# WAVELET ANALYSIS OF THE BOUNDARY LAYER DETACHMENT IN A CURVED DUCT FLOW

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Abstract. The present work analyzes the effect of the boundary layer separation in a curved duct of rectangular section which is part of a boiler in a thermal power plant. The test section is a scale model of the duct, that connects the two economizer heat exchangers. Visualization methods and hot wire anemometry were applied. The data were analyzed using Fourier and wavelet transforms. The visualization of the flow put in evidence the boundary layer separation at the wall region near the strongest convexity and suggests a transient detachment and reattachment of the flow. Fourier analysis is unable to detect the phenomena, due to its bulk approach. The wavelet transform, conversely, made a time frequency analysis, more suitable for transient phenomena. Indeed, the wavelet spectrum show energy peaks near 20 Hz, denoting the passage of a large vortex through the probe region. The phenomenon occurs at a time interval of 10 seconds observed also in the visualizations

Keywords: Turbulent flow, Hot wires, Wavelets, Fourier transform

# 1. INTRODUCTION

The phenomenon of boundary layer separation in curved ducts, may cause structural and maintenance problems in heat exchangers of thermo-electric power plants (Indrusiak, 1997). The turbulent flow impinging on the bank of tubes of the energy saver heat exchanger (economizer) may produce unwanted vibration of the tubes leading to a possible break. Furthermore, particles with high kinetic energy carried by the flow can produce erosion of the tubes and the subsequent shutdown of the boiler. For these reasons is important to know which are the frequency, intensity and energy of the vortex caused by the detachment of the flow.

The main parameter for understanding the phenomenon that occurs in the flow is the velocity and its behavior along time. The mean velocity can be determined if the flow rate and the duct cross section are known. The measurement of the velocity with hot wire anemometers, which gives the instantaneous velocity in selected flow positions, allows the analysis of the velocity fluctuations along time.

The usual method of analysis of random data as velocity in turbulent flows is the Fourier analysis. Nevertheless, Fourier analysis fails when dealing with transient phenomena and the use of wavelet analysis is more suitable, Indrusiak ET. AL., 2005. The Fourier spectrum can show the energy content of the phenomena in frequency domain but the information related to the time in which they occur is lost. To obtain higher resolution on the analysis is used the wavelet spectrum, which maintains the time information content of the signal.

### 2. FUNDAMENTALS

### 2.1. Fourier and Wavelet Transforms

The Fourier transform of a discrete time series enables the study of the bulk spectral behavior of the random phenomenon represented by the series. For a finite function x(t), given as a discrete time series, is defined as:

$$\hat{x}(f) = \frac{1}{2\pi} \sum_{0}^{T} x(t) e^{-ift}$$
(1)

The Fourier spectrum gives the energy distribution of the signal in the frequency domain evaluated over the entire time interval:

$$\mathbf{P}_{xx}\left(\mathbf{f}\right) = \left|\hat{\mathbf{x}}\left(\mathbf{f}\right)\right|^{2} \tag{2}$$

In practice, in order to minimize the random error, the power spectral density function (PSD) is used, which is the Fourier spectrum of the series smoothed over frequency intervals and over an ensemble of estimates (Bendat and Piersol, 1971).

While the Fourier transform uses trigonometric functions as basis, the bases of wavelet transforms are functions named wavelets. A wavelet is a finite energy function,  $\psi$  (t), with a zero average that generates an entire set of wavelet basis:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right)$$
(3)

The parameters *a* and *b* are respectively scale and position coefficients  $(a, b \in \Re)$  and a > 0. The continuous wavelet transform of a function x(t) is given by:

$$\widetilde{X}(a,b) = \int_{-\infty}^{\infty} x(t) \psi_{a,b}(t) dt$$
(4)

The respective wavelet spectrum is defined as:

$$\mathbf{P}_{xx}(\mathbf{a},\mathbf{b}) = \left| \widetilde{\mathbf{X}}(\mathbf{a},\mathbf{b}) \right|^2 \tag{5}$$

In the wavelet spectrum, Equation (5), the energy is related to each time and scale (or frequency) (Daubechies, 1992). This characteristic of the wavelet transform allows the representation of the distribution of the energy of the transient signal over time and frequency domains, this representation is called spectrogram.

According to Percival and Walden (2000), the discrete wavelet transform (DWT) is a judicious sub sampling of the continuous wavelet transform (CWT), dealing with dyadic scales, and given by:

$$d(j,k) = \sum_{i} x(t) \psi_{j,k}(t)$$
(6)

where the scale and position coefficients  $(j, k \in I)$  are dyadic sub samples of (a, b).

The Fourier transform of a finite series gives only a finite number of coefficients, depending on the length of the time series, and therefore neglects the coefficients related to the higher frequencies, which are previously filtered at the acquisition process, to prevent aliasing. In the wavelet transform of a finite series, the length of the series also restricts the number of computable coefficients but, unlike the Fourier transform, the remaining coefficients are related to the lower frequencies, including the mean value of the signal, and cannot be disregarded. In practice, the DWT of a series with more than 2J elements is computed for  $1 \le j \le J$ , being J a convenient arbitrary choice. The remaining part of the signal, containing the mean values for a scale J, is given by:

$$c(J,k) = \sum x(t)\phi_{J,k}(t)$$
<sup>(7)</sup>

where  $\phi(t)$  is the scaling function associated to the wavelet function.

Any discrete time series with sampling frequency Fs can be represented by:

$$x(t) = \sum_{k} c(J,k)\phi_{J,k}(t) + \sum_{j \le J} \sum_{k} d(j,k)\psi_{j,k}(t)$$
(8)

where the first term is the approximation of the signal at the scale J, which corresponds to the frequency interval [0, Fs/2J+1] and the inner summation of the second term are details of the signal at the scales j ( $1 \le j \le J$ ), which corresponds to frequency intervals [Fs/2j+1, Fs/2j].

### 2.2. Hot Wire Anemometry

According to Möller (2004), hot wire anemometers measure the fluid velocity through the variation of the heat transfer in a thin sensor exposed to a flow. Its small size and its fast response in frequency, make the hot wire anemometers very adequate to the study of turbulent flows.

The hot wire anemometer used in this work is the straight type, which measures only one component of the velocity. The hot wire probe is coupled to a Wheatstone bridge in such a way that the temperature of the wire is kept constant. The air flow cools the wire, thus unbalancing the Wheatstone bridge and producing an electric response which is read by the acquisition system coupled to the computer.

#### 3. CASE STUDY

The case study in this work is related to a curved duct in the flue gases exit of a boiler before reaching the energy saver heat exchanger. This duct presents a strong curvature, where an adverse pressure gradient is formed, leading to boundary-layer separation with flow reversion, forming a large recirculation vortex. In Fig. 1, a schematic view of the duct and the heat exchanger is presented.



Figure 1 - Section studied in this work.

In order to investigate the phenomenon, ten hot wire measurements were performed, departing from the convex wall to a position 156 mm downstream this location. The distances of the measurement points to the convex duct wall were 15 mm, 26 mm, 36 mm, 46 mm, 56 mm, 76 mm, 96 mm, 116 mm, 136 mm and 156 mm. Two acquisition frequencies were used: 1 kHz and 3 kHz, each with 65536 points and a low pass filter of one third of the acquisition frequency.

The equipment used in measurements is listed in table 1.

Hot wire calibration provides the relation between the flow velocity and the voltage read in the computer, Fig. 3.

Table 1 - Equipment used for measurements.

According to Idelchik (1986) and Kim et al. (1994), in the curve inlet there is an increasing of the pressure of the flow near to the concave wall of the duct and a corresponding decreasing of the pressure in the convex wall. Conversely, in the curve outlet (the vortex formation zone, shown in Fig. 1), the pressure near to the concave wall decreases and in the convex wall there is a strong increase of the pressure. This generates an inverse pressure gradient, which is a proper condition for detachment of the boundary layer, and subsequent vortex formation.

In a scale model of the duct, a flow visualization experiment was made using the classic tuft method. Wool wires where attached to the back wall. The air flow is driven by a blower in suction operation, Fig. 3.

The test section used is geometrically similar, in scale (15.8:1), of one of the boilers of an actual coal thermal power plant in the State of Rio Grande do Sul, Brazil. The inlet cross section is of 265.8 x 273.4 mm and the outlet cross section is of 379.7 x 273.4 mm. In the Fig. 2 is possible to see that the test section is equiped with a venturi for mass flow measurements, with two piezometric rings for pressure measurement, and a flow rate regulator, which was not used in this experiment, leaving the mass flow rate as maximum and calculated as 0.655 kg/s of air.

With the flow rate the inlet mean velocity, equal to 7.5 m/s, and the outlet mean velocity, equal to 5.3 m/s were determined.



Figure 2 - Response curve of the anemometer.



Figure 3 - (a) View of the Test Section (b) Schematic representation of the complete test apparatus.

### 4. ANALYSIS AND RESULTS

The mean velocity values obtained at the measurement points ranges from 4.81 m/s to 5.99 m/s, being consistent with mean velocity of the section of 5.3 m/s in the outlet.

Figure 3 shows record of the instantaneous velocity of the flow for the position of 46 mm from the wall at the acquisition frequency of 3 kHz.

In the tuft method flow visualization it is possible to see the reversion of the flow which happens in the zone of recirculation, this phenomenon characterizes the boundary layer separation. However this reversion seems to be intermittent, without a defined frequency.

For the same position and acquisition frequency, the Fourier and the wavelet spectra are shown in Fig. 5 (a) and (b) respectively.

In Fig. 5 (b) it is possible to see peaks of energy in a time-frequency space, having two significant energy peaks, one with a frequency of about 20 Hz at 5 seconds, and other with the same frequency at 15 seconds. These peaks are clearly distinguishable in relation to other peaks nearby in the figure.

Figure 6 shows Fourier and wavelet velocity spectra for the position of 15 mm away from the wall with an acquisition frequency of 1 kHz. Figure 6(a) shows again the typical turbulent pattern, without any significant peak of energy; and in Fig. 6(b) is possible to see again peaks between 3 and 20 Hz, being the most significant the one who appears between 40 and 50 seconds.

In Fig. 7(a) it is possible to see the Fourier spectrum, where again the typical turbulent flow behavior appears, but in this case, there is a little peak near 30 Hz. In the figure (b) the wavelet analysis shows several peaks between 3 and 25 Hz being the most important near 15 Hz and 5 seconds. Other peaks are visible but no apparent pattern is detected.



Figure 4 - Velocity for the position of 46 mm away of the wall.



Figure 5 - Fourier spectrum (a) and wavelet spectrum (b) in 46 mm and acquisition frequency of 3 kHz.



Figure 6 - Fourier analysis (a) and wavelet analysis (b) for 15 mm and 1 kHz



Figure 7 - Fourier (a) and wavelet (b) spectra of the velocity signal acquired 136 mm away from the wall with acquisition frequency of 1 kHz.

Error analysis was performed for calibration errors, measurement errors and statistical errors.

The calibration error is equal to 4.98%, calculated using a velocity relation. The second error type was approximate to the most unreliable measure instrument, a water column manometer, which presents an uncertainty of 1 mm of water column. And at last the statistical errors were calculated according to Bendat e Piersol (1971), where the square error has an inverse relation with the product of the band width and the acquisition time, in this case the error for 65.536 seconds of acquisition and a frequency of 1 kHz is 17.77% and for 3 kHz is 17.68%.

### 5. CONCLUDING REMARKS

The purpose of this work was to detect the presence of large scale turbulence structures and their downstream evolution in a curved duct model of coal power plant. The Fourier spectra showed the usual turbulent behavior, but no special features were detected.

In the wavelet analysis, peaks of energy where detected but a cyclical pattern was not observed. Instead it was observed that there were vortices who absorbed energy and in some instant went forward carried by the flow, leaving space for the formation of a new vortex. The phenomenon is evidenced by the peaks in all wavelet spectra presented in this work. The energy carried out by the flow is significant in those peaks. The periodicity of the phenomenon, if existent, is perceptible neither in the Fourier analysis nor in the wavelet analysis.

The lack of periodicity in the vortex detachment process could be detected only through wavelet analysis, more adequate than Fourier transform to the analysis of non stationary phenomena.

The continuation of this study with simultaneous velocity measurements using two hot wire probes or velocity and pressure fluctuations by means of hot wire and pressure transducers may give additional information for better understanding of the phenomena involved and for the determination of the periodicity characteristics of the vortex detachment.

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