method is employed to determine a solution to the unconstrained optimization problem. Damage is represented by reduction in the elastic extensional modulus of the element, and those are designated as the design variables of the problem (Hajela and Soeiro, 1990). The use of static structural displacements as the measured response is a departure from the standard practice of using eigenmodes alone for the identification problem (Soeiro and Hajela, 1992). Numerical evidence clearly indicates that when eigenmodes alone are used for identification, the location and extent o damage predicted by the optimization approach is dependent on the number of modes used to match the measured and the predicted response. Higher modes are difficult to determine and to measure. In this work incomplete sets of displacements and modes are used to simulate real measured data. The approach of treating the modulus of each structural element as an independent design variable results in a large dimensionality problem. This results in a very nonconvex design space probably with several local minima. The gradient-based nonlinear programming algorithms may have difficulties to find the global optimum. In this work the Ant Colony Optimization (ACO) method which a global optimization metaheuristic is used for the solution of the resulting optimization problem

#### 2. STRUCTURAL DAMAGE ASSESSMENT

In a finite element formulation, structural characteristics are defined in terms of the stiffness, damping, and mass matrices [K], [C] and [M], respectively. The governing equation of equilibrium for a dynamical system involves each of these matrices, and can be written as,

$$[M]\ddot{x} + [C]\dot{x} + [K]x = P(t)$$

Where  $\{x\}$  is the displacement vector and P(t) is the vector of applied loads. The static load-deflection relation only involves the system stiffness matrix,

$$[K](x) = \{P\}$$
<sup>(2)</sup>

The analytical model describing the eigenvalue problem for an undamped system can be stated in terms of the system matrices defined above, the i-th eigenvalue  $\omega_i^2$ , and the corresponding eigenmode  $X_i$  as follows:

$$([K] - \omega_i^2[M]) \{X_i\} = \{0\}$$
(3)

It is clear from these equations that a change in the system matrices results in a different response, and this difference can be related to changes in specific elements of the system matrices. Since internal structural damage typically does not result in a loss of material, it will be assumed that the mass matrix is constant. The stiffness matrix can be expressed as a function of the thickness 't', the length 'L', the cross-sectional area 'A', the Young's modulus 'E' and the flexural and torsional stiffness EI and GJ, respectively,

$$[K] = [K(t, L, A, E, EI, GJ)]$$

$$\tag{4}$$

In the present work, changes in these quantities are lumped into a damage coefficient  $d_i$ , that is used to multiply the stiffness matrix of a particular element. The coefficients  $d_i$  constitute the design variables for the damage assessment problem and vary from 0 (undamaged element) to 1 (completely damaged element). The values of the coefficients  $d_i$  give the location and the extent of damage in the structure.

The approach was applied to simple truss structures with different levels of damage. The stiffness matrix of the truss element modified to include the damage coefficient is

$$K_{i}^{(.)} = \frac{(1-d_{i})E_{i}A_{i}}{L_{i}} \begin{bmatrix} C^{2} & CS & -C^{2} & -CS \\ CS & S^{2} & -CS & -S^{2} \\ -C^{2} & -CS & C^{2} & CS \\ -CS & -S^{2} & CS & S^{2} \end{bmatrix}$$
(5)

Where  $C = \cos \alpha$  and  $S = \sin \alpha$ , and the truss element is shown in Fig. 1.



Figure 1 - Truss element

If the measured and analytically determined static displacements or vibration modes are denoted by  $\{Y_m\}$  and  $\{Y_a\}$ , respectively, the optimization problem can be formulated as determining the vector of design variables  $d_i$  that minimize the scalar objective representing the difference between the analytical and experimental response and stated as follows:

$$F = \left\| Y_m^{ij} - Y_a^{ij} \right\| \tag{6}$$

Where *i* represents the degree of freedom and *j* denotes a static loading condition or a particular vibration mode. One important advantage of this approach is that the complete set of modes or displacements is not needed since the objective function involves only the difference between components of those vectors. Some of the components may be neglected according to its importance in the behavior of the structure. For example the 11 bar truss shown in Fig. 4.1 is subjected to a vertical load applied at the free end. For damage assessment purposes, only the vertical displacements were used. They are dominant and easier to measure. The other components are relatively small and neglecting them in the objective function does not affect the process of damage detection. The approach still works and becomes more realistic since in large structures only few dominant displacements can be obtained accurately. Also in the cases where eigenmodes were used for damage assessment purposes, only the first four modes and the respective eigenvalues (natural frequencies) were used in the objective function.

#### 3. ANT COLONY OPTIMIZATION (ACO)

The Ant Colony Optimization method was proposed by Dorigo in his Ph.D. thesis (1992). Real ants are capable of finding the shortest path from a food source to the nest without using visual cues. Also, they are capable of adapting to changes in the environment, for example finding a new shortest path once the old one is no longer feasible due to a new obstacle.

It is well-known that the main means used by ants to form and maintain the line is a pheromone trail. Ants deposit a certain amount of pheromone while walking, and each ant probabilistically prefers to follow a direction rich in pheromone rather than a poorer one. This elementary behavior of real ants can be used to explain how they can find the shortest path which reconnects a broken line after the sudden appearance of an unexpected obstacle has interrupted the initial path.

In fact, once the obstacle has appeared, those ants which are just in front of the obstacle cannot continue to follow the pheromone trail and therefore they have to choose between turning right or left. In this situation one can expect half the ants to choose to turn right and the other half to turn left.

It is interesting to note that those ants which choose, by chance, the shorter path around the obstacle will more rapidly reconstitute the interrupted pheromone trail compared to those which choose the longer path. Hence, the shorter path will receive a higher amount of pheromone in the time unit and this will in turn cause a higher number of ants to choose the shorter path. Due to this positive feedback process, very soon all the ants will choose the shorter path.

ACO was developed initially for combinatorial optimization only. Particularly good results were obtained in the solution of the Problem of the Traveling Salesman (Dorigo and Gambardella, 1997). The damage assessment problem deals with continuous variables. In this work an extension of the ACO algorithm applied to continuous variables is used (Pourtakdoust and Nobahari, 2004) and (Socha, 2004). A population of  $n_{pop}$  ants is expressed as  $n_{pop} x n_{dv}$  matrix, where  $n_{dv}$  is the number of design variables in the problem. Each ant, referred to as an individual, has  $n_{dv}$  numerical values associated with it. The path of each ant is related to the value of the objective function. The pheromone trail corresponds to an amount of pheromone laid on the path. For the *i-th* dimension of the design space the pheromone trail,  $\tau_i$ , is given by:

$$\tau_i(x) = e^{\frac{(x - x_i^*)^2}{2(\sigma_i)^2}}$$
(7)

where:

- $x_i^*$  is the *i-th* coordinate of the best point found by the optimization task within the design space until the current iteration;
- $\sigma_i$  is an index related to the aggregation of the population around the current minimum for the *i-th* coordinate of the design space and is given by:

$$\sigma_i = \sqrt{\frac{1}{n_{pop} - 1} \sum_{j=1}^{n_{pop}} \left( z_j - \overline{z} \right)} \tag{8}$$

- z is a vector corresponding to the *i*-th column of the population matrix ; and
- $\overline{z}$  is the mean value of the vector  $\mathbf{z}$ .

The updating process of the values of each design variable for all individuals is based on the probability distribution given by Eq. (7). Also it can be seen that the concentration of pheromone increases in the area of the candidate to the optimum. This approach (also called as positive update) reinforces the probability of the choices that lead to good

solutions. However, for avoiding premature convergence, negative update procedures are not discarded. A simple method to perform negative update is by dissolving certain the amount of pheromone in the path. The idea of this scheme is to decrease the amount of pheromone by changing the current standard deviation (Eq. 8) for each variable. The dissolving rate affects the exploration capabilities, and consequently, the convergence of the algorithm. The flowchart of the ACO algorithm is presented in Fig. 2.



Figure 2 - Flowchart of the ACO algorithm applied to continuous variables

#### 4. DISCUSSION OF RESULTS

The procedure described in previous sections was implemented using the Simple Optimization Toolbox developed by Viana (2008). This toolbox presents several heuristic methods of optimization using the MATLAB environment. A finite element analysis program, ANSYS, was used for response analysis. The measured data were simulated using the finite element solution obtained for the damaged structure, corrupted by a random noise signal. The flow between the various processors was controlled by a MATLAB main program. In all of the examples presented in this paper the same parameters were used in the ACO algorithm. The population size was ten times the number of design variables. For the ten bar truss a population of 100 ants was used. The number of iterations considered for each example was also ten times the number of design variables. For the same ten bar truss a total of 10000 objective function evaluations were performed. In all the cases presented these settings worked very well.

The method for damage detection was applied to a series of trusses with an increasing number of members. For the eleven bar truss shown in Fig. 3, damage was introduced in members 3 and 10 by reducing their Young's modulus by 30% and 70% of the original values, respectively ( $d_3 = 0.3$  and  $d_{10} = 0.7$ ). In this case, a reduced set of static displacements was used. As measured response, only the vertical displacements were used in the objective function. Noise was not considered. Figure 4 shows that damage was clearly identified in both members. Also, the extent of those damages was assessed with very good accuracy. Figure 5 shows the good convergence performance of the optimization algorithm.



Figure 3 - An eleven bar truss

Another example using the same eleven bar truss is that were element 1 was damaged with  $d_1 = 0.5$ . In this case, the synthetic measured static displacements were corrupted by a noise of 2%. Figure 6 shows that the results are still very good. The location and extent of damage were clearly identified, although some other members present also very small values of damage. One can see also in Fig. 7 that the final objective function is not as low as the previous one.



Figure 4 - Results obtained for the eleven bar truss using static displacement



Figure 5 - Objective function values



Figure 6 - Results for the eleven bar truss using static displacements and 2% noise



Figure 7 - Objective function values

The ten bar truss of Fig. 8 was damaged in members 2 and 5 by a reduction of 80% and 20%, respectively, in the values of their Young's modulus ( $d_2 = 0.8$  and  $d_5 = 0.2$ ). The first two eigenmodes were used as measured response. In this case the measurements were noiseless. Figures 9 and 10 show the results. Even with a small numbers of modes damage was detected with good accuracy.



Figure 8 - A ten bar truss



Figure 9 - Results for the ten bar truss using two eigenmodes



Figure 10 - Objective function values

The same ten bar truss was damaged in member 3 with 50% of reduction in the Young's modulus. In this example, noise was introduced in the measured data with a level of 2%. One can see by the results shown in Fig. 11 that damage in member 3 was correctly detected but some other members present also some damage. The final value of the objective function is also high compared to values of the objective function in the previous examples (Fig. 12). If the level of noise is very high the problem may become very difficult. More information about the damaged structure is needed.



Figure 11 - Results for the ten bar truss using two eigenmodes and 2% noise



Figure 12 - Objective function values



Figure 13 - Nineteen bar truss

A larger structure with nineteen members is now considered (Fig. 13). Damage of 50% was introduced in member 9 and noise was not considered. In this case, four eigenmodes were necessary to provide sufficient information to the optimizer to find the correct solution that can be seen in Figures 14 and 15.



Figure 14 - Results for the nineteen bar truss using four eigenmodes



Figure 15 - Objective function values

#### 5. CONCLUSIONS

This paper presents a method to solve structural damage detection using a system identification technique and the Ant Colony Optimization method. The approach was applied to simple truss structures with different levels of damage and presented promising results. The application of this methodology to large structures is the continuation of the present work.

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