PERSISTENCE OF STRESSING IN A 4:1 ABRUPT CONTRACTION OF OLDROYD-B AND PTT VISCOELASTIC FLUIDS

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Abstract. *This paper presents a numerical investigation using the commercial software Polyflow on the criterion to classify motions proposed by Thompson (2008) which is based on the dynamics of a material element. This criterion can be applied to second order tensor fields chosen to describe the relation between material response, forces and kinematics. In our case, the focus of the investigation will be the deviatoric part of the stress tensor. An aim of this criterion is to provide a measure of the local instantaneous tendency of the material to persist on stressing the same material line. An important concept that is used is a decomposition of a tensor into in-phase and out-of-phase parts with respect to a second tensor. A persistence-of-stressing parameter is calculated using the self-correlation of the convected time derivative that vanishes the Elastic Strain Measure of the correspondent model. The case to be resolved is the planar contraction flow of Oldroy-B and PTT fluids with the exponential stress function and with a solvent viscosity ratio equal to 1/9. When comparing the results for each model, by fixing the Deborah number, we find an increase in the region of persisting of stressing as the PTT model departs from the Oldroyd-B one.*

Keywords: *Persistence of stressing; Abrupt contraction; Viscoelastic fluids; Polyflow*

1. INTRODUCTION

1.1 Extensional motion

Even with one hundred years of existence, since the work of Trouton (1906), extensional flows are still an intriguing and challenging subject Petrie (2006b). Extensional rheometry is an experimental task which has not been precisely addressed (specially for mobile liquids), in contrast to viscometric rheometry. Unfortunately material functions obtained from viscometric flows cannot be, in general, extended or translated into extensional rheological functions. Although there are precise definitions of what are the extensional material functions (e.g. Dealy (1995)), it is difficult (if not impossible) to impose an extensional kinematic to a mobile fluid which is steady in the Lagrangian and Eulerian sense. Difficulties come mainly from two sources: 1) the walls of an apparatus intended to produce an extensional kinematics “contaminate" the flow with shear; 2) growth of material lines are exponential in time and there is no room in a common laboratory to wait for achieving a steady-state flow. Therefore, researchers of extensional field have, until the moment, to be satisfied with $appearent$ extensional viscosities (James and Walters (1990), Petrie (2006a)). For this reason, experiments on extensional motion are considered indexers of extensional response, i.e. their measurements can only provide a material classification order which discriminates which material resists more to a certain flow that is predominantly extensional. Experiments which are considered to have a strong extensional character are: fiber spinning, contraction and convergent flows, flow in opposed nozzle device, four-roll mill apparatus and filament stretching.

In addition to what was discussed above is the fact that extensional flows constitute a broader class of motions, comparing to viscometric ones (Thompson and Souza Mendes (2005)), since, depending on the kind of extensional flow the material is submitted, the response can be different. In fact there are an infinity of different kinds of extensional flows. Attention of researchers is, however, dedicated to the three main representative extensional motions, namely, uniaxial, biaxial and planar extension. However, since extensional flows appeared as a concept (Trouton (1906)), till nowadays (e.g. McKinley and Sridhar (2002)), uniaxial extension has received the great part of efforts on extensional rheometry and the technique of filament-stretching seems to be the most reliable.

Since obtaining a truly extensional motion is an intrinsical challenge for general materials, and it would be interesting to know how these materials react when submitted to this kind of motion, Pountney and Walters (1978) and Huilgol (1979) defined, independently, the concept of nearly extensional flow and analyzed the stress response of incompressible simple fluid undergoing a motion which is resulted from a perturbation of an uniaxial-extensional strain history. In their analysis they determined the non-zero linear functionals, interrelations among the components of the functional and the relations with uniaxial extensional viscosity. A different approach was considered by Astarita (1979) and Thompson and Souza Mendes (2005), (2007), who have developed criteria to be $applied$ to motions in order to $measure$ how close they are from an extensional flow.

As discussed above, it is not an easy task to produce steady extensional motions. Therefore, one can conclude that what is needed to be known about materials is how they react when submitted to transient extensional flows.

In this connection Coleman (1968) defined a “motion of extension" of a particle X by the following: “$the\ history\ of$

X up to time t has been an **extension** *if there is (at least) one orthonormal basis $\mathbf{e}_i^u$ independent of s, such that the matrix of the components of $\mathbf{U}_t(t-s)$ with respect to $\mathbf{e}_i^u$ has the form*

$$[\boldsymbol{U}_t(t-s)] = \begin{bmatrix} \lambda_1^U(t-s) & 0 & 0 \\ 0 & \lambda_2^U(t-s) & 0 \\ 0 & 0 & \lambda_3^U(t-s) \end{bmatrix} \quad (1)$$

for all s, $0 \le s < \infty$. Where $\mathbf{U}_t(t-s)$ is the Right-relative-stretch tensor defined by $\mathbf{U}_t(t-s) = \sqrt{\mathbf{F}_t^T(t-s)\mathbf{F}_t(t-s)} = \sqrt{\mathbf{C}_t(t-s)}$. $\mathbf{F}_t(t-s)$ is the Relative-deformation-gradient tensor. Coleman (1968) also concluded that the Left-relative-stretch tensor, $\mathbf{V}_t(t-s)$, and $\mathbf{C}_t(t-s)$ have also null off-diagonal components in this basis, i.e. share the same eigenvectors. A direct consequence of this definition is that all R-E tensors share the same eigenvectors with $\boldsymbol{C}_t(\tau)$ and therefore, commute with each other.

Analyzing a *simple fluid* undergoing that motion, Coleman (1968) also proved that the stress tensor $\mathbf{S}(t)$ share the same proper directions or

$$\left[\boldsymbol{S}^{U_t}(t)\right] = \begin{bmatrix} \lambda_1^S(t) & 0 & 0 \\ 0 & \lambda_2^S(t) & 0 \\ 0 & 0 & \lambda_3^S(t) \end{bmatrix} \quad (2)$$

In Thompson (2008), some kinematic concepts like MWCRPSH, extensional motion, and persistence of straining tensor were adapted to a dynamic framework. Then, a criterion based on the dynamics of a material was developed to classify motions. Although the criterion can be applied to any second order tensor fields, $\mathbf{\Gamma}$, chosen to describe the relation between material response, forces and kinematics, in the present work we are concerned with the deviatoric part of the stress tensor. In this case, the present criterion intends to provide a measure of the local instantaneous tendency of the material to persist on stressing the same material line. The present criterion encompasses a large variety of materials including ones with no explicit constitutive equations, since it only needs the fields of tensor $\mathbf{\Gamma}$ and velocity vector to calculate the scalar field of the measurer considered. In order to construct the present criterion two important concepts introduced in Thompson (2008) are used. The first is a decomposition of a tensor into *in-phase* and *out-of-phase* parts with respect to a second tensor. And the second is the $\boldsymbol{\mathcal{T}}$-natural convected time derivative, $\frac{D_{\mathcal{T}}()}{Dt}$. This operator, which can be applied to any second order tensor, is a time derivative measured from a frame attached to the eigenvectors of the symmetric part, $\boldsymbol{\mathcal{T}}^S$, of a second order tensor $\boldsymbol{\mathcal{T}}$.

2. DYNAMICALLY PERSISTENT MOTIONS

There are interesting consequences on considering a finite interval $[t_0, t]$ where an extensional kinematics holds for a material element. First, this is a more realistic approach, since in a laboratory we cannot subject a material to an extensional motion since ever. Another worth noticing fact, when we consider that the motion of particle X is not an extensional motion for $\tau \in (-\infty, t_0) \cup (t, \infty)$, is related to other dynamic evolution of this material element, as measured by stress for example. If we assume that a certain material, is undergoing such a motion we would have to consider that the stress tensor (a non-kinematic quantity) is not given, in general, by an equation like Eq.(2) for the same interval $[t_0, t]$.

For the purpose of analyzing this kind of situation, let us define a *Common Transient Towards Extension Process*.

Definition 4.1: *A Common Transient Towards Extension Process is a process which, at time t_0^E, starts in a situation where the eigenvectors of the symmetric part of the velocity gradient, $\mathbf{D}$, and the eigenvectors of the stress tensor, $\mathbf{S}$, are not aligned; evolves in time monotonically to a situation, at time t_1^E, where $\mathbf{D}$ and $\mathbf{S}$ are coaxial; and stays in this last situation for a finite interval of time, till t_2^E.*

In a mathematical form, Def.(4.1), can be given as

$$\frac{d}{d\tau}\left\|\tilde{\Xi}\left[\mathbf{D},\mathbf{S}\right]\right\| < 0,\ \tau \in [t_0^E, t_1^E)$$

$$\left\|\tilde{\Xi}\left[\mathbf{D},\mathbf{S}\right]\right\| = 0,\ \tau = t_1^E$$

$$\frac{d}{d\tau}\left\|\tilde{\Xi}\left[\mathbf{D},\mathbf{S}\right]\right\| = 0,\ \tau \in (t_1^E, t_2^E] \quad (3)$$

It is worth noticing that during the stage where $\tau \in [t_1^E, t_2^E]$ the material element that is undergoing a Common Transient Towards Extension Process is not necessarily experiencing a steady motion from a Lagrangian point-of-view. Since the eigenvalues of $\mathbf{D}$ or $\mathbf{S}$ can change in this interval of time, this process for $\tau \in [t_1^E, t_2^E]$ is *Lagrangian-steady only through the perspective of the eigenvectors of these tensors* ($\mathbf{D}$, $\mathbf{S}$ and others related to the history). Hence, concerning the problem cited above in which the material starts an extensional motion at $\tau = t_0 = t_0^E$ there is a time-lag, $\Delta t_{stress} = t_1^E - t_0^E$ in which stresses accommodate and, eventually, reach the state of Eq.(2). Said differently, for $\tau \in [t_0, t_0 + \Delta t_{stress}]$, the material element keeps a memory of its history previous to time t_0, and, the stress tensor still carries this information even when the strain history does not. The material, therefore, starts a process of stress relaxation *at the directions which are orthogonal to the attractor eigendirections of rate-of-strain*, eventually vanishing, therefore, these stress components. This implies that during Δt_{stress} if we compute stresses only, we would not know that the material is undergoing an extensional motion. Moreover, if $\Delta t_{stress} > t - t_0$, it is possible that the stress tensor never reaches a state where it commutes with that extensional strain history. The time lag Δt_{stress} is a function of the material (relaxation time), but is also dependent on the previous history ($\tau < t_0$), the higher is the distance between $\mathbf{C}_t(\tau)_{\tau<t_0}$ and $\mathbf{C}_t(\tau)_{\tau\in[t_0,t]}$, the higher is Δt_{stress}.

A situation that could lead to out-of-phase stress-strain evolution, in contrast to Eqs.(1) and (2), even for a time interval of extensional motion greater then Δt_{stress} is if we consider, for example, anisotropic fluids. If the orientation director of a nematic, for example, that is exposed to an aligning magnetic field, is not coincident to any proper strain direction, an extensional motion would not guarantee stress persistent evolution of the material elements of the fluid considered. In this case, the material considered subjected to the external forces considered cannot undergo a Common Transient Toward Extension Process.

Another situation where this can occur is if we consider a plastic deformation which leads to residual stresses at the material that remain when the deformation is reversed. In this case, stresses cannot relax completely in the orthogonal directions of a new motion.

The next step to be developed is to define a dynamically persistent evolution in time (τ, $t_0 \leq \tau \leq t$) of a material element in a general framework. For this purpose it is necessary to use a history measurer of this material element.

Below we define a dynamically persistent motion of a material element using the generic relative dynamic history, $\mathfrak{J}_t(\tau)$. For the reasons discussed previously, we require that $\mathbf{\Gamma}_0 \equiv \mathbf{\Gamma} \neq \mathbf{1}$ and $\mathfrak{J}_t(\tau)$ is once differentiable[1].

Definition 4.2: *A particle X experiences a dynamically persistent motion from the point-of view of a chosen relative dynamic history, $\mathfrak{J}_t(\tau)$ between times t_0 and t if there is (at least) one orthonormal basis $\mathbf{e}_i^{\mathfrak{J}_t}$ independent of τ, such that the matrix of the components of $\mathfrak{J}_t(\tau)$ with respect to $\mathbf{e}_i^{\mathfrak{J}_t}$ has the form*

$$[\mathfrak{J}_t(\tau)] = \begin{bmatrix} \lambda_1^{\mathfrak{J}_t}(\tau) & 0 & 0 \\ 0 & \lambda_2^{\mathfrak{J}_t}(\tau) & 0 \\ 0 & 0 & \lambda_3^{\mathfrak{J}_t}(\tau) \end{bmatrix} \tag{4}$$

for all τ, $t_0 \leq \tau \leq t$. A direct consequence of this definition is that the Generic-Convected-Dynamic tensors, defined by Eq.(**??**), share the same eigenvectors as $\mathfrak{J}_t(\tau)$ and therefore, commute with each other. In other words, these tensors are coaxial. For a dynamically persistent motion it is necessary and sufficient that

$$\left\{\forall\tau,\ \tau\in[t_0,t],\ \widetilde{\mathbf{\Xi}}\left[\mathfrak{J}_t(\tau), \frac{d\mathfrak{J}_t(\tau)}{d\tau}\right] = \mathbf{0}\right\} \tag{5}$$

The special case of the rate-of-strain measure $\boldsymbol{H}_t(\tau)$ is recovered for $\mathbf{\Gamma} = \mathbf{A}_1$ and the history being the covariant one, $\mathfrak{J}_t(\tau) \equiv \mathfrak{J}_t^{cov}(\tau)$.

The above definition gives a logic structure, inside the paradigm that force is cause and kinematics is consequence, to address the dual problem of the one presented at the beginning of this section, by identifying $\mathfrak{J}_t(\tau)$ as a stress history. Hence, we can think of a common transient toward extension process through which a material element is undergoing, that, at time t_0^E, starts being stress persistent, or, equivalently, at time t_0^E starts an stress evolution that can be represented by Eq.(4). If we assume that previously to time t_0^E this material element was not undergoing an extensional motion, at time t_0^E the strain history will not begin to evolve as in Eq.(1). Analogously to its dual case, it will take a finite time, Δt_{strain}, for stress and strain to equalize their eigenvectors and complete the common transient toward extension process. During this interval time, information about the history of the material previous to time t_0 is stored (available) in a relative strain tensor. During Δt_{strain}, there is a process of reverse deformation in which the material gradually stops to deform in the directions orthogonal to the stress eigendirections. If the material will undergo a general extensional motion or a EMWCRPSH, depends on the existence of a fixed triad of eigenvalues attractor (besides the triad of eigenvectors attractor) , which in the case of viscoelastic materials is given by the condition of a vanishing natural time derivative of stress.

As it was done when we considered the kinematic quantity $\boldsymbol{H}_t(\tau)$, we are able to give a definition of a local instantaneous persistent dynamic evolution. This happens when the tendency of the dynamic history as measured by $\mathfrak{J}_t(\tau)$, in a vicinity of arbitrarily *small* $s = t - \tau$ is to maintain its orthonormal eigenvectors. So, we can define an instantaneous dynamically persistent motion as

Definition 4.3: *A local instantaneous dynamically persistent evolution at time t of a particle X occurs when the orthonormal eigenvector basis of $\mathfrak{J}_t(\tau)$ and its derivative relative to τ, $\frac{d}{d\tau}\mathfrak{J}_t(\tau)$, are the same when τ is,*

[1] The case where one identifies the identity tensor as the special dynamic tensor at the present time to $\mathbf{\Gamma} = \mathbf{1}$ can be treated in a similar way as done with the strain history $\boldsymbol{C}_t(\tau)$, i.e. requiring that the history is twice differentiable and working with the first and second derivatives.

arbitrarily close to t.

$$\lim_{\tau \to t} \widetilde{\boldsymbol{\Xi}} \left[\mathfrak{J}_t(\tau), \frac{d}{d\tau} \mathfrak{J}_t(\tau) \right] = \mathbf{0} \tag{6}$$

And the corollary analogous to Corollary 3.1 is given by

Corollary 4.1: *A particle* X *at time* t *experiences a local instantaneous dynamically persistent evolution if and only if* $\boldsymbol{\Gamma}$ *and* $\boldsymbol{\Gamma}_1$ *commute.*

And the theorem that can be proven by the same procedure as Theorem 3.1, is stated as follows

Theorem 4.1: *A motion between times* t_0 *and* t *of a particle* X *is dynamically persistent if and only if this particle* X *experiences a local instantaneous dynamically persistent motion at every instant of time which belongs to the that finite interval* $[t_0, t]$.

3. PROBLEM FORMULATION AND NUMERICAL IMPLEMENTATION

3.1 Conservation equations

The velocity and pressure fields are defined by the governing equations that impose conservation of mass and momentum for an incompressible fluid, together with the appropriate boundary conditions. In the present work, the main hypothesis considering the flow conditions are

1. The fluid is incompressible.
2. Steady-state laminar regime.
3. Body forces are conservative.
4. Isothermal flow.

With these hypothesis, the conservation of mass is given by

$$\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial u}{\partial r} = 0, \tag{7}$$

while the conservation of momentum is given by

$$\rho \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} \right) = \frac{1}{r}\frac{\partial}{\partial r}(rT_{xr}) + \frac{\partial}{\partial x}(T_{xx}) \tag{8}$$

$$\rho \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial r} \right) = \frac{1}{r}\frac{\partial}{\partial r}(rT_{rr}) - \frac{T_{\theta\theta}}{r} + \frac{\partial}{\partial x}(T_{rx}) = 0 \tag{9}$$

Where u and v are respectively the axial and radial components of the velocity field $\mathbf{u}$ and the quantities T_{xx}, T_{xr}, T_{rx}, T_{rr} and $T_{\theta\theta}$ are the components of the stress tensor $\mathbf{T}$.

3.2 Constitutive equations

In order to close the set of unknowns and equations given by Eqs. (7), (8), and (9), addition equations are needed. Generally they come from an assumption on the behavior of the material when it is subjected to a certain state of stress. This constitutive equation is, therefore, a relation between the stress tensor $\mathbf{T}$ and kinematics. In the present work, we two viscoelastic materials models: Oldroyd-B and Phan-Thien-Tanner. The two models are based in a split of the extra-stress part of the stress into a solvent and a polymeric contributions as

$$\mathbf{T} = -p\mathbf{1} + 2\eta_s\mathbf{D} + \boldsymbol{\tau}_p \tag{10}$$

where p is the pressure, η_s is the solvent viscosity, $\mathbf{D} \equiv 0.5\left(\nabla\mathbf{v} + \nabla^T\mathbf{v}\right)$ is the symmetric part of the velocity gradient, and $\boldsymbol{\tau}_p$ is the stress that comes from the polymeric molecules. In the Oldroyd-B fluid the evolution of $\boldsymbol{\tau}_p$ is given by

$$\boldsymbol{\tau}_p + \lambda\overset{\triangledown}{\boldsymbol{\tau}}_p = 2\eta_p\mathbf{D} \tag{11}$$

where $\overset{\triangledown}{\boldsymbol{\tau}}_p$ denotes the contravariant convected time derivative of $\boldsymbol{\tau}_p$.

while in the PTT model the evolution of $\boldsymbol{\tau}_p$ is given by

$$f(\mathrm{tr}\boldsymbol{\tau}_\mathrm{p})\boldsymbol{\tau}_\mathrm{p} + \lambda\overset{\triangledown}{\boldsymbol{\tau}}_\mathrm{p} + \xi\left(\boldsymbol{\tau}_\mathrm{p}\cdot\mathbf{D} + \mathbf{D}\cdot\boldsymbol{\tau}_\mathrm{p}\right) = 2\eta_\mathrm{p}\mathbf{D} \tag{12}$$

where tr denotes the trace operator, ξ is a parameter related to the slipage of the polymeric network. Depending on this parameter, different convected time derivatives are obtained (the whole spectrum of the Gordon-Schowalter convected time derivative). The function of the trace $f(\mathrm{tr}\boldsymbol{\tau}_{\mathrm{p}})$ can be given by two expressions

$$f(\mathrm{tr}\boldsymbol{\tau}_{\mathrm{p}}) = \exp\left(\frac{\epsilon\lambda}{\eta}\mathrm{tr}\boldsymbol{\tau}_{\mathrm{p}}\right) \tag{13}$$

or

$$f(\mathrm{tr}\boldsymbol{\tau}_{\mathrm{p}}) = 1 + \frac{\epsilon\lambda}{\eta}\mathrm{tr}\boldsymbol{\tau}_{\mathrm{p}} \tag{14}$$

In the present work, all the PTT results were obtained with $\xi = 0$ and the exponential version of $f(\mathrm{tr}\boldsymbol{\tau}_{\mathrm{p}})$ given by Eq.(13)

3.3 Abrupt contraction

In order to measure the persistence of stressing a planar 4:1 abrupt contraction was considered. The boundary conditions for this problem are

1. fully-developed flow at the inlet and outlet.
2. no-slip condition at the wall.
3. symmetry at the middle plane.

3.4 Solution of the equation system by Galerkin / Finite Element Methods

The commercial software Polyflow $3.11.0$ is used to solve the differential equations that govern the problem. They are solved in a coupled manner by the Galerkin/Finite Element Method. Biquadratic basis functions ϕ_j are used to represent the velocity and nodal coordinates, while linear discontinuous functions χ_j are employed to expand the pressure field. The velocity and pressure are represented in terms of appropriate basis functions

$$u = \sum_{j=1}^{n} U_j\phi_j \quad ; \quad v = \sum_{j=1}^{n} V_j\phi_j \quad ; \quad p = \sum_{j=1}^{m} P_j\chi_j \quad ; \tag{15}$$

The coefficients of the expansions are the unknown of the problem

$$\underline{c} = \left[\begin{array}{ccc} U_j & V_j & P_j \end{array}\right]^T$$

The corresponding weighted residuals of the Galerkin method related to conservation of momentum, mass and mesh generation are:

$$R_c^i = \int_{\bar{\Omega}} \left[\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial u}{\partial x}\right]\chi_i r||J||d\bar{\Omega} \tag{16}$$

$$\begin{aligned} R_{mx}^i \quad = \quad & \int_{\bar{\Omega}} \left[\frac{\partial \phi_i}{\partial x}T_{(xx)} + \frac{\partial \phi_i}{\partial r}T_{(xr)}\right] r||J||d\bar{\Omega} \\ & - \int_{\bar{\Gamma}} \mathbf{e}_x \cdot (\mathbf{n}\cdot\mathbf{T})\phi_i r\frac{d\Gamma}{d\bar{\Gamma}}d\bar{\Gamma} \end{aligned} \tag{17}$$

$$\begin{aligned} R_{mr}^i \quad = \quad & \int_{\bar{\Omega}} \left[\frac{\partial \phi_i}{\partial x}T_{(xr)} + \frac{\partial \phi_i}{\partial r}T_{(rr)} + \frac{\phi}{r}T_{(\theta\theta)}\right] r||J||d\bar{\Omega} \\ & - \int_{\bar{\Gamma}} \mathbf{e}_r \cdot (\mathbf{n}\cdot\mathbf{T})\phi_i r\frac{d\Gamma}{d\bar{\Gamma}}d\bar{\Gamma} \end{aligned} \tag{18}$$

3.5 Solution of the non-linear system of algebraic equation by Newton's Method

As indicated above, the system of partial differential equations, and boundary conditions is reduced to a set of simultaneous algebraic equations for the coefficients of the basis functions of all the fields. This set is non-linear and sparse. It is solved by Newton's method. The linear system of equations at each Newton iteration was solved using a frontal solver.

4. RESULTS

4.1 Newtonian

Figures 1, 2, 3, and 4 the persistence of straining and persistence of stressing of the Newtonian case.

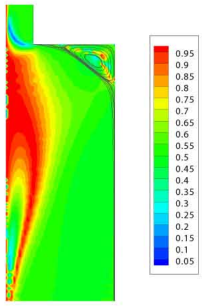

Figure 1. Persistence of straining with a contravariant measurer.

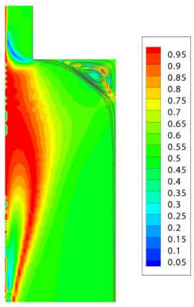

Figure 2. Persistence of straining with a covariant measurer.

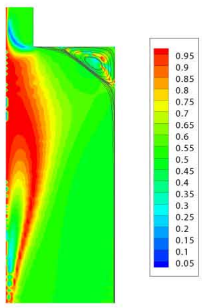

Figure 3. Persistence of stressing with a contravariant measurer.

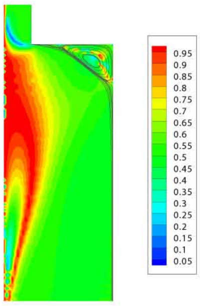

Figure 4. Persistence of stressing with a covariant measurer.

4.2 Oldroyd-B

Figures 5, 6, 7, and 8 the persistence of straining and persistence of stressing of the Oldroyd-B case for a value of the Deborah number $De = 2$.

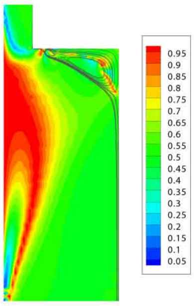

Figure 5. Persistence of straining with a contravariant measurer.

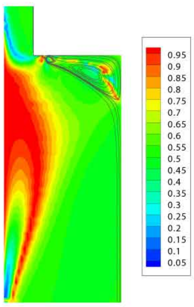

Figure 6. Persistence of straining with a covariant measurer.

4.3 PTT

Figures 9, 10, 11, and 12 the persistence of straining and persistence of stressing of the PTT case for a value of the Deborah number $De = 2$.

4.4 Discussion

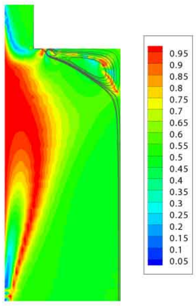

Figure 7. Persistence of stressing with a contravariant measurer.

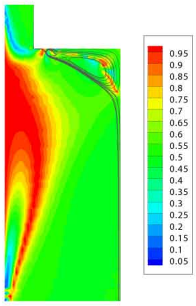

Figure 8. Persistence of stressing with a covariant measurer.

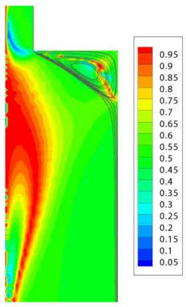

Figure 9. Persistence of straining with a contravariant measurer.

From our study realized there was a region of red near the contraction means that there the fluid persists in pulled or extended depending on the type of analysis that is being made. We note that the region near the wall we have the green that mean that there is occurring the shear of the fluid and blue next to the center of the vortex where we have the motion of rigid body.

We can see that for the Newtonian case as expected the kinematic and dynamic fields were equal. This occurs because the stress tensor is simply proportional to the rate of deformation tensor in the fluid constitutive equation, ie the eigenvectors are the same for these two tensors for a given position in the field. In other words the same result shows us that whatever study by a kinematic or dynamic vision the tendency of movement to continues is the same for the Newtonian case.

There is a green area within the region in red for all kinematic results, ie independent of the model or that are derived used this region always appears, which does not occur in any of the dynamic results. The results of kinematic Oldroyd B and PTT models are similar but not identical. At the end of the red region for the case of De iqual to 2 realize a more

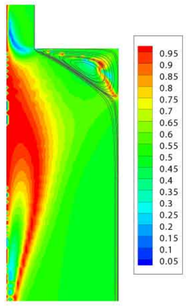

Figure 10. Persistence of straining with a covariant measurer.

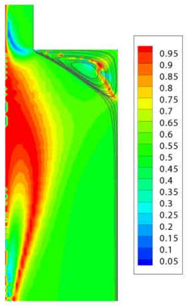

Figure 11. Persistence of stressing with a contravariant measurer.

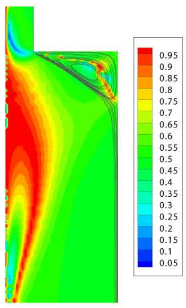

Figure 12. Persistence of stressing with a covariant measurer.

curve to the PTT case.

About the size of vortex realize that the Newtonian vortex does not reach the contraction, ie the size of the vortex Newtonian is smaller than models of PTT and OldroydB. In the case of model OldroydB near the lower pipe have the appearance of the second vortex.

Now for the case of the dynamic results we noticed that the region is where the traction is lower for the case of the PTT to OldroydB.

5. FINAL REMARKS

This is a preliminary work on the subject and therefore we still have to investigate other conditions/geometry to have a picture of the classification. The persistence of stressing can be a measure that gives important information of the flow as compared to the persisitence of straining. However, for this to appear in a more pronounced manner, we should go to higher Deborah numbers.

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8. Responsibility notice

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