NUMERICAL MODELING OF WATER WAVE IMPACT ON RESERVOIRS

Paolo Contim Nicolato
Roger Matsumoto Moreira
[1]Laboratório de Dinâmica dos Fluidos Computacional (LabCFD). Escola de Engenharia, Universidade Federal Fluminense. R. Passos da Pátria 156, bl.D, sl.563A, Niterói, R.J., Brasil. CEP: 24210-240.

Abstract. *When a partially filled reservoir is submitted to external exciting forces, the phenomenon known as wave sloshing may occur, with a high pressure impulse being measured at its walls. This paper aims to model numerically the generation and propagation of free surface waves and examine the influence of the vessel's geometry on water wave impact. The numerical model is based on a finite volume method with a homogeneous multiphase condition and solves the full Navier-Stokes and mass conservation equations for 2D and 3D geometries with impermeable walls. A good agreement was found comparing the numerical results with analytical solutions developed by Moiseyev (1958) for resonant frequencies and with experiments carried out by Bredmose et al. (2003), who studied the coexistence of large amplitude sloshing motions and violent brief impacts on the container. Changes in the design of the reservoir are suggested, which prove to be efficient, reducing the pressure peaks at the wall. The implemented model allows to predict non-linear free surface flow phenomena for different motions of the reservoir, including the double peaked pressure or "church roof" profile, commonly observed when highly energy waves impact at vertical walls during run-up*

Keywords*: sloshing, computational fluid dynamics, wave impact*

1. INTRODUCTION

It is known that when partially filled reservoirs are submitted to external exciting forces may perform wave motions relative to its hydrostatic equilibrium, resulting in the phenomenon called *sloshing*. This wave generation may cause high pressure impulse at its walls.

Sloshing has become an interesting problem since mid-1950s. On a first approach the studies were concentrated on aeronautics application due to the fuel motion in the fuel tanks that could affect the dynamics of a plane. Besides that, partially filled moving tanks are met on rockets, nuclear reactors and liquefied natural gas (LNG) storage, studied by *Lee et al.* (2006) for example. These waves, when reflected on the walls of a reservoir, impose hydrodynamic forces that may cause instability, loss of maneuverability and occasionally rupture of the container. Further fields of interest include aerodynamics and seismic stabilization of high structures. Sloshing in mobile tanks is typically generated by guidance and control systems commands, manoeuvres and structural vibrations.

Floating Production, Storage and Offloading vessels (FPSOs) are floating offshore units where the petroleum extracted or received from other platforms may be processed and stored in huge tanks to be exported by pipelines or by relief vessels. These watercrafts and platforms are submitted to the natural wave motion (see Fig. 1) that can lead to the sloshing phenomena resulting in high impact pressure at the time of collision, what is an important task in the design of internal cargo structures. Many studies on the ship sloshing problem were carried out in the 1970s and early 1980s for the design of LNG carriers, and recently, for FPSO vessels.

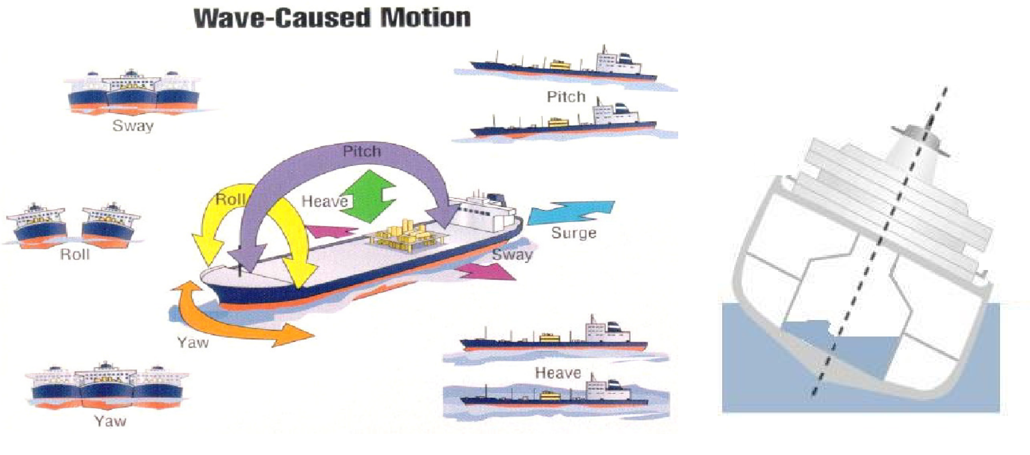


Figure 1. Schematic representation of a vessel submitted to ocean motions

Sloshing induced loads can be divided principally into non-impulse and impulse dynamic pressures. In case of a shallow filled and large excitation tank motion, a vertical front may be generated resulting on a very large impact on the tank walls. By the way, in a nearly full compartment, a progressive wave may cause stress acting on the roof of the tank. The impact is localized in time and space, with high pressure gradient on a very small scale of time. These characteristics turn sloshing into a complex situation which requires fine mesh grids and small time steps for numerical models, in order to avoid inaccurate results and numerical instabilities.

Violent and brief wave impacts have been studied by Cooker & Peregrine (1992) focusing on the pressure measured on a vertical wall. The pressure distribution graph presents the formation of two peaks, the first one due to the inertia of the fluid, acting on a very short period of time (which may last only some milliseconds) turning the gravitational effects negligible, and the second due to hydrostatic pressure created by the downward wave motion just after the maximum steep surface wave near the vertical wall, known as *run up*.

The sloshing phenomenon is a classic eigenvalue problem of the Fluid Mechanics, characterized by the nonlinearity due to the convective term of liquid acceleration, from the Navier Stokes equations, and the nonlinear surface boundary conditions. Recent references for sloshing studies include Attari & Rofooei (2008) which studied the lateral response of a single degree of freedom structural system containing a circular cylindrical tank under harmonic and earthquake excitations; Chen *et al.* (2008) which sloshing in a rectangular tank excited by horizontal harmonic motion is assessed numerically at different filling levels and excitation frequencies and Yonghwan (2007) who considers the experimental and numerical observations of strongly nonlinear sloshing flows in ship cargo and their coupling effects with ship motion. Godderidge *et al.* (2009) study a near resonant sloshing flow in a rectangular tank in order to compare a homogeneous and inhomogeneous multiphase approach for fluid density and viscosity in a commercial CFD code.

Bredmose *et al.* (2003) analyzed experimentally and numerically the generation of nonlinear waves on a 1480 x 400 x 750 mm^3 (length x width x height) reservoir. On the first situation studied (H10 experiment) horizontal oscillations are imposed in order to investigate the wave impact on lateral tank walls. In this study was observed two types of response that may coexist: long lasting, large amplitude sloshing motions and violent brief impact of the liquid on the container wall. In this case, a Boussinesq extended equations model is used and a good agreement between the numerical and experimental results is found, except for the *run up* prediction, where it does not represent the downward jet.

Moiseev (1966) finds for a 2D box of length πL and depth hL undergoing harmonic horizontal oscillations, a discrete spectrum of frequencies:

$$\omega = \sqrt{\frac{ng}{L}\tanh\left(\frac{nh}{L}\right)} \tag{1}$$

Where g is the gravity acceleration and n is the wave number. Resonance occurs when the fluid is forced to oscillate at these frequencies. Cox *et. al.* (2005) studied the effect of slowly changing the length of a tank on the nonlinear standing waves (free vibrations) and resonant forced oscillations of shallow water in the tank.

In view of preventing structural failure, extensive experimental and theoretical studies have been undertaken. In particular, NASA design criteria are based on subdividing the container by longitudinal walls or installing baffles. Craig *et al.* (2006) and Cho *et al.* (2005) illustrate some strategies for preventing the impacts at walls. Gavrilyuk *et al.* (2006) centers around fundamental solutions of linearised problem on fluid sloshing in a vertical circular cylindrical tank having a thin rigid-ring horizontal baffle.

2. GOVERNING EQUATIONS

In this study, a fluid domain is chosen by taking a perpendicular plane to the motion of the reservoir. Cartesian coordinates are defined with the x axis on the stationary surface, such that the fluid occupies the region $y \leq 0$ when in rest - initial condition.

The homogeneous fluid interaction model applied for the numerical simulation of sloshing uses the volume fraction ϕ of each fluid to determine the fluid mixture properties. Air is taken as an ideal gas while the liquid phase, water, is considered incompressible. The full Navier Stokes equation may be written by:

$$\frac{D(\phi\rho\vec{u})}{Dt} = \nabla \cdot \left(\phi\vec{\vec{T}}\right) + \rho\phi\vec{f} \tag{2}$$

Where $\vec{\vec{T}}$ represents the sum of the pressure, expansion and viscous forces, For a Newtonian fluid with viscosity μ, expansion viscous k and submitted to a dynamic pressure p_E , this tensor may be written as:

$$\vec{\vec{T}} = -p_E\delta_{ij} + \left(k - {}^{2}\!/_{3}\,\mu\right)\nabla\cdot\vec{u}\,\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{3}$$

The vector $\vec{u}=(u,v,w)$ represents the fluid velocity, ρ is the fluid density and $\vec{f}$ is the sum of the external forces, e.g. gravity $\vec{g}=(0,-g,0)$.

For the homogeneous multiphase flow:

$$\begin{cases} \rho = \sum_{l=1}^{2}\phi_l\rho_l \\ \mu = \sum_{l=1}^{2}\phi_l\mu_l \end{cases} \tag{4}$$

The Mass Balance for the multiphase flow is written as:

$$\frac{\partial(\phi\rho)}{\partial t} + \nabla\cdot(\phi\rho\vec{u}) = 0 \tag{5}$$

None heater transfer is considered on the simulations developed for this study.

The kinematic boundary condition to be satisfied on the free surface is based on the idea that the fluid particle, described for the position vector $\vec{r}=(x,y,t)$, lies on the x axis, then:

$$\frac{D\vec{r}}{Dt} = \nabla\Phi \tag{6}$$

Where Φ is the velocity potential of the fluid. The dynamic condition of the free surface is given by the Bernoulli equation:

$$\frac{D\Phi}{Dt} = \frac{1}{2}\left|\nabla\Phi\right|^2 - g\,y - \frac{p}{\rho} \tag{7}$$

On the walls, the Neumann condition shall be satisfied. The bottom and the side walls are considered rigid and impermeable:

$$\vec{u}\Big|_{walls} = 0 \tag{8}$$

3. OBJECTIVE

This paper aims to model numerically the flow with free surface caused by a horizontal motion of a partially water filled reservoir – 1480 *x* 480 *x* 750 mm^3 (length *x* width *x* height) – studied by Bredomose *et al.* The height of the water interface was defined as 155 mm. The tank is first represented by a 2D box, with symmetric conditions applied on the front and back walls (normal to z axis).

The mesh was refined near the free surface and near the walls in order to better predict the nonlinear profile of the fluid motion. For the top of the tank the free mass transfer condition was applied.

For a first approach, the solution of resonant waves for shallow flow (Eq. 1), developed by Moiseyev (1958) was implemented to validate the numeric model with analytical results. For this case the resonant frequencies was applied for a reservoir with the same aspect ratio of the one used for H10 experiment.

On a second approach, the displacement (Fig. 2) and conditions used at Bredmose *et al.* experiment is used for a 2D reservoir. A fine time grid is required due to the high pressure gradients in a short period of time while the wave impacts on the walls, as discussed before, with time steps of 1 *ms*. To simulate the sloshing phenomenon the finite volume method based software ANSYS CFX was used. After those, the three-dimensional box was simulated.

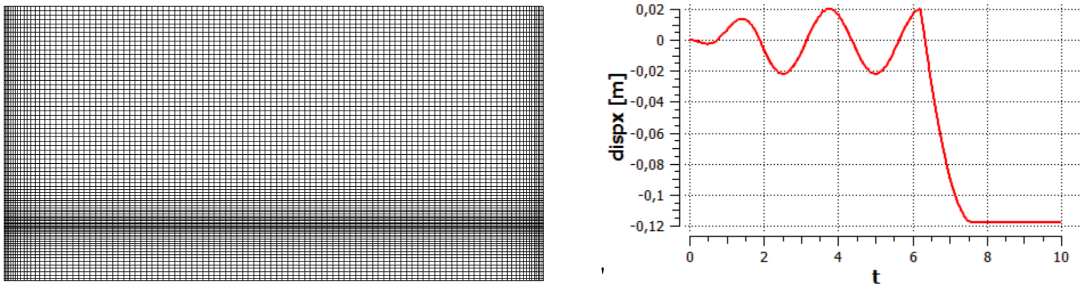


Figure 2. On the left, the mesh used for the 2D simulations (18.000 nodes) and, on the right the horizontal displacement of the reservoir

4. NUMERICAL RESULTS

The numerical results are subdivided into three studies, analyzed on the following sessions:

4.1. Ressonance Frequencies

The analytical validation of the numerical model used on this study was made by classic works from Moiseyev (1958) and Chester (1968). The profiles found on Fig. 3 verify the resonant linear wave formation when the system oscillates on the frequencies defined by the Eq. 1. The frequencies were applied to a sinusoidal harmonic horizontal motion (Eq. 9) on the 2D reservoir used for the H10 experiment, where h/L = 0,105. The amplitude of the displacement (U_0/ω) used was $2.\omega^{-1}$ cm. High displacement amplitudes cause abrupt displacement of the fluid on the reservoir, generating turbulent flow and divergence of the numerical solution.

$$x = \frac{U_0}{\omega} sen(\omega t) \tag{9}$$

Where the dimensionless properties are written as:

$$\begin{cases} t' = t\sqrt{g/L} \\ U'_0 = U_0/\sqrt{(gL)} \\ \omega' = \omega\sqrt{L/g} \end{cases} \tag{10}$$

With the increase of the resonance frequencies were observed the increase on the number of waves, as expected. The results demonstrated that the kinetic energy provided to the reservoir, dissipated by fluid forces as viscous forces, for example, prompt the stationary resonant waves formation, after a initial transient period. The boundary conditions and initial conditional used for these cases are the same then those used for H10 experiment. On the figure bellow the vertical axis is amplified in order to better observe the resonant waves.

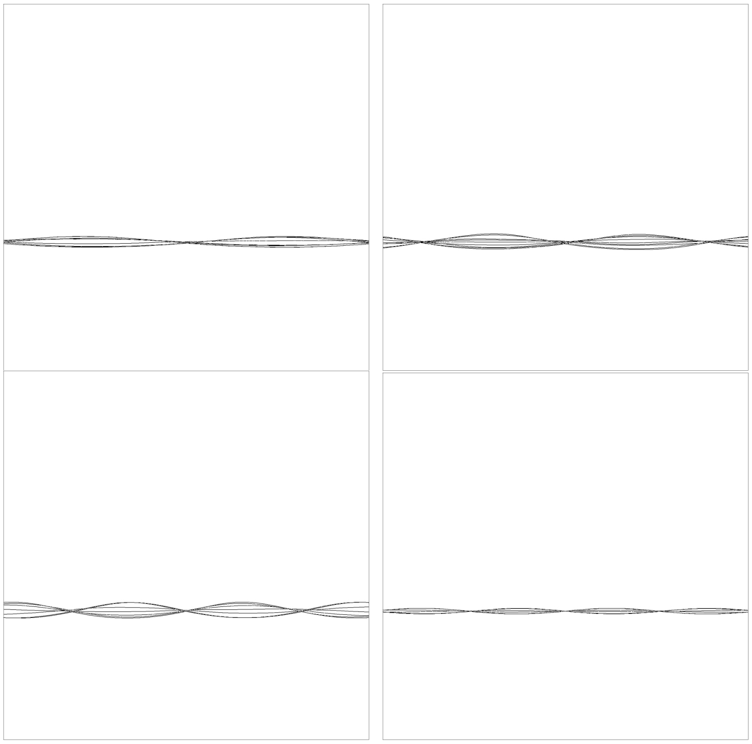


Figure 3. Resonant waves formed from harmonic horizontal excitation.

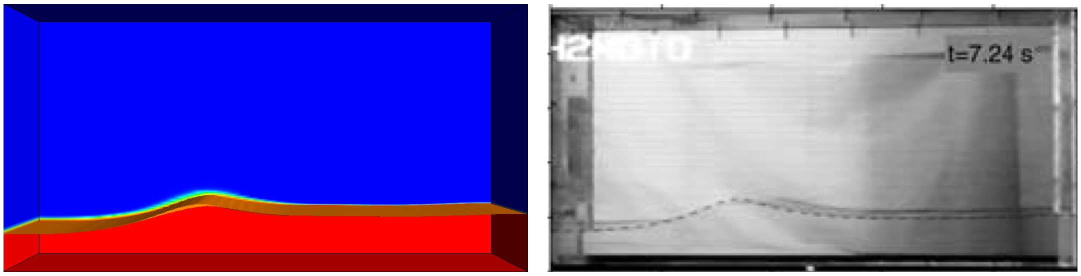
4.2. H10 Experiment

The *Bredmose* experiment described before, where transient horizontal high amplitude oscillations are imposed on the partially filled reservoir, has been modeled and a good agreement of the numerical and experimental results were found. The Figure 4 represents, on the right, a sequence of pictures gotten from the H10 experiment, while the dashed lines correspond to the numerical solution using Boussinesq model. On the left of the same figure, the results obtained using ANSYS CFX are presented. The images show the free surface motion resulted from the lateral impulse caused by the reservoir walls.

The dashed lines - Boussinesq model - represent with good agreement the experimental results until the run up the wall. The CFX numerical results, on a general view, show excellent accordance between the experimental e numerical studies, including for the non linear phenomenon. The generated wave is followed until hits the left wall, proceeded by the run up and the reflection. The full simulation illustrate the wave amplitude decay while approximates to the right vertical wall, where turns more steep and starts its advancement. Another aspect not reproduced by Boussinesq model is the vertical downwind jet observed at 8.04 seconds, directed to the bottom of the reservoir.

After the good agreement obtained for the 2D geometry, the 3D reservoir model was simulated with the same boundary and initial conditions. The mesh required for the tridimensional simulation is times heavier than the previous one. In order to reduce the computational demand, the time steps used were 10 times bigger than the 2D model, obtaining results for each 10 *ms* for the 12 seconds of the experiment. Good accordance with the experimental results was obtained once again.

The Fig. 5 illustrates the wave motion for the three-dimensional simulation for the H10 experiment.



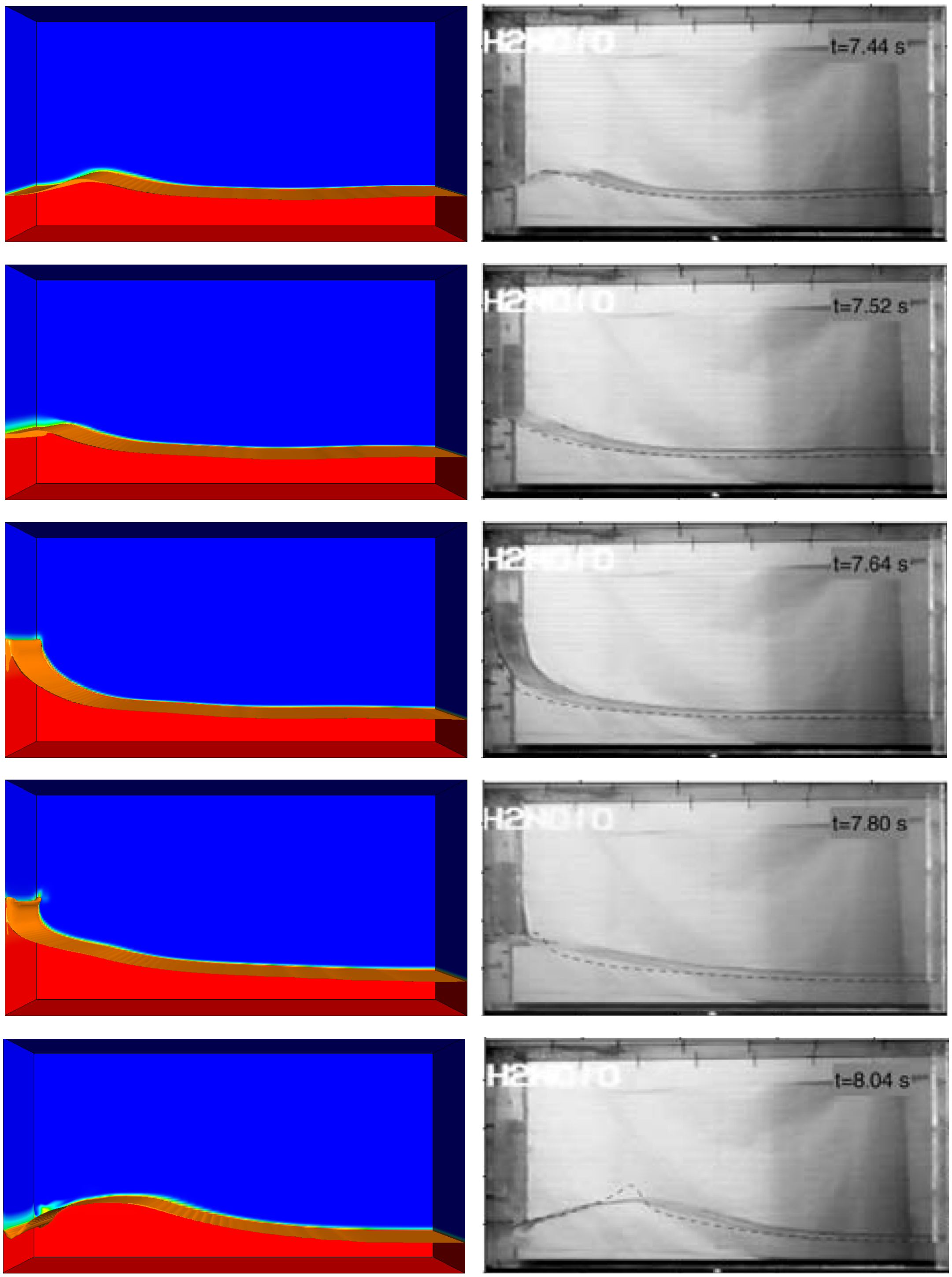


Figure 4. On the left the results obtained from the simulations, on the right a photo sequence of the H10 experiments

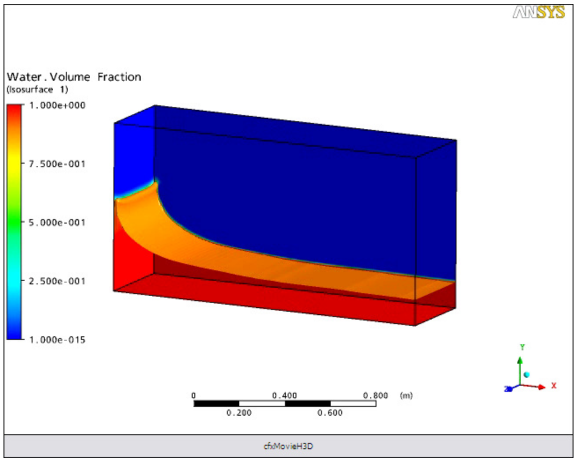
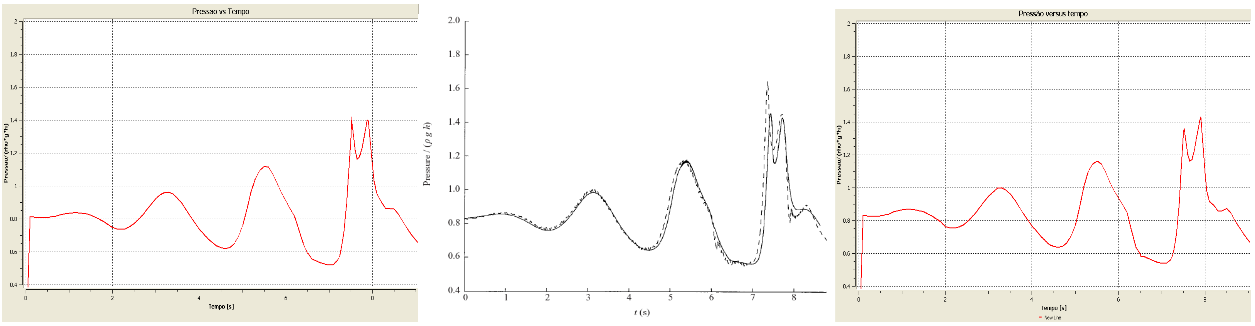


Figure 5. The run up for the three-dimensional simulation

The pressure profile from CFX simulations are compared with the measurements and calculus from Bredmose *et al.* On the Fig. 6 it is possible to observe exactly the same pressure profile for nine seconds of the experiment. It is clearly noted the traditional double peak pressure formation when the occurrence of the run up on the vertical wall.

The vertical axis for pressure scale is dimensionless, dividing the pressure incident on the wall by the liquid phase density, gravity and liquid height product (ρgh).

For the numerical simulation is observed a little difference on the first peak of the double peak formation for the run up. Its magnitude is a little under predicted. It may be explained by the high pressure gradient imposed on the wall on a very small period of time as discussed before, especially observed on the 3D geometry due to its loss of time grid refinement.



(a) (b) (c)

Figure 6. (a) Pressure distribution from ANSYS CFX (2D). (b) Comparison between the measured pressures (- - -) and calculated (——) by Bredmose et al (2003). (c) Pressure distribution from CFX for the 3D simulation

4.3. Geometry Adaptation

The reduction of the level of sloshing in a partially filled container is a challenge while designing equipments and structures partially filled with a liquid. In order to illustrate the effects of the strategy of incorporating baffles or obstacles on the direction of the wave generation, the following case was studied. On this tridimensional simulation, vertical walls were added in the center of the reservoir from H10 experiment. The non-slip condition was applied for all the walls. The same lateral displacement illustrated on the Fig. 2 was used and the graphic results of the dimensionless pressure impact at the left wall are presented on the Fig. 8 and compared with the experimental results for the same motion without the vertical structures.

The Fig. 7 presents the surface wave generation, and its movement inside the tank. The simulation showed that run up is not recorded on the vertical walls for the same motion described on the session 3, for this reason the pressure double peak is not observed as well. The pressure high gradients, that could cause serious impacts on the structure of the reservoir due to its huge energy, are prevented, testifying the efficiency of this building stratagem.

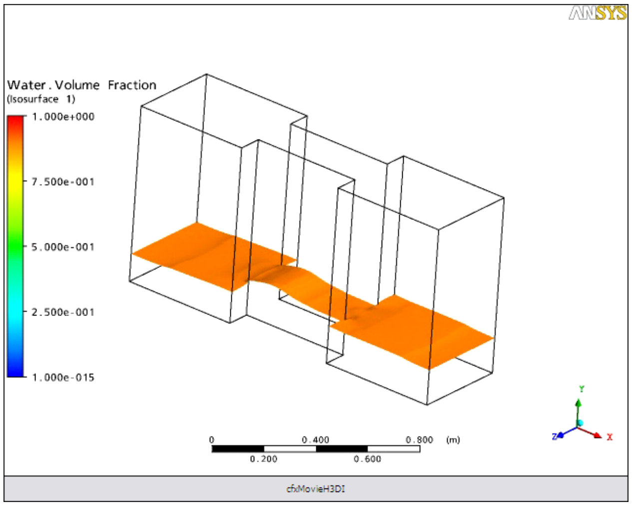


Figure 7. Wave motion for the horizontal sloshing with vertical walls

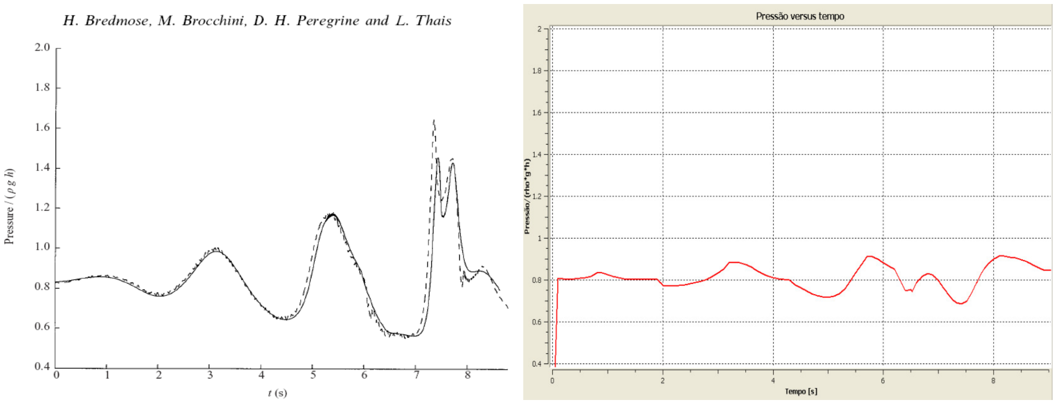


Figure 8. Pressure distribution comparing the efficiency of vertical obstacles in the path of the surface wave, for the same horizontal excitation

5. CONCLUSIONS

The application of numerical methods for free surface flows has shown good efficiency for forecast linear and non linear phenomena. The validation of the numerical solutions obtained for this study has been made by analytical and experimental results. The analytical solution, in spite of being generally applied for simplest conditions and geometries, presents simple and good results. The resonance frequency developed by Moseyev was taken as parameter for the comparison of the numerical solutions with the analytical ones. The experimental procedure is usually more expensive and may inflict hardness regarding representing real aspects of practical situations or due to safety issues. The Bredomse experiment H10 was simulated with analogue conditions so as the numerical model could be validated with experiments, with an excellent agreement.

Future developments are suggested for the validation of vertical acceleration motion and prediction of *table top* phenomenon.

6 .REFERENCES

Attari, N.K.A., Rofooei, F.R., 2008, “On Lateral response of structures containing a cylindrical liquid tank under the effect of fluid/structure resonances”, Journal of Sound and Vibration, Vol. 318, pp.1154-1179.

Bredmose, H, Brocchini, M, Peregrine, D.H., Thais, L., 2003, “Experimental Investigation and Numerical Modelling of Steep Forced Water Waves”, Journal Fluid Mechanics, pp. 217-249.

Chen, Y.G./ Djidjeli, K., Price, W.G., 2008, “Numerical simulation of liquid sloshing phenomena in partially filled containers”, Comput Fluids, doi:10.1016/j.compfluid.2008.09.003

Chester, W., Jul. 30, 1968, “Resonant Oscillations of Water Waves”, Proc. Royal Soc, Series A, Mathematical and Physical Sciences, Vol. 306, 5. 1968, pp.5-22.

Cho, J.R., Lee, H.W., Ha, S.Y., 2005, “Finite element analysis of resonant sloshing response in 2-D baffled tank” Journal of sound and Vibration 288, pp. 829-845.

Cooker, M,J., Peregrine, D.H., 1992, “Wave Impact Pressure and Effect Upon Bodies Lying on the Bed”, Coastal Engng, Vol.18, pp.205-229.

Cox, E.A., Gleeson, J.P., Mortell, M.P., 2005, “Nonlinear Sloshing and Passage Through Resonance in a Shallow Water Tank”, Z. angew. Math.. Phys. 56. p.645-680.

Craig, K.J., Kingsley, T.C., 2006, “Design optimization of containers for sloshing and impact”. *Springer-Verlag.*

Godderidge, B., Turnock, S., Tan, M., Earls, C., 2009, “An investigation of multiphase CFD modelling of a lateral sloshing tank”, Computers & Fluids 38, pp.183-193.

Kyoung, J.H., Hong, S.Y., Kim, J.W., Bai, K.J., 2005, “Finite-element computation of wave impact load due to a violent sloshing”, Ocean Engineering 32, pp-2020-2039.

Lee, D.H., Kim, M.H., Kwon, S.H., Kim, J.W., Lee, Y.B., 2007, “A parametric sensitivity on LNG tank sloshing loads by numerical simulation”, Ocean Engineering 34, pp. 3-9.

Gavrilyuk, I., Lukovsky, I., Trotsenko, Yu, Timokha, A., 2006, “Sloshing in a vertical circular cylindrical tank with an annular baffle. Part 1. Linear fundamental solutions”, Journal of Engineering Mathematics 54, pp. 71-88.

Ma, Q.W., Yan, S., 2006, “Quasi ALE finite element method for nonlinear water waves”, Journal of Computational Physics 212, pp.52-72.

Moiseyev, N.N., 1958, “On the theory of nonlinear vibrations of a liquid”. Prikl. Mat. Mekh. 22, pp. 612–621.

Moiseev, N.N., Petrov, A.A., 1968, “The Calculation of Free Oscillations of a Liquid in a Motionless Container” Adv.Appl. Mech., v.9, pp.91-154.

Ockendon, H., Ockendon, J.R, Johnson, A.D., 1986, “Resonant Sloshing in Shallow Water”, Journal Fluid Mechanics, Vol.167, pp. 465-479.

Yonghwan, K., 2007, “Experimental and numerical analyses of sloshing flows”, J Eng Math 58, pp. 191-210.

7. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.