USE OF THE GENERALIZED INTEGRAL TRANSFORM METHOD FOR SOLVING EQUATION OF MASS TRANSFER IN FOOD DRYING

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Abstract. Dried fruit and vegetables have gained commercial importance and their growth on a commercial scale has become an important sector of the agricultural industry. Plots of drying curves are used in the determination of the drying process behavior inside the samples and of the optimal drying conditions, taking into account the quality of the dried product and also the economical aspects. This research was developed with the objective of studying and modeling the phenomenon of the mass transfer in the agricultural products drying process, using the diffusional model (Fick's Second Law of Diffusion) adapted to infinite flat plate geometry. The model of the unsteady state diffusion neglects the effects of temperature and total pressure gradients, describes the moisture content transport during the food drying process that takes place in the falling rate period and gives the transient distribution of the moisture (GITT), which is an hybrid numerical–analytical solution methodology and powerful tool to solve partial differential equations. The predict results were compared with experimental data for the drying of mushroom slices of the species Agaricus blazei dried at different operating conditions. Graphics and tables permit the phenomenological analysis. The analysis showed no significant differences between the data obtained by GITT and those obtained experimentally. Thus, the model is capable for predicting the moisture distributions in the drying process.

Keywords: Mass transfer, Mathematical Modeling, GITT

1. INTRODUCTION

Foodstuffs quality and the cost of their manufacture are the most important factors to be considered when choosing a food preservation method. Water, being one of the main food components, has a decisive direct influence on the quality and durability of foodstuffs through its effect on many physico-chemical and biological changes (El-Aouar *et al.*, 2003). Agricultural products drying is a widely spread method offering physico-chemical stabilization by taking away part of the moisture content, producing different products with new qualitative properties, distinct nutritional and economical values (Babalis and Belessiotis, 2004). Dried fruit and vegetables have gained commercial importance and their growth on a commercial scale has become an important sector of the agricultural industry (Karim and Hawlader, 2005). Therefore, in the course of the last few years a number of researches have been accomplished regarding the drying process of food products (Silva *et al.*, 2009; Alencar Junior *et al.*, 2008; Giri and Prasad, 2007; Jambrak *et al.*, 2007; Ruiz-López *et al.*, 2007; Bialabrzewski, 2006; Wald *et al.*, 2006; Cao *et al.*, 2003; Krokida *et al.*, 2003).

Analytical and numerical solutions of partial differential equations systems that describe these drying processes are used for designing new or improving existing drying systems or even for the control of the drying process. All parameters (transfer coefficients, drying constants, etc.) used by the simulation models are directly related to the drying conditions, i.e. temperature and velocity of the drying medium inside the mechanical dryer (Babalis and Belessiotis, 2004).

The Generalized Integral Transform Technique (GITT) is a well-known hybrid numerical–analytical approach that can efficiently handle diffusion and convection–diffusion partial differential formulations. It is based on expansions of the original potentials in terms of eigenfunctions and the solution is obtained through integral transformation in all but one of the independent variables, thus reducing the partial differential formulations to an ordinary differential system for the expansion coefficients, which can be then solved using numerical techniques or in some special cases, analytical procedures (Almeida *et al.*, 2008).

The GITT has quite recently appeared in the literature as an alternative to conventional discrete numerical methods, for various partial differential formulations (Venezuela *et al.*, 2009, Naveira *et al.*, 2007, Barros *et al.*, 2006, Macêdo *et al.*, 1999). Its hybrid numerical - analytical structure permits the automatic control of the global error in the simulation, which avoids the need for several computer program runs to inspect for the convergence on the final results, and therefore yields codes that automatically work towards user prescribed accuracy targets. A number of applications, using the GITT, have been considered within the last few years dealing with drying in capillary porous media (Dantas *et al.*, 2007; Dantas *et al.*, 2003; Dantas *et al.*, 2002). However, the application of this technique in problems of food drying is still scarce. Therefore, the present work addresses the solution via GITT of a transient one-dimensional diffusion formulation describing the drying process.

2. PHYSICAL PROBLEM AND MATHEMATICAL MODELLING

2.1. Model assumptions

The physical problem involves a mass transfer inside food products subjected to air drying processes, initially at uniform moisture content. Fick's second law of the unsteady state diffusion, resulting by neglecting the effects of temperature and total pressure gradients, can describe the transport of moisture during the food drying process that takes place in the falling rate period (Crank, 1975).

The following assumptions were adopted in order to simplify the model:

- The product is represented by the geometrical form of a plate of thickness 2L;
- Moisture transfer is predominantly unidirectional;
- The initial moisture content is uniformly distributed throughout the product;
- Shrinkage is considered negligible;
- The diffusion coefficient is considered constant and homogeneous during drying.

2.2. Mathematical modeling

Based on the above assumptions, the equation describing moisture transfer in foods during heat treatment is given as follows:

$$\frac{\partial X(z,t)}{\partial t} = D_{ef} \frac{\partial X^2(z,t)}{\partial t^2} \quad at \ 0 < z < L$$
(1a)

The initial and boundary conditions are listed below to complete the numerical formulation of the problem:

• Boundary conditions

Symmetry of moisture:

$$\frac{\partial X(0,t)}{\partial t} = 0 \quad at \ z = 0 \ and \ t > 0 \tag{1b}$$

Equilibrium moisture at surface:

$$X(L,t) = X_e \quad at \ z = L \ and \ t > 0 \tag{1c}$$

• Initial condition

Uniform initial moisture:

$$X(z,0) = X_0$$
 at $0 < z < L$ and $t = 0$ (1d)

where X_0 is the initial moisture content, X_e is the equilibrium moisture content and X is the moisture content (kg/kg), D_{ef} is the diffusion coefficient (m²/s), and z and t are the independent variables, i.e., position (m) and time (s), respectively.

The Eqs. (1) are given in dimensionless form by:

$$\frac{\partial \theta(z^*,\tau)}{\partial \tau} = \frac{\partial^2 \theta(z^*,\tau)}{\partial {z^*}^2} \quad 0 < z < 1, \quad \tau > 0$$
(2a)

with boundary and initial conditions

$$\frac{\partial \theta(0,\tau)}{\partial z^*} = 0 \quad z *= 0, \qquad \tau > 0 \tag{2b}$$

$$\theta(1,\tau) = 0 \quad z = 1, \quad \tau > 0$$
 (2c)

$$\theta(z^*, \tau) = 1 \quad 0 < z^* < 1, \ \tau = 0 \tag{2d}$$

where the dimensionless groups are:

$$\tau = \frac{D_{ef}t}{L^2}; \quad z^* = \frac{z}{L}; \quad \theta(z^*, \tau) = \frac{X(z, t) - X_{eq}}{X_0 - X_{eq}}$$

The objective, now, is to determine the dimensionless moisture content field, in the food drying process. The mathematical model is solved here applying the Generalized Integral Transform Technique (GITT) as following described.

2. METHOD OF SOLUTION FOR THE MATHEMATICAL MODEL

The Generalized Integral Transform Technique (GITT) is a powerful hybrid numerical-analytical approach, which has been successfully applied to obtain *benchmark* solutions for different classes of linear and non-linear diffusion/convection problems (Dantas *et al.*, 2002). Such a technique, as employed to time dependent problems, includes the following basics steps:

(i) Selection of an associated auxiliary eigenvalue problem, that retains the highest capacity of information of the original problem;

(ii) Develop the appropriate transform/inverse formulae pair;

(iii) Integral transform the original problem by substituting the inverse formula into non-transformable terms or by using the integral balance approach;

(iv) Solve the resulting coupled system of ordinary differential equations in the time variable;

(v) Apply the inverse formula to the transformed field in order to obtain the solution for the original problem.

System (2) is solved using the Generalized Integral Transform Technique (GITT) (Özisik, 1980). The first step is to choose the so-called auxiliary problem. If we consider the differential Eq. (2) with its boundary conditions, this can be expressed as (Özisik, 1980)

$$\frac{d^2\psi_i(z^*)}{d{z^*}^2} + \mu_i^2\psi_i(z^*) = 0$$
(3a)

$$\left. \frac{d\psi_i(z^*)}{dz^*} \right|_{z^*=0} = 0 \tag{3b}$$

$$\psi_i(1) = 0 \tag{3c}$$

Equations (3) represent a classical Sturm-Liouville problem. Its solution is obtained in the form of eigenfunctions and eigenvalues, respectively.

$$\psi_i(z^*) = \cos(\mu_i z^*)$$
$$\mu_i = \left(i - \frac{1}{2}\right) \quad i = 1, 2, 3 \cdots$$

The next step is to define the Transformed/Inverse pair

$$\overline{\theta_i}(\tau) = \int_0^1 \widetilde{\psi_i}(z^*) \theta(z^*, \tau) dz^* \qquad Transformed \qquad (4a)$$

$$\theta(z^*,\tau) = \sum_{i=1}^{\infty} \widetilde{\psi_i}(z^*) \overline{\theta_i(\tau)} \qquad Inverse$$
(4b)

where $\overline{\theta_i}(\tau)$ is the transformed dependent variable and the norm N_i is given by:

$$N_i = \int_0^1 \psi_i^2(z^*) dz^*$$
 (4c)

The normalized eigenfunctions are defined by:

$$\widetilde{\psi_i}(z^*) = \frac{\psi_i(z^*)}{N_i^{1/2}}$$
(4d)

Moreover, the eigenfunctions have the following orthogonality property:

$$\int_{0}^{1} \widetilde{\psi_{i}}(z^{*})\widetilde{\psi_{j}}(z^{*})dz^{*} = \begin{cases} 0, i \neq j \\ 1, i = j \end{cases}$$

$$(4e)$$

At this point, the integral transformation can be applied. Eq. (2) is integrated using the operator

$$\int_{0}^{1}\widetilde{\psi_{\iota}}(z^{*})dz^{*}$$

On the basis of the orthogonality property (4e), the Integral Transform (4a) and the Inverse Formula (4b), from Eq. (2) we get

$$\frac{d\overline{\theta_i}(\tau)}{d\tau} = -\mu_i^2 \overline{\theta_i}(\tau) \tag{5a}$$

The integral transform of the entry condition (Eq. 4a) produces the following transformed initial condition:

$$\overline{\theta_i}(0) = \int_0^1 \widetilde{\psi_i}(z^*) dz^*$$
(5b)

The system of ordinary differential equations presented in Eqs. (5) for the transformed potentials was solved in the Matlab R2008a. For computational purposes this system is truncated to a sufficiently large finite order, N, for the required convergence control.

3. RESULTS

3.1. Validation

In this section, a comparison is performed between the results obtained through the hybrid numerical-analytical solution and the experimental results given by Kurozawa (2005), whose aim was the validation of the model. The mushrooms (Agaricus blazei), with a half-thickness of approximately 2.5×10^{-3} m, were dried on a fixed bed dryer under different conditions of temperature (45 and 75°C) and air velocity (1.20 and 2.30 m/s).

The curves presented in Fig. 1, for three considered tests, show that the existent moisture content at the beginning of the drying process is exponentially reduced until reaching the equilibrium moisture content. Such behavior demonstrates the inexistence of the period of constant drying, thus, the process of drying of the product just happened in the decreasing period of drying, being controlled for the internal diffusion of the liquid to the surface where the evaporation happens.

The residuals or the difference between the values given by the measurement and the model were used to estimate the quality of the model (Beck and Arnold, 1977). The residuals can be expressed as $r = X_{exp} - X_{comp}$.

The consistency of the optimization method presented in this paper can be also confirmed through the residues dispersion, as presented in Fig. 1. The magnitude of the oscillations had a minimum value of -0.26082 kg/kg, -0.41042 kg/kg, -0.13618 kg/kg, and a maximum of 0.56375 kg/kg, 0.80888 kg/kg, and 0.39113 kg/kg respectively for the tests 1, 2 and 3. Analyzing these residues, it may be seen that low values, centered around zero and with a relatively random distribution were found, indicating that the measurement was reliable and the hybrid solution represented the physical phenomenon.



Figure 1. Comparison of measured and theoretical moisture content and related residuals. (a) Test 1, (b) Test 2 and c) Test 3.

3.2. Convergence analysis

We focus the following analysis on the effects of the temperature and air velocity. The convergence ongoing in the eigenfunction expansion is illustrated in Tabs. 1-3 below, for the moisture content profile, for test 1, 2 and 3 at two different values of time, t=0.03 and 0.25 h, and at three distinct dimensionless longitudinal positions, $z^* = 0.1$, $z^* = 0.5$ and $z^* = 0.9$.

The convergence progress can be observed in the following tables through the increase in the system truncation order, denoted by the letter N. A maximum of 20 terms in the moisture content series were considered for this demonstration with excellent convergence characteristics. Even for the lower values of the time variable, four converged digits are achieved for truncation orders. As expected, during the drying process, was observed that at positions close to the center of the body, a greater number of terms to achieve a converged solution to the four digits were required. In the worst case, related to test 1 and t = 0.03 h, the moisture content reached a precision of four digits from N = 15 terms in the series.

t = 0.03 h				t = 0.250h			
N	Moisture Content (kg/kg)			N	Moisture Content (kg/kg)		
IN	z* = 0.1	z* = 0.5	z* = 0.9	IN	z* = 0.1	z* = 0.5	z* = 0.9
1	12.1007	8.6721	1.9432	1	10.7244	7.6868	1.7252
2	8.9739	11.1535	3.5364	2	9.6728	8.5213	2.2610
3	10.0791	10.0484	4.6415	3	9.7264	8.4678	2.3146
4	9.7549	9.5436	5.2776	4	9.7255	8.4664	2.3163
5	9.8028	9.7599	5.5797	5	9.7255	8.4665	2.3163
6	9.8214	9.8439	5.6971	6	9.7255	8.4665	2.3163
7	9.8027	9.8149	5.7337	7	9.7255	8.4665	2.3163
8	9.8116	9.8060	5.7426	8	9.7255	8.4665	2.3163
9	9.8086	9.8084	5.7441	9	9.7255	8.4665	2.3163
10	9.8094	9.8089	5.7443	10	9.7255	8.4665	2.3163
15	9.8093	9.8088	5.7442	15	9.7255	8.4665	2.3163
16	9.8093	9.8088	5.7442	16	9.7255	8.4665	2.3163
20	9.8093	9.8088	5.7442	20	9.7255	8.4665	2.3163

Table 1. Convergence behavior of the test 1 (45°C and 1.20 m/s)

Table 2. Convergence behavior of the test 2 (75°C and 1.20 m/s)

t = 0.03 h				t = 0.250h			
N	Moisture Content (kg/kg)			N	Moisture Content (kg/kg)		
IN	z* = 0.1	z* = 0.5	z* = 0.9	IN	z* = 0.1	z* = 0.5	z* = 0.9
1	11.7951	8.4457	1.8722	1	7.9422	5.6873	1.2619
2	9.6166	10.1746	2.9822	2	7.8803	5.7364	1.2934
3	10.008	9.7829	3.3738	3	7.8803	5.7364	1.2935
4	9.9665	9.7180	3.4556	4	7.8803	5.7364	1.2935
5	9.9681	9.7252	3.4657	5	7.8803	5.7364	1.2935
6	9.9683	9.7257	3.4664	6	7.8803	5.7364	1.2935
7	9.9682	9.7257	3.4664	7	7.8803	5.7364	1.2935
8	9.9682	9.7257	3.4664	8	7.8803	5.7364	1.2935
9	9.9682	9.7257	3.4664	9	7.8803	5.7364	1.2935
10	9.9682	9.7257	3.4664	10	7.8803	5.7364	1.2935
15	9.9682	9.7257	3.4664	15	7.8803	5.7364	1.2935
16	9.9682	9.7257	3.4664	16	7.8803	5.7364	1.2935
20	9.9682	9.7257	3.4664	20	7.8803	5.7364	1.2935

t = 0.03 h				t = 0.250h			
N	Moisture Content (kg/kg)			N	Moisture Content (kg/kg)		
IN	z* = 0.1	z* = 0.5	z* = 0.9	IN	z* = 0.1	z* = 0.5	z* = 0.9
1	10.2916	7.3755	1.6523	1	8.3397	5.9781	1.3431
2	7.9105	9.2651	2.8655	2	7.9829	6.2612	1.5249
3	8.5851	8.5905	3.5401	3	7.9863	6.2578	1.5284
4	8.4431	8.3693	3.8188	4	7.9863	6.2578	1.5284
5	8.4566	8.4302	3.9039	5	7.9863	6.2578	1.5284
6	8.4596	8.4438	3.9229	6	7.9863	6.2578	1.5284
7	8.4581	8.4414	3.9260	7	7.9863	6.2578	1.5284
8	8.4584	8.4410	3.9263	8	7.9863	6.2578	1.5284
9	8.4583	8.4411	3.9264	9	7.9863	6.2578	1.5284
10	8.4584	8.4411	3.9264	10	7.9863	6.2578	1.5284
15	8.4583	8.4411	3.9264	15	7.9863	6.2578	1.5284
16	8.4583	8.4411	3.9264	16	7.9863	6.2578	1.5284
20	8.4583	8.4411	3.9264	20	7.9863	6.2578	1.5284

T 11 2 C		6.1	(1500 1000 ())
Table 3. Convergence	e behavior c	of the test 3 ($(45^{\circ}C \text{ and } 2.30 \text{ m/s})$

To visualize the behavior of drying process, the 3D curves of moisture content distribution for each studied test are obtained from converged values and represented by Figs. (2-4), below showed. The curves indicate that the variation of moisture content is more significant in cases performed with highest values of temperature and air-drying velocity, however, they show the similar behavior. It is also possible to realize that drying process achieved with higher temperatures and air speeds provide minor equilibrium moisture content and the equilibrium condition is reached in a shorter period of time. Physically, this denotes that the mass transfer increases with increasing temperature and air-drying velocity.



Figure 2: Visualization in 3D the moisture content variation inside the body, X, for test 1, using Matlab, for N = 20



Figure 3: Visualization in 3D the moisture content variation inside the body, X, for test 2, using Matlab, for N = 20



Figure 3: Visualization in 3D the moisture content variation inside the body, X, for test 2, using Matlab, for N = 20

Figures (2-4) illustrate that the equilibrium moisture content is reached faster at the product surface, as expected. This can be explained by the fact that, in the beginning of the process, a significant part of the moisture content is free on the surface of the body and thus is easily removed. For others times of drying, the differences of moisture content between different positions increase, due to internal resistance to the transport of moisture. During this period, water

interacts with polar groups of the constituents' molecules. Therefore, the higher are the temperature and velocity of the drying air, the water is removed with greater ease.

4. CONCLUSIONS

In this work, the Generalized Integral Transform Technique (GITT) was successfully employed in a mass transfer problem. The results illustrating moisture content distribution were presented graphically for different operating conditions, varying temperature and drying air speed. Results obtained in this study show good agreement with literature ones, this modeling allows the moisture content profile determination within food products submitted to a drying process. The 3D curves represent the moisture content transient behavior which varies with position, indicating that equilibrium condition at surface is reached faster than at the center of the product. Analyzing the convergence of the series, during resolution procedure, only 15 terms were required to get a converged response, for the worst situation (low temperature and drying air velocity). A posterior research, already in progress, includes the modeling development taking into account simultaneous heat and mass transfer and also variable properties.

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