NUMERICAL SIMULATION OF THE BIOMASS CONCENTRATION OF MICROALGAE CULTIVATED IN A SELF-SUSTAINABLE PHOTOBIOREACTOR

Robert Luis Lara Ribeiro, robertlarabr@yahoo.com.br
Departamento de Engenharia Mecânica, Universidade Federal do Paraná, C. P. 19011, Curitiba, PR 81531-980, Brasil
André Bellin Mariano, E-mail: andrebmariano@gmail.com
Departamento de Engenharia Mecânica, Universidade Federal do Paraná, C. P. 19011, Curitiba, PR 81531-980, Brasil

Jeferson Avila Souza, E-mail: jasouza@furg.br
Escola de Engenharia, Universidade Federal de Rio Grande, Av. Italia, km 08 S/N, Rio Grande, RS, 96201-900 - Brasil

José Viriato Coelho Vargas, E-mail: jvargas@demec.ufpr.br
Departamento de Engenharia Mecânica, Universidade Federal do Paraná, C. P. 19011, Curitiba, PR 81531-980, Brasil

Juan Carlos Ordonez, E-mail: ordonez@caps.fsu.edu
Department of Mechanical Engineering and Center for Advanced Power Systems, Florida State University, Tallahassee, Florida, 32310, USA

Abstract. *Studies concerning microalgae cultivation are calling attention scientific research in several countries such as Brazil, United States, China, Italy, Spain and Iraq due to its high productivity of oil when compared with crops Microalgae can be used in many important applications such as to obtain compounds of interest in food, chemicals and pharmaceuticals. The cultivation of microalgae in photobioreactors is an effective way of producing microalgae biomass. Inside the bioreactor it is possible to control the conditions of cultivation inducing the production of higher concentrations of some products of interest in a particular enterprise, such as proteins, pigments, fatty acids and carbohydrates. In this paper, the microalgae growth is modeled based upon a mathematical relationship with the light intensity. The model is utilized in a novel self-sustaining compact photobioreactor with physicochemical parameters of the microalgae Phaeodactylum tricornutum. The numerical results are capable of predicting the biomass concentration in the photobioreactor in space and time with low computational time. As a result, the model is expected to be a useful tool for simulation, design, and optimization of compact photo-bioreactors. The model uses a mathematical relationship proposed by Grima et al., (1994), where the growth of biomass depends only on the light intensity that focuses on the photobioreactor pipes. This mathematical model is being applied to the design of a photobioreactor proposed by Vargas, (2007) using the microalgae Phaeodactylum tricornutum (Bacillariophyceae). The numerical solution of this computational model allows visualizing the concentration of biomass and the production volume. Several simulations were made with a light intensity of* $1620\mu E m^{-2} s^{-1}$ *on the surface of the pipe and with different rates of dilution, made in different local of concentration. The parameters that led to the best performance were: rate of dilution 0.04*h^{-1} *and point of dilution of 500* g/m^3

Keywords: *Numerical Simulation, Microalgae Growth, Light Intensity.*

1. INTRODUCTION

The microalgae are present in all aquatic systems where the incidence of sunlight occurs. This happens because the light is a factor of great importance for its growth. Because of its high biodiversity there are many characteristics that are attributed to the microalgae. In some cases, like the *Phaeodactylum tricornutum* (Bacillariophyceae), the main characteristic is the high density of lipid in its structure (Xu, 2006). Due to the fact that microalgae are capable of producing more tons of vegetable oil per hectare per year than any oleaginous (Pérez, 2007) a large amount of research has been concentrated on it. Many companies have been investing time and money on the algae technology, allowing the researchers to improve the way in this culture growth, increasing the production in large scales. The oil produced with the algae biomass can be used in different ways. In the present work, a photobioareactor has been designed to work on a plant where the produced oil will be processed into biodiesel and consumed in an internal combustion generator. Part of the energy produced will be used to supply the plant needs characterizing in this way, a self sustainable unit.

The development of a mathematical model to correctly describe (detailed) the growth of microalgal and its consequent numerical solution is of great importance for the development of this area. This model should take into consideration the design and engineering of a self-sustainable photobioreactor.

In the present work, the mathematical model to be implemented computationally is based on the model presented by Grima *et al.* (1994). The model is applied to a tubular photobioreactor of continuous cultivation, where the growth of microorganisms is a function of the average light intensity inside the pipe. When physical parameters like pH and temperature are not limiting the growth of biomass, light intensity becomes the most important factor.

Richmond (1992) noted that the availability of light for each cell in a photoautotrophic culture is a function of the intensity and duration of the incident light and the concentration of cells, or population density, which affects the process of mutual growth through shading. Serenotti et al. (2004) comment that under appropriate conditions of light, cells can store energy and produce intermediate products (such as ATP) that are used for fixation of carbon dioxide and the biomass synthesis.

The numerical solution will be able to determine in which stage of the concentration the system should activate or disable the dilution rate in the culture. With this, a good production of biomass will possibly be obtained.

The graphics of growth rate, average light intensity within the photobioreactor pipes and biomass concentration during the five days of cultivation were analyzed after the end of each simulation. Every simulation of 120 hours of cultivation was performed with a different dilution rate. For each one of them, many simulations were performed for different average dilution concentration points. From the large data set generated through simulation, the combination of light intensity and dilution leading to the largest microalgae production were selected.

2. MATHEMATICAL MODEL

Several mathematical models of photobioreactors based in the scheme of light decay can be found in literature. The general problem of photobioreactor design considering light attenuation is extensively discussed by Bernardez et al. (1987). Several mathematical descriptions of photobioreactors had taken into consideration the distribution of light in the volume of the culture, either using an averaged value of the illuminance, or averaging the growth rate (Dermoun, 1992; Evers, 1991; Fernández, 1997; Frohlich, 1983; Grima, 1994).

In the present work, the mathematical model used to formulate the concentration growth rate of the biomass culture is given by Grima et al. (1994).

$$\frac{dC}{dt} = C(\mu - m - D) \tag{1}$$

where C represents the concentration of biomass$[m^2/g]$, μ the specific growth rate $[h^{-1}]$, m the maintenance rate $[h^{-1}]$ and D the dilution rate $[h^{-1}]$. This mathematical model was designed for a continuous tubular photobioreactor.

The specific growth rate of biomass (μ) is modeled with Eq. (2) (Grima, 1994). In this equation, the average light intensity within the photobioreactor, is modeled as an independent variable. The light availability inside the culture is determined by the solar irradiance, the design of the reactor, the biomass concentration, and the pigment content in the culture which leads to self-shading (Grima, 1994).

$$\mu = \frac{\mu_{MAX} I^n}{I^n + I_k^n} \tag{2}$$

It may be noticed that the specific growth rate (μ) uses two constants: μ_{MAX} which represents the maximum growth rate and I_k, that represents the affinity of the algae with the light. The average light intensity within the photobioreactor pipe is represented by I. The average light intensity is given as a function of concentration in Eqs. (3) and Eq. (4).

$$I(C) = \frac{I_0}{\pi R} \int_0^R \int_0^\pi e^{-CK_a\left((R-S)\cos\phi + \sqrt{R^2-(R-S)^2 \sin^2\phi}\right)}\, d\phi\, dS \tag{3}$$

$$K_a = Y_p^{'}(1{,}12.10^{-2} - 8{,}6.10^{-6}C + 1{,}6.10^{-6}C^2) + Y_b \tag{4}$$

where K_a means the biomass absorption coefficient $[m^2 g^{-1} biomass]$, R the radius $[m]$, S is the distance from vessel surface to an internal point $[m]$ and φ is the angle of incidence of the path of light.

3. NUMERICAL SIMULATION

The computational model uses data related to the microalgae *Phaeodactylum Tricornutum* withdrawn from Grima et al. (2001), Fernández et al. (2000) and the photobioreactor parameters proposed by Vargas (2007). These are summarized in Table-1.

Table 1. Constants used in numerical simulation.

$C_0 = 200\ g/m^3$	$Y_b = 0.0105 m^2/g$
$t_{final} = 120\ h(5\ days)$	$Y'_p = 2.99 m^2/g$
$m = 0.00385 h^{-1}$	$I_k = 120 \mu E m^{-2} s^{-1}$
$I_0 = 1620 \mu E m^{-2} s^{-1}$	$D_i = 0 h^{-1}$
$\mu_{MAX} = 0.075 h^{-1}$	$n = 2.02$

A Fortran 95 code was developed for the computational implementation of the mathematical model represented by Eqs (1-4), while the Gnuplot [*http://www.gnuplot.info*] application is used for an interactive graphical plot of results. The Gnuplot is called from inside the main Fortran program.

The current computational model simulates day light cultivation cycle with 12 hours of light and 12 hours in the dark. During the 12 hours of darkness there is only the loss of biomass caused by the rate of maintenance.

The dilution rate is an important operating variable in continuous and semi-continuous cultures of microorganisms. When the dilution rate balances with the specific cell growth rate, a stationary state can be achieved and the cell concentration is theoretically maintained at a constant value. However, in some specific occasions, it is necessary to have a stationary concentration value where the dilution rate is such that it is possible to economize labor and nutrients.

The dilution rate is responsible for the culture renewal, for example, using a dilution rate of $0.1h^{-1}$ means that every hour 10% of the total culture volume is removed from the reactor. Simultaneously, the same amount of water rich with the necessary nutrients for the culture growth are entering the reactor. The adequate determination of the dilution rate can provide a significant savings in the water and used nutrients.

In this computational model a semi-continuous culture is simulated numerically. The cell concentration is kept between two, reference points: the dilution point and the recovery point. The dilution point sets the moment when dilution is initialized, while the recovery point will mark the end of dilution process, i. e., the moment when the cell concentration starts to increase again. This interval was chosen in order to guarantee a high growth rate. A third parameter, the midpoint concentration, is the concentration rate calculated as the arithmetic average between the concentration values at the dilution and recovery points. The overall process can be summarized as: the cell concentration grows up to the dilution point. At this moment, the dilution starts and the system starts to be renovated. This will occur until the moment that the concentration reaches the recovery point and the dilution is interrupted.

Several simulations varying the dilution rate and the midpoint concentration were performed. The dilution point and the recovery point were set as 20 g/m³ above and below the midpoint concentration, respectively. Thus, the overall range interval of the dilution rate for each chosen midpoint concentration is 40 g/m³. Simulations were performed for the following dilution rates: $0.01h^{-1}$, $0.025h^{-1}$, $0.04h^{-1}$, $0.08h^{-1}$, $0.1h^{-1}$, $0.4h^{-1}$, $0.9h^{-1}$ and midpoint concentration: $205g/m^3$, $250g/m^3$, $300g/m^3$, $350g/m^3$, $400g/m^3$, $425g/m^3$, $450g/m^3$, $475g/m^3$, $500g/m^3$, $550g/m^3$, $567g/m^3$, $575g/m^3$, $600g/m^3$, $650g/m^3$, $700g/m^3$, $800g/m^3$, $900g/m^3$, $950g/m^3$.

Equation (1),which predicts the evolution if the concentration in time is a first order ordinary differential equation and was solved with a 4th order Runge-Kutta. Equation (3), which is a double integral, was approximated with Riemann sum algorithm, which is an approximation of the actual volume by the volume of "n" columns, i. e., Eq. (5) is a approximation of the real value given by Eq. (3).

$$I'(C) = \frac{I_0}{\pi R} \sum_{i=1}^{m} \sum_{j=1}^{n} e^{-CK_a\left((R-S_i)\cos\phi_j + \sqrt{R^2-(R-S_i)^2 \sin^2 \phi_j}\right)} \Delta\phi \Delta S \quad (5)$$

where $m = \frac{R}{\Delta S}$ and $n = \frac{\pi}{\Delta \phi}$.

4. RESULTS AND DISCUSSIONS

Figures 1 and 2, illustrate the numerical results for the in batch microalgae cultivation. The 5 days cultivation generated a total production of 1100.4 g/m^3. In Figure 3 the productivity during the 120 hours of the process is shown.

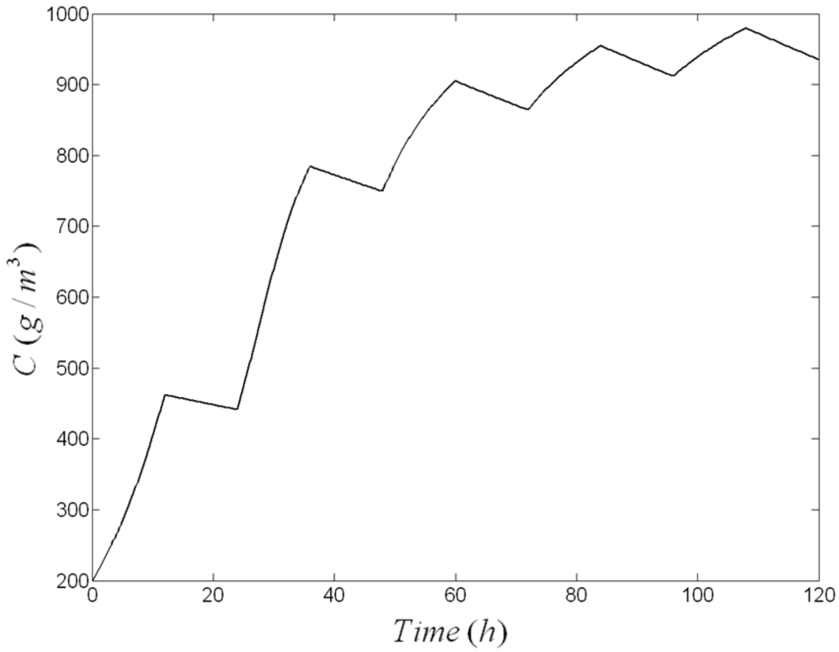

Figure 1. Concentration of biomass versus simulation time of a cultivation in batch.

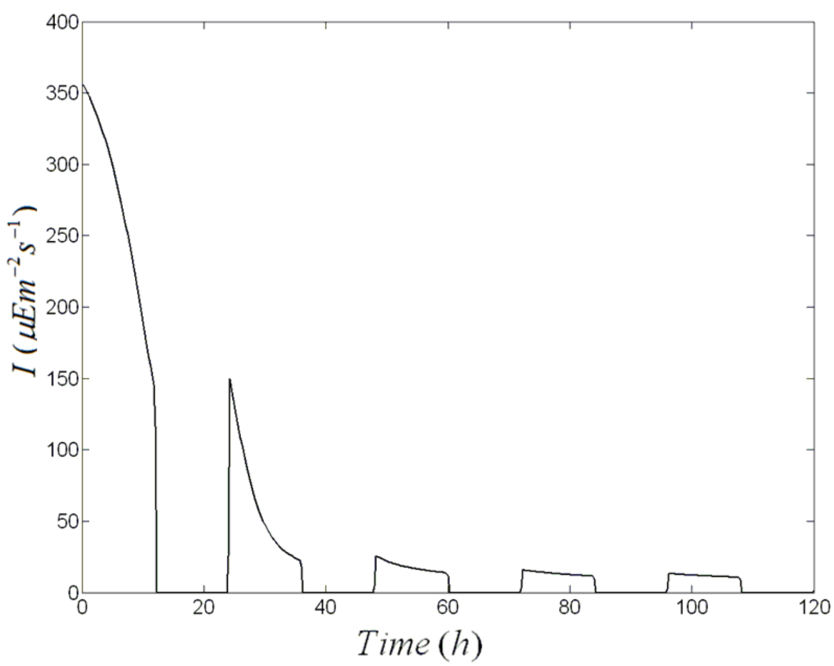

Figure 2. Average light intensity within the pipes of the photobioreactor versus simulation time of a cultivation in batch.

The 12 hours cycles of darkness (night), can be observed in Fig. 1. In these periods, the concentration rate is negative and a small decrease in the microalgae concentration occurs. With the absence of sunlight, the system keeps working only with the maintenance rate. In this period, there is no microalgae growth, but only the subtract consumption due to the maintenance of the microorganisms vital functions.

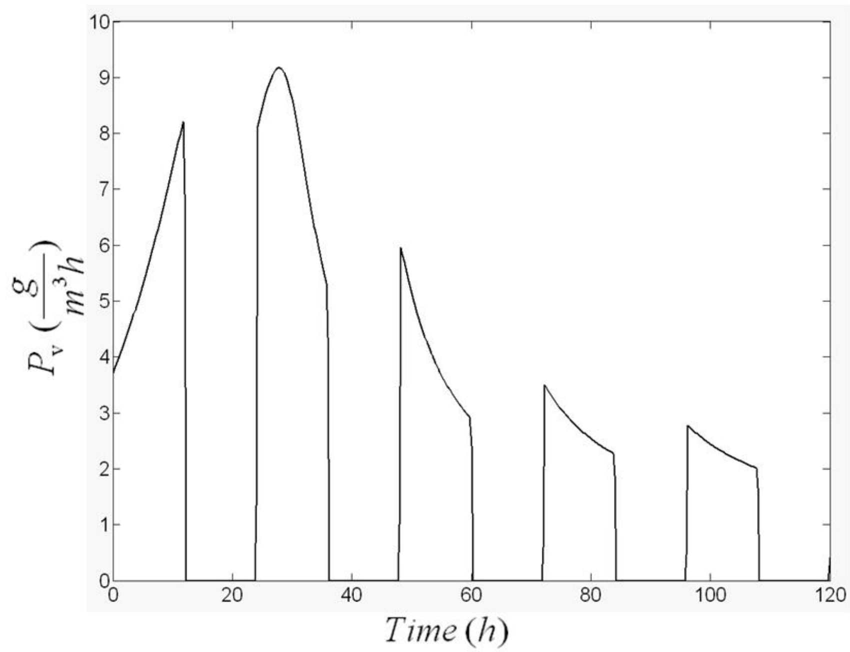

Figure 3. Volumetric production versus simulation time of a in batch cultivation.

During the 12 hours of darkness the total production is zero. It is due to the fact that without the solar irradiation there is no growth of biomass.

The results of all simulations of semi-continuous culture are summarized in Table 2.

Table 2. Volumetric production during the 5 days of semi-continuous culture, varying the dilution rates and the midpoint concentration.

Dilution rates / **Concentrations**	**0.01**h^{-1}	**0.025**h^{-1}	**0.04**h^{-1}	**0.08**h^{-1}	**0.1**h^{-1}	**0.4**h^{-1}	**0.9**h^{-1}
205 g/m^3	1558,8	1737,1	1127,7	964,1	922,2	903,2	867,4
250 g/m^3	**1560**	1797,9	1334,6	1156,3	1124,5	1085,8	1100,2
300 g/m^3	1559,8	1840,4	1554,1	1343,8	1327,2	1317,4	1288,5
350 g/m^3	1558,1	1921,8	1744,8	1528,9	1513,7	1485	1465,3
400 g/m^3	1555,4	1952,3	1894	1694,9	1691,8	1655,1	1634,1
425 g/m^3	1544	**1957**	1948,7	1775,5	1773,2	1730,8	1706,8
450 g/m^3	1519,1	1955,7	1992,3	1837,9	1849,2	1807,3	1779,1
475 g/m^3	1516,5	1953,2	2007,2	1918,1	1916,3	1872,1	1845,7
500 g/m^3	1513,7	1950	**2010,2**	1958,3	1966,7	1936,7	1906,9
550 g/m^3	1507,6	1885,4	1983	2028,6	2025,2	2025,2	1990,7
567 g/m^3	1505,3	1861	1966,2	**2033,4**	**2033,2**	**2033,1**	2003,6
575 g/m^3	1504,2	1849,1	1953,2	2031,8	2031,6	2031,6	2008,6
600 g/m^3	1500,6	1807,3	1919,5	2010	2011,5	2013,4	**2018,4**
650 g/m^3	1493,1	1710,7	1827,3	1906,6	1906,1	1919	1970,4
700 g/m^3	1453,7	1605	1714,9	1732,2	1736,7	1760,2	1860,1
800 g/m^3	1280,8	1367,7	1373,6	1375	1374,3	1399,9	1522,9
900 g/m^3	1141,3	1149,5	1146	1157,5	1154,9	1172,2	1222,2
950 g/m^3	1101,7	1103	1104,1	1105	1104,4	1109,7	1125,2

A graphical representation of Table 2 is shown in Fig. 4

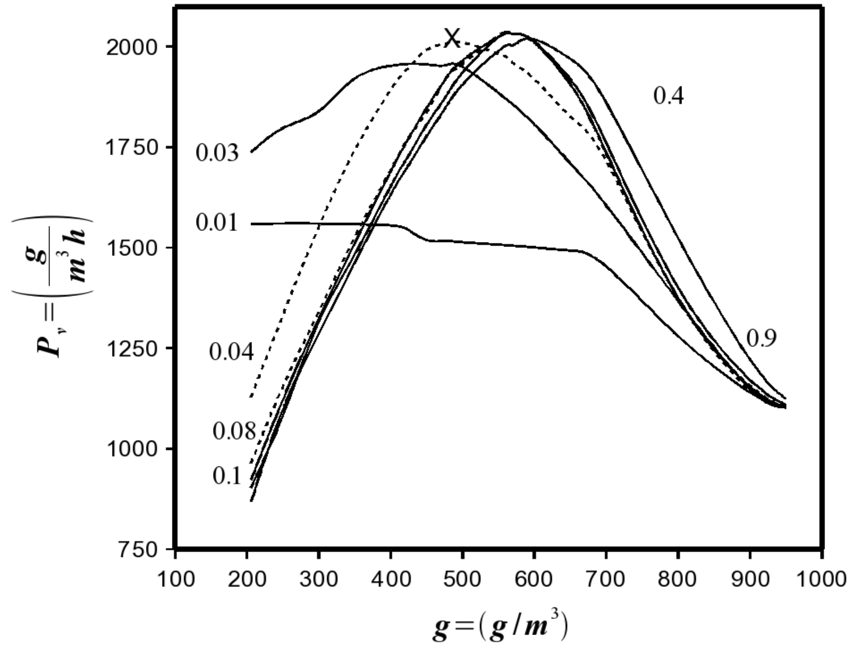

Figure 4. Biomass production versus midpoint concentration for Dilution rates varying from 0.01 to 0.9

The results in Table 2 refer to the total volumetric productions obtained by the several simulations performed. These runs generated the data set from which the best midpoint concentration and the best dilution rate values were selected. In bold in Table 2, it can be seen the total volumetric production for each maximum dilution rate used in the simulations.

In Figure 4, it can be seen that there is an optimum operating condition, where the biomass production is maximized as a function of the biomass midpoint concentration and the dilution rate. Another important conclusion that can be obtained with the analysis of Fig. 4 is that, if economical aspects were taken into account, a good choice for the unit operating condition would be the $0.04h^{-1}$dilution rate with a midpoint concentration of $500\, g/m^3$. This point is highlighted with an X on Fig. 4. At this point, the biomass production is only 1.14% smaller than the maximum obtained with the dilution rate of $0.08h^{-1}$and concentration of $567\, g/m^3$. The 1.14% increase in the biomass production will not justify the increase in the production cost. With the $0.08h^{-1}$dilution rate, 8% of the total crop will be renewed each hour while with the $0.04h^{-1}$ dilution rate, only 4% of the total crop will be renewed. These numbers mean that the $0.04h^{-1}$ dilution rate will require only half of the water and nutrients and a smaller pumping capacity than the $0.08h^{-1}$dilution rate.

In Figures 5, 6, 7 and 8 the results for the biomass concentration, total volumetric production, instantaneous growth rate, and average light intensity within the photobioreactor pipes are presented. The operating conditions used in these figures were: dilution rate of $0.04h^{-1}$ and biomass concentration for $500\, g/m^3$.

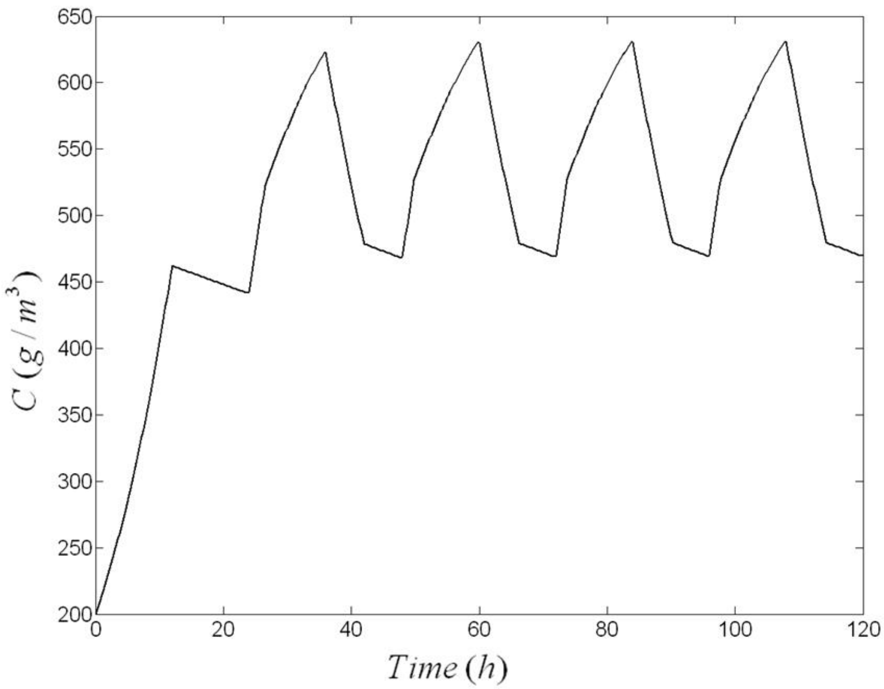

Figure 5. Concentration of biomass in relation to time: dilution rate of $0.04h^{-1}$ and midpoint concentration of $500\ g/m^3$.

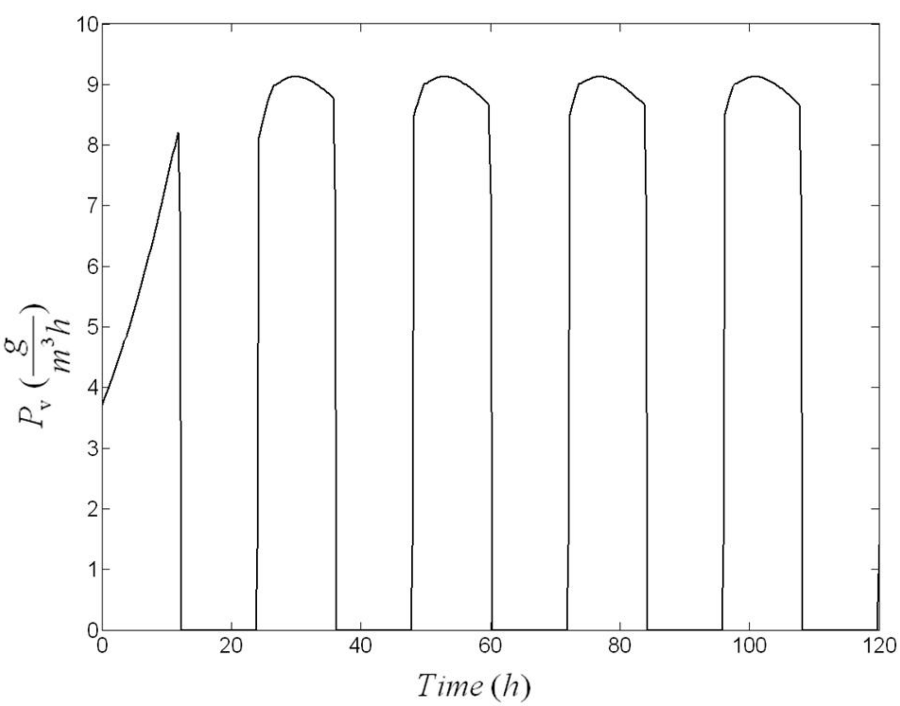

Figure 6. Volumetric production in relation to time: dilution rate of $0.04h^{-1}$ and midpoint concentration of $500\ g/m^3$.

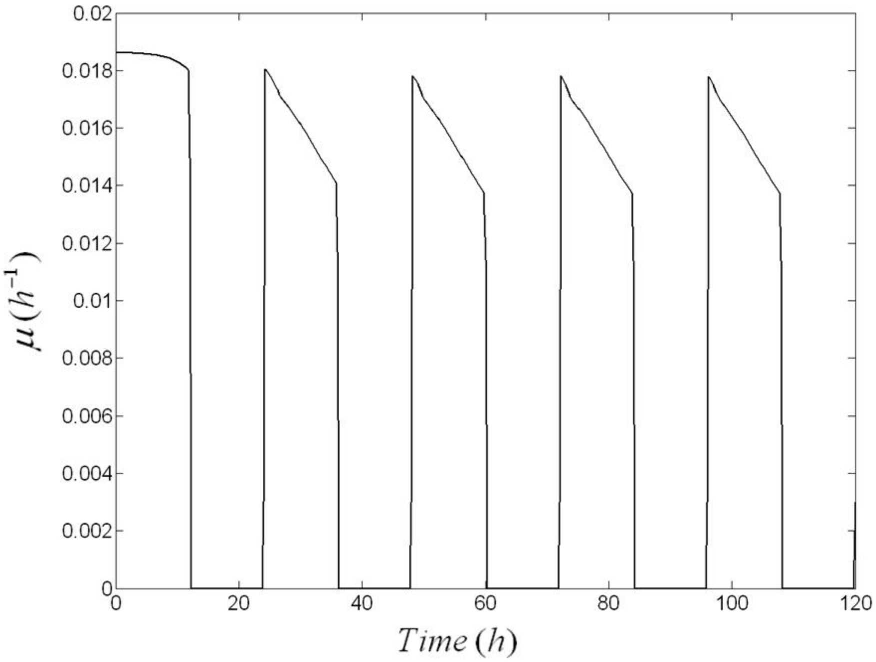

Figure 7. Instantaneous rate of growth in relation to time: dilution rate of $0.04h^{-1}$ and midpoint concentration of $500\ g/m^3$.

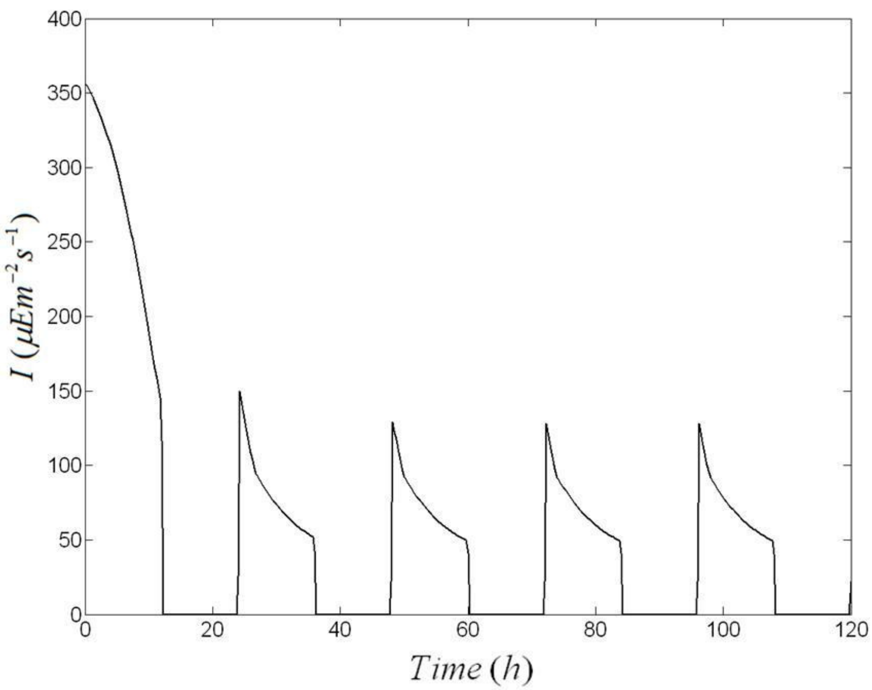

Figure 8. Average light intensity within the tubes of photobioreactor in relation to time: dilution rate of $0.04h^{-1}$ and midpoint concentration of $500\ g/m^3$.

5. CONCLUSION

This work presented the numerical modeling of a novel self-sustainable microalgae photobioreactor. A mathematical model (Grima, 1994) which formulates the microalgae growth as a function (among others) of the light intensity was used to optimize the biomass production in a batch microalgae cultivation reactor. The model was adapted to be used with a reactor currently under design in our group (Vargas, 2007). Results showed that the operating conditions can be optimized in such a way that the resulting set of operating variables would keep high levels of biomass production (1.14 % below the possible maximum) with a significant reduction in the dilution volumetric flow rate (about 50%), reducing in this way, the unit operating costs.

6. ACKNOWLEDGMENT

The authors gratefully acknowledge support from CNPq, NILKO, FURG, UFPR, CAPS and IESES.

7. REFERENCES

Bernardez E. R., Claria M.A. and Cassano A.E. 1987. Analysis and design of photoreactors. In: Carberry J.J. and Varma A. (eds), Chemical Reaction and Reactor Design. Marcel Dekker, NY. USA.

Dermoun D., Chaumont D., Thebault j. and Dauta A. 1992. Modeling of growth of Porphyridium cruentum in connection with two interdependent factors: light and temperature. Biore. Technol. vol. 42. pp. 113-117.

Evers E.G. 1991. A model for light limited continuous cultures. Biotechnol. Bioengng. Vol. 38. pp. 254-259.

Fernández, F.G.A., Pérez, J.A.P., Sevilla, J.M.F., Camacho, F.G. and Grima, E.M. 2000. Modelintog of Eicosapentaenoic Acid (EPA) Production from Phaeodactylum tricornutum Cultures in Tubular Photobioreators. Effects of Dilution Rate, Tube Diameter, and Solar Irradiance. Biotechnology and Bioengineering. Vol. 68, number 2, April 20.

Fernández, F.G.A., Camacho, F.G., Pérez, J.A.P., Sevilla, J.M.F. and Grima, E.M.E. 1997. A Model for Light Distribution and Average Solar Irradiance Inside Outdoor Tubular Photobioreactors for the Microalgal Mass Culture. Biotechnology and Bioengineering. Vol. 55, number 5, September 5.

Frohlich B.T., Webster I.A., Ataai M.M. and Shuler M.I. 1983. Photobioreactors: Models for interaction of light intensity reactor design and algal physiology. Biotech. Bioengng. Symp. 13: 331-350.

Grima, E.M., Camacho, F.G., Perez, J.A.S., Sevilla, J.M.F., Fernandez, F.G.A. and Contreras Gomez, A. 1994. A mathematical model of microalgal growth in light-limited chemostat culture. J. Chem. Tech. Biotechnol, vol. 61, pp. 167–173.

Pérez, H. E. B., 2007. Biodiesel de Microalgas – Parte 1. Energia Verde - Biodiesel, MDL e Tecnologia em Microalgas.

Richmond, A. 1992. Mass culture of cyanobacterium. In MAN, N.H. & CARR, N.G. (eds) Photosynthetic Prokaryotes, Plenum Press, New York.

Serenotti, F., Crespi, B.A. and Torres, L. 2004. Contribuição à modelagem da produção de Spirulina máxima em Fotobioreatores. Rev. Univ. Rural Sér. Ci. Exatas e da Terra Seropédica, RJ. EDUR. vol. 23. pp. 08-17.

Vargas, J.V.C. Núcleo de pesquisa e desenvolvimento de energia auto-sustentável a partir do biodiesel e outras fontes. Projeto financiado pelo Conselho Nacional de Desenvolvimento Científico e Tecnológico do Edital 039/2007, UFPR.

Xu, H., Miao, X. and Wu, Q. 2006. High quality biodiesel production from a microalga *Chlorella protothecoides* by heterotrophic growth in fermenters.Journal of biotechnology. Journal of Biotechnology, vol. 126, pp. 499-507.

NOMENCLATURE

C	$Biomass\ concentration\ (\ gm^{-3})$
μ	$Specific\ growth\ rate\ (h^{-1})$
m	$Specific\ maintenance\ rate\ (h^{-1})$
D	$Dilution\ rate\ (h^{-1})$
μ_{MAX}	$Maximum\ specific\ growth\ rate\ (h^{-1})$
$I, I(C)$	$Mean\ light\ intensity\ (\mu Em^{-2}s^{-1})$
R	$Vessel\ radius\ (m)$
S	$Distance\ from\ vessel\ surface\ to\ an\ internal\ point\ (m)$
ϕ	$Angle\ of\ incidence\ of\ the\ light\ path$
I_0	$Incident\ light\ intensity\ on\ culture\ surface\ (\mu Em^{-2}s^{-1})$
C_0	$Initial\ Biomass\ concentration\ (gl^{-1})$
I_k	$Constant\ representing\ the\ affinity\ of\ algae\ to\ light\ (\mu Em^{-2}s^{-1})$
K_a	$Biomass\ absorption\ coefficient\ (m^2g^{-1})$
Y_b	$Absorption\ coefficient\ normalized\ to\ pigment-free\ biomass\ (m^2g^{-1})$
Y_p'	$Absorption\ coefficient\ normalized\ to\ total\ pigment\ content\ (m^2g^{-1})$