A PROPOSAL OF NEW METRIC FOR OPTIMALITY FOR SETS OF SOLUTIONS OF MULTI-OBJECTIVES PROBLEMS

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Abstract. *A new optimality metric for multi-objective problems, called Normalized Dominance Weighted Distance (NDWD), is proposed. The metric is based on Error Ratio and on Generational Distance metrics. The objective of this proposal is to evaluate the distance between the set of solution and a reference set of previous solutions. The metric is applied to an aerodynamics optimization problem. The reference set of solutions was obtained from literature. The optimization is performed by an NSGA-II implementation. The results show that the proposal metrics gives a good parameter for compare two different sets of solution and to observe the evolution of the optimization.*

Keywords: *Genetic Algorithm, NSGA-II, Optimality metrics, Transonic airfoil*

1. Introduction

Over the years, more emphasis in the area of multi-objective evolutionary algorithms (MOEA) has been laid in the development of performance metrics. A recent study of Zitler et al.(2002) has shown that for an M-objective optimization, at least M-performance metrics must be used. However, the metrics have to measure, for example: efficiency, accuracy, convergence, diversity, optimality, robustness and scalability of an optimization. In his book, Deb(2001) has classified the metrics in three classes: metrics of convergence, metrics of diversity and metrics of convergence and diversity.

The performance metrics are very useful for running monitor the optimization algorithm or to statistical analysis of and experiment of any MOEA implementation. Coello et al.(2007) describe the motivation and methodology of MOEA experimental analysis.

The present study is organized as follow: in section 2is defined the multi-objective problem; in section 3is presented the proposed metric; in section 4is presented the multi-objective problem applied to aerodynamic design; in section 7are presented the results of optimization using the proposed metric; and in section 8is presented the conclusion of study.

2. Multi-objective Problem

The development of computation techniques gives good tools for engineering designer for prediction and analysis of design. At the last decades, the mathematical development of optimization theory and techniques allows the formalization of design problems, even for several performance parameters and concurrent ones.

The Multi-objective problem (MOP) is formally written at the form:

$$
\begin{aligned}
\text{Maximition:}\quad & f_1(x) \\
& \vdots \\
& f_M(x) \\
\text{Minimition:}\quad & f_{M+1}(x) \\
& \vdots \\
& f_{M+N}(x) \\
\text{Subject to:}\quad & \\
& g_1(x) \\
& \vdots \\
& g_P(x)
\end{aligned}
$$

where M is the number of objectives to be maximized, N is the number of objectives to be minimized, P is the number of constraints, x is the variable vector, $f_i(x)$ is the i^{th} objective and $g_j(x)$ is the j^{th} constraint.

At engineering design problem, $f_i(x)$ functions are the performance of design, for example: weight of an structure and speed of an airplane. The $g_j(x)$ functions are restrictions or requirement, for example: environmental operation conditions and performance specifications. The design variable vector x are parameter value of design, for example: material, dimensions of structure and geometry.

The solution of these problem is a set of solutions called Pareto front (PF) and the solutions of set are called not dominated (Deb, 2004). These solution dominates the others possible solutions of problem. If there is a problem that the PF is known, it is possible to measure how near a set of solution, known as Pareto Known (PF_{known}), is from it and this measure is called optimality metric. But, for many engineering problems, it is not possible to define the exact solution of MOP. Thereby there is a difficulty to define how near a set of solution is from the PF.

3. Performance Metric

Leading to define a metric capable to measure the optimality of a set of solution, Veldhuizen(1999) proposed the metric Error Ratio (ER) that is the fraction of solution of a set that is dominated by PF. Veldhuizen & Lamont(1998) defined the Generational Distance (GD) to measure how far the set of solution is from the PF. The set is getting closer to PF when the metrics go to zero. Both metrics are defined based on exact solution of the MOP. Leading to engineering problems, where the exact solution is no allowed, the author proposed the metric Normalized Dominance Weighted Distance (NDWD) based on metrics ER and GD.

The metric NDWD uses a reference solution set to define the distance used at GD and this value is weighted by a factor that: is 1 if the solution dominates any solution of the reference set, -1 if it is dominated by any solution of reference set and 0 other wise. Calling f^j_{ref} the j^{th} solution of reference and f^i_{known} the i^{th} solution of PF_{known}, the metric is defined as:

$$NDWD = \sum_{i=1}^{K} \frac{e_i d_i}{K} \quad (1)$$

$$d_i = min_j(||f^j_{ref} - f^i_{known}||_2) \quad (2)$$

where K is the number of solutions on PF_{known} set. The objectives on both sets are normalized by the range of value from reference solutions taking all value at the range $[0, 1]$, to calculate the distance.

4. Aerodynamic Optimization

For the development of modern aviation, researchers are required to improve the aircraft performance. The most important motivation for this is related with the fuel consumption reduction. According to Mair and Birdsall (1998), in transonic airplanes, the specific range of an airplane is defined by Eq.3:

$$r_a = \frac{V}{cW} \cdot \frac{L}{D} \quad (3)$$

where r_a is the specific range, W is the airplane weight and c is the specific consumption. Considering the air temperature constant, V is proportional to Ma, where Ma is the Mach number (ratio between air and sound speed). So to improve r_a in a fixed Ma, is necessary to maximize $\frac{L}{D}$, where L is the lift force and D is the drag force. Thinking at a constant weight and speed, the lift coefficient (Cl, the admensionalization of L) of an airplane is defined at cruise speed. So to make the $\frac{L}{D}$ better is necessary the reduction of drag coefficient (Cd).

To perform the stabilization of the the airplane, its is necessary to use a horizontal tail. This device of aircraft generate a down force that reduce the total lift and generate the called trim drag. To obtain the reduction of this drag, therefor of the down force, it is necessary to reduce the intensity of the pitching moment (Cm).

So, leading to improve the range of aircraft, it is defined the multi-objective problem:

Maximize: $f_1 = Cl$
Minimize: $f_2 = Cd$
$f_3 = |Cm|$
e
Subject to:
camber $\leq 20\%$ of chord
thickness $\leq 30\%$ of chord
numerical stability

that represented the maximization of aircraft range. To define the fitness function that represent the airfoil, is adopted the control-points parametrization with Jameson finite volume method, as used by Cuenca(2009).

5. Genetic Algorithm

Deb et al.(2000)(a) and Deb et al.(2000)(b) have proposed the Elitist Non-dominated Sorting Genetic Algorithm II (NSGA II). Looking for the usage of elitism, it is proposed the creation of a new population from the joint of parents and sons. The ranking by dominance is applied to the created population, than the individuals of same dominance level are ordered by Crowd distance criteria (Deb, 2004) at crescent order. When the order is made, it is selected by elitism the new population. Fig.(1) shows the procedure here described. For more details about the technique, see Deb(2004).

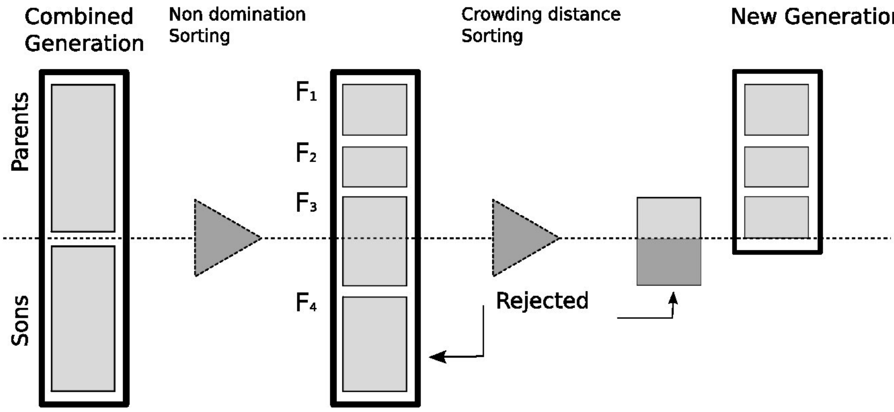

Figure 1. Elitism selection scheme of NSGA II.

This algorithm is adopted at the study because the good representation of Pareto Front that is obtained, how shown in literature.

6. Verification and validation

To perform the fitness function, it was necessary to use a computational fluid dynamics code. The one used in this study has the centered Jameson method described by Jameson et al.(1981) and the validation of code is presented at (Cuenca, 2009).

To guarantee the performance of genetic code, 2 benchmark problems were used: one with 1 variable and 2 objectives and one with 2 variables and 2 objectives.

The benchmark problem with one variable is defined by:

$$\text{Minimization:} \quad f_1 = \begin{cases} -x & ; \text{ se } x \leq 1 \\ -2 + x & ; \text{ se } 1 < x \leq 3 \\ 4 - x & ; \text{ se } 3 < x \leq 4 \\ -4 + x & ; \text{ se } 4 < x \end{cases}$$
$$f_2 = (x-5)^2$$

Subject to:

$$-5 \leq x \leq 10$$

The solution domain is $1 \leq x \leq 2$ and $4 \leq x \leq 5$. The Fig.(2)(a) shows the Pareto Front and the Fig.(2)(b) the algorithm solution.

The benchmark problem with two variable is defined by:

$$\text{Minimization:} \quad f_1 = (x-2)^2 + (y-1)^2$$
$$f_2 = 9x - (y-1)^2$$

Subject to:

$$x^2 + y^2 - 225 \leq 0$$
$$x - 3y + 10 \leq 0$$
$$-20 \leq x, y \leq 20$$

The solution domain is shown at Fig.(3)(c). The Fig.(3)(a) shows the Pareto front of problem and the Fig.(3)(b) shows the solution found by the algorithm.

7. Results

The NSGA-II was applied to the multi-objective optimization problem 10 times, all of than started with feasible random population. 200 generations were performed and the genetic algorithm has used with 25% of probability of mutation and 75% the probability of crossover. The CFD requires a long processing time and the GA requires several runs to converge. So full optimization needs long running time, because of that it was used the restart technique for

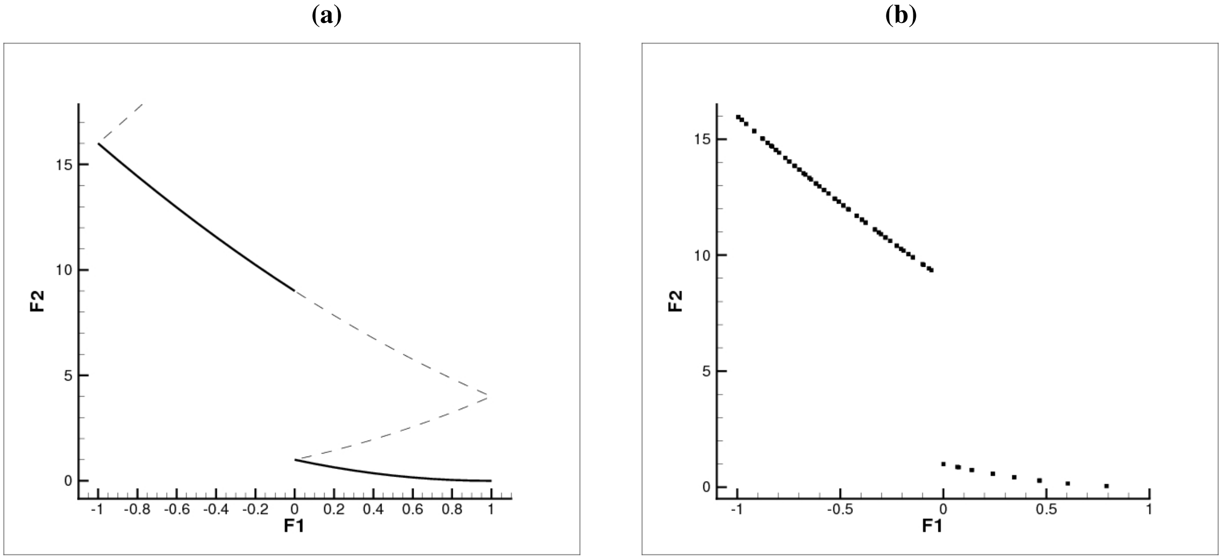

Figure 2. Exact solution (a); Approximated solution (b).

security. At every generation end the optimization state is saved allowing the algorithms restart the optimization if any problem happens, like a system shutdown.

The Fig.(4) shows the evolution of ER metric, where Fig.(4)(b) shows the mean and standard deviation of the 10 runs at the generation. The Fig.(5) shows the evolution of NDWD metric, where Fig.(5)(b) shows the mean and standard deviation of the 10 runs at the generation.

It is possible to see at figures that the new metric is consistent with ER metric. At the beginning of optimization, the metric ER and NDWD show that the solutions rise the optimality, in addition, the metric NDWD shows that the distance between the Pareto set and the reference one reduce just a little and shows a tendency to stabilise its distance.

Around the generation 65, it is possible to see a soonden loss of optimality. At this generation happened a run stop and the restart was used. Because of sole problems at restart implementation, the solutions had a loss of precision at its genes and the optimality of set was lost. After this generation the distance between the two sets of solution backs to rise. The ER metric shows that after the restart the ratio of dominated solutions stay stable but the NDWD metric shows a tendency of the PF_{known} goes far from the reference to the dominated region.

8. Conclusion

A new optimality metric for multi-objective optimization is presented. This metric is compared with Error Ratio metric and both ones are applied to an aerodynamic optimization performed by NSGA-II. 10 optimizations are evaluated, all with random initial population. The results shows that the new metric is consistent with Error Ratio and NDWD metric was able to show the behavior of Pareto front along the generations giving some informations that ER doesn't. In addition it was able to show the effect of the implementation problem in restart technique.

9. ACKNOWLEDGEMENTS

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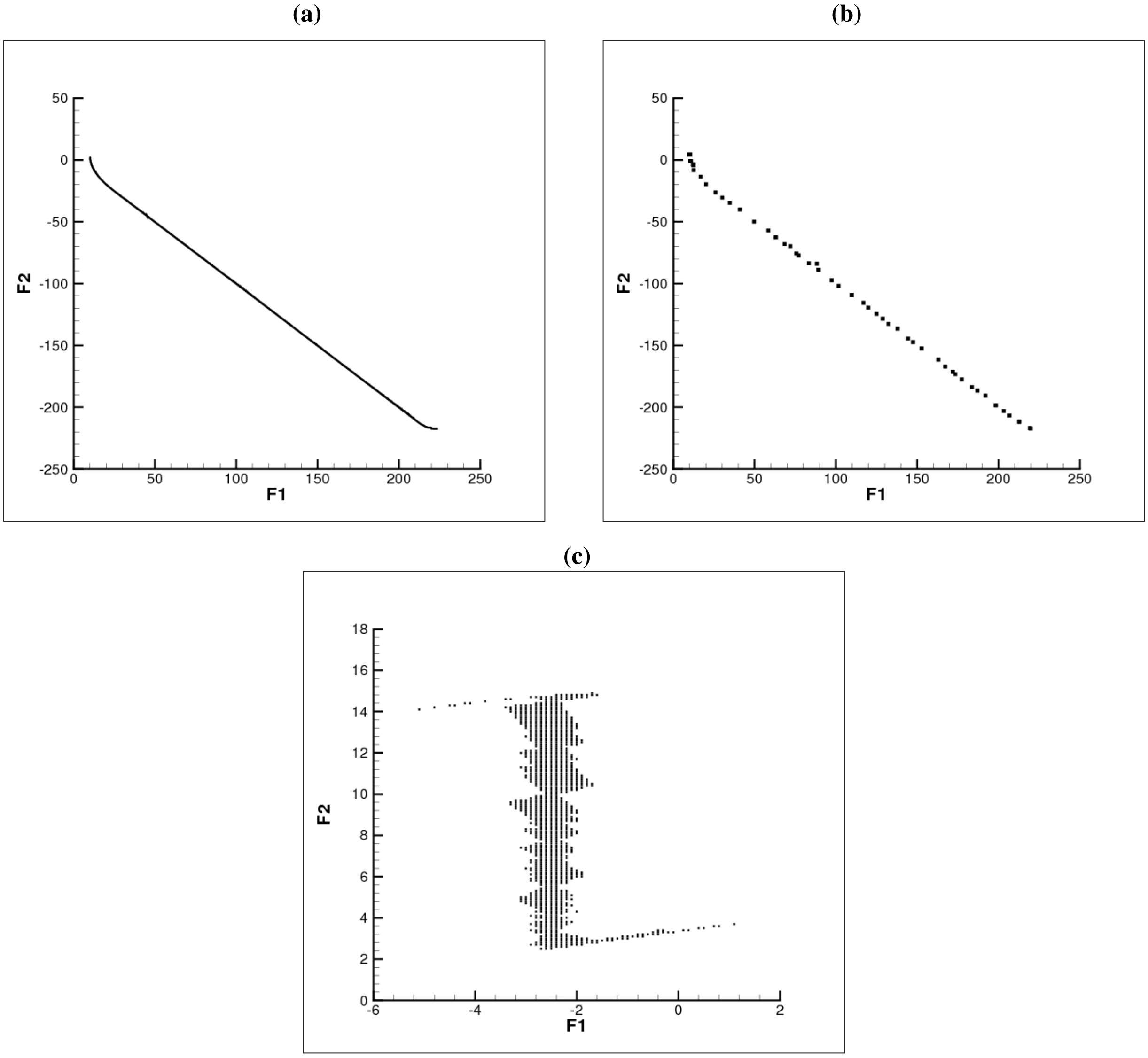

Figure 3. Exact solution (a); Approximated solution (b); Domain (c).

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(a) **(b)**

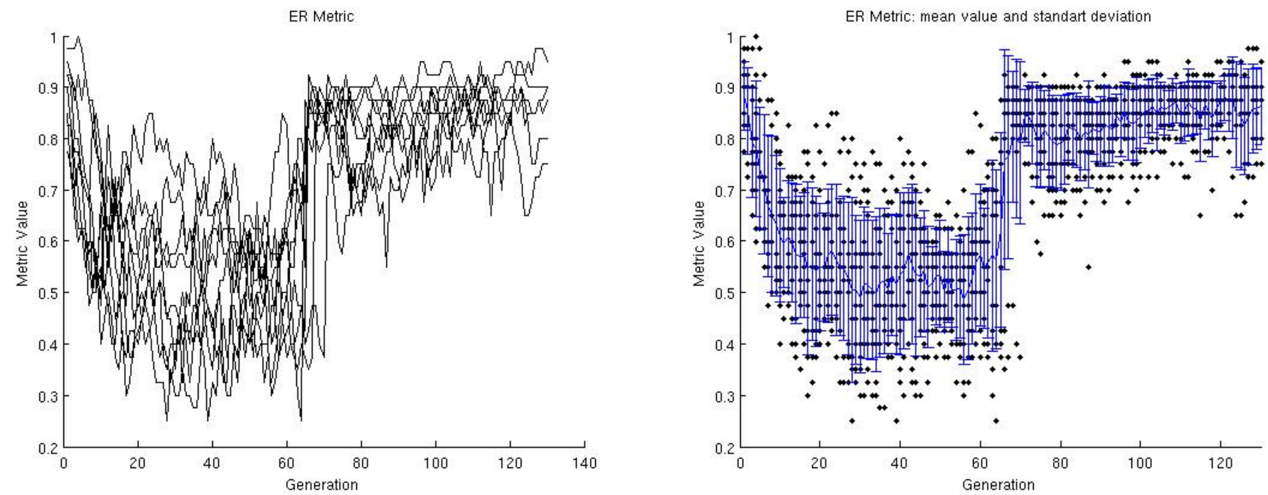

Figure 4. ER metric: (a) Evolution; (b) Mean and standard deviation.

(a) **(b)**

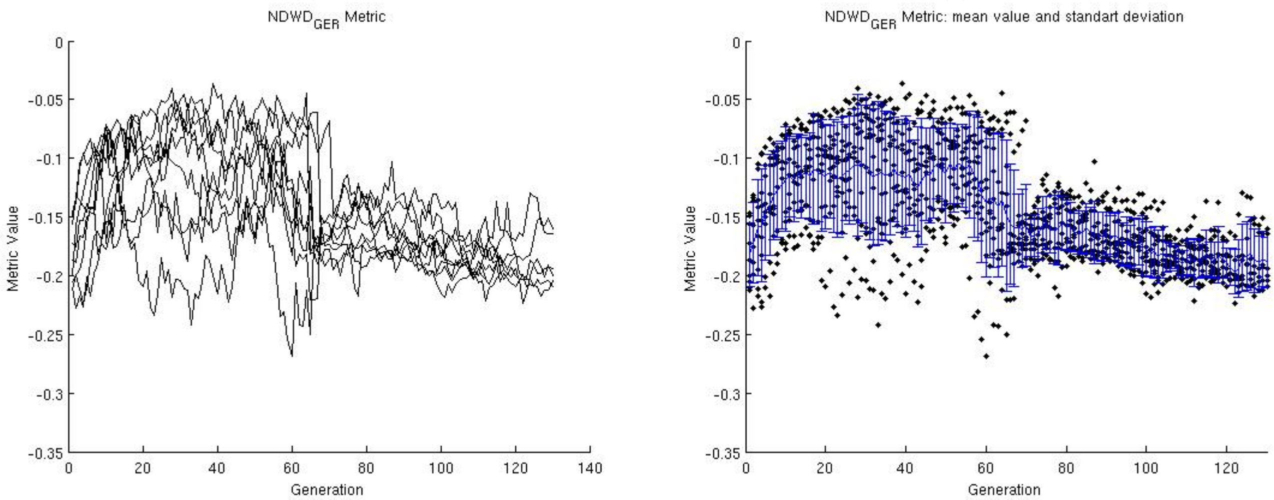

Figure 5. $NDWD$ **metric: (a) Evolution; (b) Mean and standard deviation.**