TEACHING AND COMPARING ADVANCED CONTROL TECHNIQUES IN A BALL AND BEAM DIDACTICAL PLANT

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Abstract. The objective of this paper is to propose a methodology of teaching control theory to undergraduate students by presenting (and comparing) a sequence of increasingly complex design techniques. At the same time, those techniques are used in order to produce controllers for a "ball and beam" system. The "ball and beam" system is a well known didactical plant, composed by a rotating beam (that is, the angle between the beam and the horizontal line can be varyied by the controllers with the aid of a DC motor) and a ball rolls over it (supposed without sliping). The closed-loop control objective is to change the equilibrium position of the ball by measuring its position (relative to the beam) and varying the torque applied (by the DC motor). Obviously, this plant is unstable. Special emphasis are given on how the controllers such difficulties, it is very suited for teaching/training in automatic control disciplines, such as classical control (or introduction to control theory), multivariable control and nonlinear control. The control laws used are PID (proportional-integral-derivative), LQG control (linear quadratic optimal control with Kalman Filter) and feedback linearization control. The plant's mathematical model (necessary for the designs) are obtained via lagrangian mechanics. The comparison among the several controllers is done via simulation in MATLAB/Simulink and the advantages and pitfalls of each technique are thoroughly stressed.

Keywords: Control Education; Nonlinear Control; LQG Control; Mechanical Systems; PID Control.

1. INTRODUCTION

Automatic control theory poses problems not only for its practicioners (in the several industries that make use of it), but also for those whom intend to teach it in undergraduate programs. As engineering disciplines, design techniques must be presented and thoroughly exercised in the traditional courses. On the other hand, as a mathematically involved human knowledge, the lecturers must, in the same courses, spend much time by teaching how to use the (mathematical) tools in order to make practical designs. Because of these, the students' understanding of 1) why to use feedback and 2) why increasingly complex design techniques could be convenient, frequently suffer.

Normally, the traditional control engineering programs begin with mathematical courses (like linear system analysis) and modeling, followed by classical control techniques (for continuous and discrete-time single-input-single-output plants, with laboratory experiences), multivariable and nonlinear control courses (adopting the state space approach) and ending with application oriented courses. Despite of being a convenient sequence, in the authors' opinion, in many cases the courses are not well coordinated, and the senior students finish the program with a "lack of understanding" feeling, that could mine their self-confidence as a professional.

In order to solve the problems above, some educators suggested that, besides the traditional courses, the fundamental control concepts should be introduced as soon as possible in the undergraduate curriculum (Djaferis, 2004), approaching concepts like disturbances, noise, model uncertainties, saturations and the ability that feedback has to attenuate these effects in a "totally devoid of mathematics way" (see Murray, 2004). This course should be offered to first-year engineering students, as well as a more diverse audience, that include biologists, economists and others.

This paper presents the experience of the authors in trying to solve (or at least contributing to the solution of) these problems, by proposing a way to integrate the traditional courses in a way that the student could learn how to correctly choose the best design technique for the problem at hand. The methodology is exemplified by a sequence of analysis and increasingly involved design techniques for a didactical (but otherwise complex) mechanical control system, which is the "ball and beam" system. Everything starts with the plant modeling, followed by the classical design, the state-space and optimal design and, finally, a nonlinear design, finishing with a comparison among all the results, as a way to show their pros and cons. The outcome is that students, at the end of the program, become firmly acquainted with the basic concepts, as well as with the necessary mathematics.

The modeling, presented in section two, is based in the lagrangean formulation and illustrates the initial phase of any model based design technique. A software implementation of this model, which is fundamental in simulation, is also presented. The classical design, in section three, follows the contents of (Ogata, 2005) or other equivalent reference. As is well known, classical control is a collection of tools, rules of thumbs and conventions, but is sufficiently simple to

provide a first contact with the subject. The linear optimal (state-space based) design techniques are successfully applied in section four, introducing new concepts, such as eigen-values and -vectors, controllability, observability, state feedback, observers (and Kalman Filters), linear quadratic regulator and robust control. At the same time, it is stressed that the classical control methodology, illustrated before, did not yield a correct controller. On the other hand, the limitations of linear controllers are presented, showing that in order to stabilize the ball in positions near the beam's extreme points, a linear controller could produce unacceptable inputs and transients. Finally, in section five, a nonlinear technique (feedback linearization) is applied in conjunction with optimal control in order to solve the problems that the linear control itself could not. Finally, conclusions are drawn.



Figure 1. Picture of the Ball and Beam Didactical Plant.

2. MATHEMATICAL MODELLING

The didactical kit to be used as example, presented in figure 1, is a "ball and beam" system, from Amira/GmbH. This kit is at the disposal of researchers and instructors in the Control Laboratory at São Paulo State University (UNESP) – Sorocaba Campus and consists of a ball that moves in a trail (beam) with 2 meters length and can have their inclination changed by a direct current (DC) motor. The ball's position is measured by a CCD camera and can be used to construct a closed-loop system, which is implemented in a personal computer. The ball rolls without slipping, what reduces by one the system's degrees-of-freedom. The DC motor's current is varied by the controllers.

The system's mathematical model is highly nonlinear and is obtained by using the lagrangean mechanical modeling, as presented in (Bloch, 2003) or (Hauser *et al.*, 1992). It is naturally an unstable system, which creates enough challenges for the student, as well as a SISO (single-input-single-output) plant, which avoids excessive mathematical complexity. A schematic representation of the plant is presented in Figure 2. The main problem hypotheses are: 1) the ball rolls without slipping; 2) The beam is massless, as adopted in (Hauser *et al.*, 1992), which means that the moment of inertia comes only from the ball, and is equal to $J_1 = Mr^2$. *M* is the ball's mass and *r* is the ball's position along the beam. The kinetic energy comes only from the ball's velocity, and is given by

$$K = \frac{l}{2} J_I \dot{\theta} + \frac{l}{2} M r^2 \tag{1}$$

where the upper dots represents time derivative. The potential energy is only gravitational and comes from the ball position, that is $U = Mgrsen(\theta)$. The lagrangean is then given by

$$L = K - U = \frac{1}{2} J \dot{\theta} + \frac{1}{2} M r^{2} - M gr sen(\theta)$$
⁽²⁾

Finally, the losses due to viscous friction (in the motor) are represented by the Rayleigh dissipation function $P=0.5 b\dot{\theta}^2$. Using the Euler-Lagrange equations (modified by including the dissipative term), that is

 $Q = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} + \frac{\partial P}{\partial \dot{q}}$

where Q represents the generalized forces, the motion equations are given by



Figure 2. Schematic representation of the plant to be controlled.

By further manipulating the equations in (3) and defining the state variables as being $x_1 = \theta - \theta_{eq}$, $x_2 = \dot{\theta} - \dot{\theta}_{eq}$, $x_3 = r - r_{eq}$ and $x_4 = \dot{r} - \dot{r}_{eq}$ and the plant's input $u = \tau - \tau_{eq}$ (where the subscript eq represents the equilibrium value), the model is written as

$$\begin{pmatrix} \dot{x}_1 = x_2 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

$$\left| \dot{x}_{2} = -\frac{1}{M(x_{3} + r_{eq})^{2}} bx_{2} - \frac{2}{(x_{3} + r_{eq})} x_{2} x_{4} - \frac{1}{(x_{3} + r_{eq})} g \cos(x_{1}) + \frac{1}{M(x_{3} + r_{eq})^{2}} (u + \tau_{eq}) \right|$$
(5)

$$\dot{x}_3 = x_4$$
 (6)

$$(\dot{x}_4 = (x_3 + r_{eq})x_2^2 - gsen(x_1))$$
 (7)

It is obvious, by equaling the state derivatives to zero, that the possible equilibrium situations only occur when $\theta_{eq} = \dot{\theta}_{eq} = \dot{r}_{eq} = 0$, where r_{eq} the equilibrium position in the beam. For each possible equilibrium position, there correspond an equilibrium torque, given by $\tau_{eq} = Mgr_{eq}$. The model's measured variable is $y = x_3$. In figure 3, the plant's mathematical model, implemented in MATLAB/Simulink, is presented.



Figure 3. Simulation Model (nonlinear plant).

3. CLASSICAL CONTROL DESIGN

In order to design a classical controller, one has to linearize the plant in a neighborhood of the equilibrium point, as described in, for example, (Garcia, 2005). This yields a linear time-invariant model with general form $\dot{x}=A x+B x$, and output equals to y=C x+D u, where the first matrices are given by:

$$\mathbf{A} = \begin{bmatrix} \frac{\partial f_{I}}{\partial x_{I}} & \frac{\partial f_{I}}{\partial x_{2}} & \frac{\partial f_{I}}{\partial x_{3}} & \frac{\partial f_{I}}{\partial x_{4}} \\ \frac{\partial f_{2}}{\partial x_{I}} & \frac{\partial f_{2}}{\partial x_{2}} & \frac{\partial f_{2}}{\partial x_{3}} & \frac{\partial f_{2}}{\partial x_{4}} \\ \frac{\partial f_{3}}{\partial x_{I}} & \frac{\partial f_{3}}{\partial x_{2}} & \frac{\partial f_{3}}{\partial x_{3}} & \frac{\partial f_{3}}{\partial x_{4}} \\ \frac{\partial f_{4}}{\partial x_{I}} & \frac{\partial f_{3}}{\partial x_{2}} & \frac{\partial f_{3}}{\partial x_{3}} & \frac{\partial f_{3}}{\partial x_{4}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{b}{Mr_{eq}} & \frac{-g}{r_{eq}} & 0 \\ 0 & 0 & 0 & 1 \\ -g & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{B} = \begin{bmatrix} \frac{\partial f_{I}}{\partial u} & \frac{\partial f_{2}}{\partial u} & \frac{\partial f_{3}}{\partial u} & \frac{\partial f_{4}}{\partial u} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 0 & \frac{1}{Mr_{eq}} & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$
(9)

Taking the plant's parameter (in SI units) as b=5, g=9.8 and m=0.111, for an equilibrium point with $\tau_{eq} = 0.5$ the plant's eigenvalues are 1.28, -0.64±1.12i and -213.21, which clearly shows that the equilibrium point is unstable (what could be deduced by physical arguments). As the additional matrices are $C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$ and D=0, calculating the transfer function, by the formula $G(s)=C(sI-A)^{-1}B+D$, one obtains:

$$G(s) = \frac{-417.9}{s^4 + 213.2s^3 - 454.6} \tag{10}$$

It is enough information for a classical design, and the student should try several unsuccessful controllers in order to acquire the engineering intuition. In the author's experience, the traditional recommendation (that is, to begin with a proportional controller and successively augment the dynamics by adding poles and zeros) is very profitable in didactical terms, because the compromise between controller complexity and improved performance becomes evident. As the design tools are graphical, a software aid is indispensable today. In particular, MATLAB / Sisotool has served well in the authors teaching experience, as a non successful design can be rapidly changed. For the sake of illustration, in figure 4, is shown, in the left, a snapshot root-locus based PID design (with the red x points representing controller poles, the blue x points representing plants poles and the O points representing zeros). By a simple examination, it is obvious that no PID practical solution is expected, as there is an entire closed-loop path in the right half complex plane (it must be emphasized that a pole-zero cancellation in this half plane is unacceptable in practice). Even a change in the controller sign would not solve the problem. In the right of the same figure, it is shown a successful design, corresponding to the controller

$$H(s) = -122.05 \frac{(1+2.13s)(1+0.483s)}{(1+3.95s)(1+19.1s)}$$
(11)

Despite the guaranteed closed-loop stability (that is, that ball would stay in the predetermined position even with small disturbances, like vibrations in the kit), the simulation in figure 5 shows that a steady state error is present such that the ball would stabilize outside the beam. The introduction of an integral action in the controller would bring back the problem that appeared in the system with PID controller.

4. MODERN LINEAR CONTROL DESIGN

Despite being relatively old, the classical design techniques have the advantage of generating low order controllers (Ogata, 2005), which could be very advantageous if one is interested in low complexity controllers. On the other hand, better stability and performance could be obtained by using modern state-space controllers (based in observers) that typically have higher order. Besides that, more quantitative ways of measuring performance indexes, as well as stability margins, are at the designer disposal.

For didactical purposes, the pole-placement design, as presented in (Chen, 1990), should be presented first, as it is the conceptually simplest design technique. If all the plant's states are available for measurement, this design consists of an arbitrary choice of the closed-loop poles, followed by the determination of the state feedback control law of the form u = -Kx, such that the closed-loop system $\dot{x} = (A - BK)x$ have those desired poles. In case of the state variables are not available, a state observer (based in the plant's model) with equation $\dot{x} = (A - HC)\hat{x} + Bu + Hy$, should also be designed (by an adequate choice of matrix *H*), and the control law would be $u = -K\hat{x}$. The controller would be the combination of the control law and state observer.

This method, however, present some difficulties, as there are much controller's parameters that must be adjusted at the same time (and the relative success of classical control design techniques is due partly to the reduced number of parameters to be determined). Besides that, some choices of closed-loop poles could produce ill-conditioned matrices (for the state feedback and observer), which cause implementation difficulties.



Figure 5. Simulation for the lead-lag design.

Time (sec)

The LQG (linear quadratic gaussian) methodology, however, came to solve this problem, as: 1) The number of parameters to be adjusted can be much reduced, and 2) it is easy to relate those parameters with the closed-loop behavior. The controller's structure is the same as in the pole-placement (that is, control law and observer), but the matrices *H* and *K* are determined in order to minimize cost criterions of the form $\int (x^T Q x + u^T R u) dt$, (one for the control law and the other for the observer, or Kalman Filter) where *Q* and *R* are chosen by the designer. In order to show that the design is simpler (and more intuitive), one could, in principle, choose Q=I and R=rI, where *I* is the identity matrix and *r* is an scalar, which means that only one parameter must be adjusted. It is also more intuitive, as the greater the parameter *r*, the lower the values the control signal *u* can achieve, which is useful when there are limits to be respected in the signal value.

In order to illustrate the design methodology, let's take the linearized plant obtained above and choose three different values for r (that is 0.01, 0.1, 1) and determine the corresponding K matrix, as well as the closed-loop poles. In Figure 5, it is shown the corresponding closed-loop and one can see that the greater the r, the closer the poles are to open-loop poles (despite one of them be in the right half-plane). It demonstrates the important fact that if the control is penalized, it is more difficult to put the closed-loop poles far from the open-loop poles. The Kalman filter poles are determined by a similar procedure, and the corresponding poles (for the best design obtained) are also shown in the figure. In figure 6, the system output (that is y)and the control signal (that is u) are shown for three different simulations (the controller used corresponds to r=0.01): 1) linearized plant and controller without observer, 2) linearized plant and controller with observer and 3) nonlinear plant and controller with observer. It is then clear that this LQG (that is, LQR

+ Kalman Filter controller) give better results than the classic one, as the ball does not fall of from the beam. Of course, the last cited simulation is the one that matters, as the observer (Kalman Filter) is indispensable (the four states are not measured and must be estimated) and the real plant is linear.



Figure 5. Optimal regulator (LQR) designs for various Q and R values.



Figure 6. Manipulated (torque) and controlled (position) variables for the optimal control.

5. NONLINEAR CONTROL DESIGN

Despite of the LQG controller has stabilized the ball in the beam (it did not fall off), there are many aspects in which the project could be improved. The last simulation started with the beam in the horizontal and with null initial velocity. If it was used initial conditions equal to $\theta_0=30^\circ$, $\dot{\theta}_0=1$, $r_0=0.5$ and $\dot{r}_0=0.5$, the simulation would show that the ball falls off. In order to solve this problem, a nonlinear design methodology, known as feedback linearization, is proposed. It is expected the cancelling of the system nonlinearities, and the design procedure consists of taking the time derivative of the output till the plant's input appears in the expression, as described in (Slotine, 1991) and (Isidori, 1995). First of all, in order to save time, the original mathematical model could be modified by defining the motor torque (until now represented by u) as a function of a new input v. The equation relating both would be:

$$u = Mr^2 v + B\dot{\theta} + 2Mr\dot{r}\dot{\theta} + Mg\,rcos\theta\tag{12}$$

Taking the time derivatives, the input appears only after three time derivations, as shown below:

$$y = x_3 \tag{13}$$

$$\begin{aligned} y &= x_3 - x_4 \\ \ddot{y} &= (x_3 + r_{eq}) x_2^2 - gsenx_1 \end{aligned}$$
(14)

$$\ddot{y} = -3x_2^2 x_4 - 3gx_2 \cos x_1 + \frac{2}{M(x_3 + r_{eq})} x_2 \tau_{eq} + \frac{2}{M(x_3 + r_{eq})} x_2 u$$
(16)

In order to transform equation (16) in a linear equation (in an exact form), the control would then be $u = 1/\beta (v-\alpha)$, where $\alpha = -3x_2^2 x_4 - 3gx_2 \cos x_1 + 2/M(x_3 + r_{eq})x_2 \tau_{eq}$ and $\beta = 2/M(x_3 + r_{eq})x_2$. Except when $\beta = 0$, the proposed control fulfills its role. However, this undesirable situation could be avoided by a slightly modification of the original method, as described in (Hauser, 1992). Deriving again the output y, one has:

$$\dot{\mathbf{y}} = \dot{\mathbf{x}}_3 = \mathbf{x}_4 \tag{17}$$

$$\underbrace{\ddot{y}}_{k_2} = \underbrace{(x_3 + r_{eq})x_2^2 \cdot \sin x_1}_{k_3}$$
(18)

$$\underbrace{\ddot{y}}_{\dot{\epsilon}_{3}} = \underbrace{-gx_{2}\cos x_{1} + x_{4}x_{2}^{2}}_{\epsilon_{4}} + \underbrace{2(x_{3} + r_{eq})x_{2}u}_{\varphi_{3}}$$
(19)

$$\underbrace{y_{\hat{k}_{4}}^{(4)}}_{\hat{k}_{4}} = \underbrace{\left(x_{3} + r_{eq}\right)x_{2}^{4}}_{\alpha} + \underbrace{\left(2x_{2}x_{4} - g\cos x_{1}\right)}_{\beta}u \tag{20}$$

As can be seen in (20), β is never zero (except in very restrict situations). Defining the state variables as ε_1 , ε_2 , ε_3 , ε_4 , the method presented in (Hauser *et al.*,1992) suggests that the term $\varphi_3 = 2(x_3 + r_{eq})x_2u$ could be neglected, if it is limited in norm. In doing so, and choosing the appropriate feedback control law, that is $u = 1/\beta(v-\alpha)$, the system would be in the Brunovsky normal form, that is, it would be linear, except for a minor term (the one neglected). The coordinate transformations are given by the functions $\varepsilon_1 = (x_1, x_2, x_3, x_4)$, $\varepsilon_2 = \varepsilon_2(x_1, x_2, x_3, x_4)$, $\varepsilon_3 = \varepsilon_3(x_1, x_2, x_3, x_4)$, $\varepsilon_4 = \varepsilon_4(x_1, x_2, x_3, x_4)$ defined in (17) to (20). To complete the design, the zero dynamic (that is, the invariant manifold such that any state trajectory in there would cause no output, that is, $y(t) \equiv 0$) must be stable, which is proved in (Hauser *et al.*,1992).

Another similar design (controller two) could be done such that new coordinate transformations are given in equations (21) to (24).

$$\underbrace{\dot{y}}_{\dot{\varepsilon}_1} = \dot{x}_3 = \underbrace{x_4}_{\varepsilon_2}$$
(21)

$$\underbrace{\ddot{y}}_{\dot{r}_2} = -\underbrace{gsenx_1}_{\dot{r}_2} + \underbrace{(x_3 + r_{eq})x_2^2}_{a}$$
(22)

$$\ddot{y} = \underline{-gx_2 \cos x_1} \tag{23}$$

$$\underbrace{\underbrace{y^{(4)}}_{\epsilon_4} = \underbrace{gx_2^2 senx_1}_{\alpha} - \underbrace{g\cos x_1}_{\beta} u}_{\beta} u \tag{24}$$

The new control law involves the new α and β functions, as in the above law. In order to compare the performance of the feedback linearization controllers and the linear optimal control (designed above), simulations were taken (for the system in (21) to (24)) for the following initial conditions $\theta_0=20^\circ$, $\dot{\theta}_0=0.5$, $r_0=0.9$ and $\dot{r}_0=0.5$, and equilibrium point equals to 0.1 and are presented in figure 7, with x_3 being the difference between the initial position and the equilibrium position. Clearly, one sees that the ball does not fall off. It is also obvious that the input signal could not be generated by a linear controller. In figure 8, the same plots are show for the controller in equations (21) to (24). It is clear that a minor adjustment in the gains should be done in order to the ball stays in the beam.

In figure 9, the complete system's block diagram is shown. The nonlinear plant is in the top of the figure. The torque transformation, presented in (12), is done in the block TORQUE GENERATOR. The two feedback linearization controllers presented above are shown at the figure's bottom, and each controller is divided in two blocks (FEEDBACK LINEARIZATION and COORDINATE TRANSFORMATION). The user selects between them by the MULTIPLEX block. The gains matrices k_1 and k_2 are determined by optimal control design (considering that, after feedback linearization, the plant is linear). In order to save space, only the blocks of controller 2 are shown in figure 10. The equation (12) is implemented by the block TORQUE GENERATOR, and is shown in figure 11.

Finally, if one wants to implement this system electronically, a nonlinear observer, like those presented in (Isidori, 1995) should be designed in order to estimate the plant's states. As nonlinear observer design is a difficult matter for undergraduate students (for any student, for that matter) it is not included in this work.



Figure 7. Control signal and system's states for the first controller.



Figure 8. Control signal and system's states for the second controller.



Figure 9. Complete block diagram.



Figure 10. Feedback linearization control law and coordinate transformation.



Figure 11. Torque transformation block.

6. CONCLUSIONS

The objective of this paper was to propose a way to better teach control system theory to engineering students. The methodology proposed consisted of presenting, for the same didactical plant, a sequence of increasingly complex design, but at the same time with better performance, such that the undesirable effects (nonlinearities, saturations, and model uncertainties, to name a few) could be better tackled. In order to exemplify the methodology, a sequence o three designs (classical, linear observer-based state space and nonlinear) for stabilize a "ball and beam" plant was showed and their results were compared. It was made clear that, in some particular cases, the simpler design techniques could not perform well (or even not perform at all, like the classical design for the plant in question) and that the time spent in the designing phase can be worthwhile. Software aided tools were widely used (both for design and simulation).

Such teaching methodology is very effective in the opinion of the authors, as it provides a complete panorama of the several automatic control techniques, and how the engineer can choose between them based in the problem at hand. In order to implement such methodology, the authors suggest two alternative ways: 1) by means of a very well coordinated sequence of classical disciplines (for example, do modeling in the classical control discipline, do classical designs in state-space control disciplines and compare both techniques and so on), or 2) create a design oriented discipline for senior students that use the techniques learned in the traditional courses. The first and third authors had the opportunity

to implement this methodology in TCC's (graduation projects) supervised by them, and the conclusion is that the student gets a complete comprehension of the subject, and become very confident in applying automatic control theory in practical problems. In many cases, he/she demonstrated interest in taking further training in the area, or even to start a research and development career.

8. REFERENCES

- Bloch, A. M., 2003, "Nonholonomic Mechanics and Control", Interdisciplinary Applied Mathematics, Springer, New York, United States of America, 483 p.
- Chen, C.T., 1999, "Linear System Theory and Design", Third edition, Oxford University Press, New York, United States of America, 352 p.
- Cruz, J. J., 1996, "Controle Robusto Multivariável", Editora da Universidade de São Paulo, São Paulo, Brazil, 163 p.
- Djaferis, T. E., 2004, "Automatic Control in First-Year Engineering Study", Control System Magazine, Vol. 24, Issue 5, p 35-37.
- Garcia, C., 2005, "Modelagem e Simulação", Second edition, Editora da Universidade de São Paulo, São Paulo, Brazil, 678 p.

Hauser, J., Sastry, S. and Kokotovic, P., 1992, "Nonlinear Control Via Approximate Input-Output Linearization: The Ball and Beam Example", IEEE Transactions in Automatic Control, number 37, vol 3, p 392-398.

Isidori, A., 1995, "Nonlinear Control Systems", Springer, London, United Kingdom, 549 p.

Murray, R. M., Waydo, S., Cremean, L.B. and Mabuchi, S., 2004, "A New Approach to Teaching Feedback", Control System Magazine, Vol. 24, Issue 5, p 38-42.

Ogata, K., 2005, "Engenharia de Controle Moderno", Prentice-Hall do Brasil Ltda, Rio de Janeiro, Brazil, 788 p.

Sanchez-Pena, R. S. and Sznaier, M., 1998, "Robust Systems: Theory and Applications", John Wiley & Sons, New York, United States of America, 258 p.

Slotine, J. J. and Li, W., 1991, "Applied Nonlinear Control", Prentice-Hall, New Jersey, United States of America, 461 p.

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