IMPROVED QUANTUM-INSPIRED EVOLUTIONARY ALGORITHM APPLIED TO ECONOMIC DISPATCH PROBLEM OF THERMAL UNITS

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Abstract. The objective of the economic dispatch problem (EDP) of electric power generation, whose characteristics are complex and highly nonlinear, is to schedule the committed generating unit outputs so as to meet the required load demand at minimum operating cost while satisfying all unit and system equality and inequality constraints. Recently, as an alternative to the conventional mathematical approaches, modern heuristic optimization techniques such as simulated annealing, evolutionary algorithms, neural networks, ant colony and tabu search have been given much attention by many researchers due to their ability to find an almost global optimal solution in EDPs. On the other hand, quantum computing is a new computing paradigm that has the potential to bring a new class of previously intractable problems within the reach of computer science. Quantum mechanical computers were proposed in the early 1980s and the description of quantum mechanical computers was formalized in the late 1980s. Many efforts on quantum computers have progressed actively since the early 1990s because these computers were shown to be more powerful than classical computers on various specialized problems. There are well-known quantum algorithms such as Shor’s quantum factoring algorithm and Grover’s database search algorithm. Research on merging evolutionary computation and quantum computation has been started since late 1990. Inspired on the quantum computation, this paper presented an improved quantum inspired evolutionary algorithm (IQEA) based on diversity information of population. A classical quantum inspired evolutionary algorithm (QEA) and the IQEA approaches are validated for a test system consisting of thirteen units, whose incremental fuel cost function takes into account the valve-point loading effects.

Keywords: economic dispatch, thermal units, electrical energy, optimization, evolutionary algorithms, quantum computing

1. INTRODUCTION

The economic dispatch optimization problem is one of the fundamental issues in power systems to obtain benefits with the stability, reliability and security. Its objective is to allocate the power demand among committed generators in the most economical manner, while all physical and operational constraints are satisfied. The cost of power generation, particularly in fossil fuel plants, is very high and economic dispatch helps in saving a significant amount of revenue (Chatuverdi et al., 2008).

Several optimization methods and techniques have been researched. In the conventional methods such as the lambda-iteration method, the base point and participation factors and the gradient methods, an essential assumption is that the incremental cost curves of the units are monotonically increasing piecewise linear functions, but the practical systems are nonlinear. However, conventional methods like lambda-iteration, quadratically constrained programming, gradient methods, among others, rely heavily on the convexity assumption of generator cost curves and usually approximate these curves using quadratic, piecewise quadratic or higher order polynomial cost functions (Wood and Wollenberg, 1984). When fuel cost function is approximated by nonsmooth or non-convex function, numerical methods are no longer applicable. For example, practical economic dispatch problems with valve-point and multi-fuel options are represented as a nonsmooth optimization problem.

Recently, a number of meta-heuristics, for example simulated annealing (Basu, 2005), genetic algorithm (Walters and Sheble, 1993), evolutionary programming (Sinha et al., 2003), differential evolution (Noman and Iba, 2008), cultural differential evolution (Coelho et al., 2008), tabu search (Lin et al., 2002), and particle swarm optimization (Panigrahi et al., 2008) have been applied to solve the economic dispatch optimization problem.

In the optimization context based on meta-heuristics, a promising area in which the combination of quantum computation and evolutionary algorithms can be useful to solve optimization problems like economic dispatch quickly with high quality solutions and stable convergence characteristics, whereas it is easily implemented. It has been shown that quantum computation can dramatically improve performance for solving problems like factoring (Shor, 1994) or...
searching in an unstructured database (Grover, 1997). On the other hand, evolutionary algorithms can be described, basically, as search algorithms.

In this paper, we present an improved quantum-inspired evolutionary algorithm (IQEA) based on the work of Han and Kim (2002) and diversity information of population (Ursem, 2002; Ursem, 2003; Coelho et al., 2009). In Han and Kim (2002) is proposed a quantum-inspired evolutionary algorithm (QEA), which is based on the concepts and principles of quantum computation, such as quantum-inspired bit (Q-bit) and superposition of states. Like the evolutionary algorithms, QEA is also characterized by the representation of the individual, the evaluation function and the population dynamics. However, instead of binary, numeric or symbolic representation, QEA uses a Q-bit as a probabilistic representation, defined as the smallest unit of information.

An economic dispatch problem is employed to demonstrate the performance of the QEA and IQEA approaches. In this context, a thirteen-unit test system (Sinha et al., 2003) with incremental fuel cost function taking into account the valve-point loading effects is used to illustrate the effectiveness of the QEA and the proposed IQEA method. Simulation results obtained with the QEA and IQEA approaches were analyzed and compared with other optimization results reported in literature.

The remainder of this paper is organized as follows: section 2 describes the formulation of the economic dispatch problem, while section 3 explains the fundamentals of QEA and IQEA approaches. Subsequently, section 4 provides the simulation results for a thirteen-unit test system. Lastly, conclusion is given in the section 5.

2. FUNDAMENTALS OF ECONOMIC DISPATCH OPTIMIZATION PROBLEM

The primary concern of an economic dispatch problem is to minimize the total fuel cost at thermal power plants subjected to the operating constraints of a power system. Therefore, it can be formulated mathematically with an objective function and two constraints. The equality and inequality constraints are represented by equations (1) and (2) given by:

\[
\sum_{i=1}^{n} P_i - P_L - P_D = 0 \quad (1)
\]

\[
P_i^{\text{min}} \leq P_i \leq P_i^{\text{max}} \quad (2)
\]

In the power balance criterion, an equality constraint must be satisfied, as shown in equation (1). The generated power should be the same as the total load demand plus total line losses. The generating power of each generator should lie between maximum and minimum limits represented by equation (2), where \( P_i \) is the power of generator \( i \) (in MW); \( n \) is the number of generators in the system; \( P_D \) is the system’s total demand (in MW); \( P_L \) represents the total line losses (in MW) and \( P_i^{\text{min}} \) and \( P_i^{\text{max}} \) are, respectively, the output of the minimum and maximum operation of the generating unit \( i \) (in MW). The objective of minimization of the total fuel cost function is formulated as follows:

\[
\min \ f = \sum_{i=1}^{n} F_i( P_i ) \quad (3)
\]

where \( F_i \) is the total fuel cost for the generator unit \( i \) (in \$/h), which is defined by equation:

\[
F_i( P_i ) = a_i P_i^2 + b_i P_i + c_i \quad (4)
\]

where \( a_i \), \( b_i \) and \( c_i \) are cost coefficients of generator \( i \).

The sequential valve-opening process for multi-valve steam turbines produces ripple like effect in the heat rate curve of the generator. This effect is included in economic dispatch problem by superimposing the basic quadratic fuel-cost characteristics with a rectified sinusoidal component. In this context, the equation (4) can be modified as:

\[
\tilde{F}_i( P_i ) = F_i( P_i ) + \left| e_i \sin \left( f_i \left( P_i^{\text{min}} - P_i \right) \right) \right| \quad \text{or} \quad (5)
\]

\[
\tilde{F}_i( P_i ) = a_i P_i^2 + b_i P_i + c_i + \left| e_i \sin \left( f_i \left( P_i^{\text{min}} - P_i \right) \right) \right| \quad (6)
\]
where \( e_i \) and \( f_i \) are valve-point loading coefficients of generator \( i \). Hence, the total fuel cost that must be minimized, according to equation (3), is modified to:

\[
\min f = \sum_{i=1}^{n} \tilde{F}_i(P_i)
\]  

(7)

where \( \tilde{F}_i \) is the cost function of generator \( i \) (in \$/h) defined by equation (6). In the case study presented here, we disregarded the transmission losses, \( P_L \) (mentioned in equation (1)), i.e., in this work \( P_L = 0 \).

3. QUANTUM-INSPIRED EVOLUTIONARY ALGORITHMS

Since Deutsch first proposed the Deutsch-Jozsa algorithm in 1985 (Deutsch, 1985), quantum computation has been widely drawing the attention of many researchers in formulation of new optimization approaches. Quantum computation is a novel inter-discipline that includes quantum mechanics and information science. This emergent research field concentrates on studying quantum computation, which is characterized by certain principles of quantum mechanics such as interference, quantum bits, coherence and superposition of states (Nedjah et al., 2008).

Quantum-inspired evolutionary algorithms can be viewed as probability optimization algorithms based on quantum computation concept and theory. Recently, some quantum-inspired evolutionary algorithms have been proposed for some combinatorial (Han and Kim, 2000, 2002, 2004) and continuous (Abs da Cruz et al., 2004) optimization problems.

The next section describes the QEA proposed by Han and Kim (2002). First, a brief overview of the QEA is provided, and finally the proposed IQEA algorithm is presented.

3.1. QEA

Han and Kim (2002) proposed the QEA introducing a Q-gate as a variation operator to promote the optimization of the individual Q-bit. QEA uses Q-bits (Q-bit is defined in Han and Kim (2002) and means quantum-inspired bit, which is different from qubit or quantum bit) as the smallest unit of information for representing individuals. A Q-bit-coded individual probabilistically represents all the states in the search space. The individuals are updated by quantum rotation gates, which can achieve an evolutionary search.

Unlike the classical bit, the Q-bit does not represent only the value 0 or 1 but a superposition of the two. Its state can be given by:

\[
|\psi\rangle = \alpha |0\rangle + \beta |1\rangle
\]  

(8)

where \( |0\rangle \) and \( |1\rangle \) represent respectively the classical bit values 0 and 1; \( \alpha \) and \( \beta \) are complex numbers that specify the probability amplitudes of the corresponding quantum state. Normalization of the state to unity always guarantees:

\[
|\alpha|^2 + |\beta|^2 = 1.
\]  

(9)

If a superposition is measured with respect to the basis \{ \(|0\rangle , |1\rangle \) \}, the probability to have the value 0 is \( |\alpha|^2 \) and the probability to have the value 1 is \( |\beta|^2 \). In classical computing, the possible states of a \( n \)-bit system form a vector space of \( n \) dimensions, i.e., we have \( 2^n \) possible states. However, in a quantum system of \( n \) Q-bits the resulting state space has \( 2^n \) dimensions. It is this exponential growth of the state space with the number of particles that suggests a possible exponential speed-up of computation on quantum computers over classical computers. Each quantum operation will deal with all the states present within the superposition in parallel.

The representation of a \( m \)-Q-bit individual is defined as follows:

\[
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_m \\
\beta_1 \\
\beta_2 \\
\vdots \\
\beta_m
\end{bmatrix}
\]

(10)

where \( |\alpha_i|^2 + |\beta_i|^2 = 1, \; i = 1, 2, \ldots, m \). Q-bit representation has the advantage that it is able to represent a linear superposition of states probabilistically. Furthermore, in this way, the \( m \)-Q-bit individual is able to represent the information of \( 2^m \) binary states simultaneously.
For the update process of QEA, suitable quantum gate $U(\Delta \theta)$ is usually adopted in compliance with optimization problems. In this work, a quantum rotation gate, such as

$$U(\Delta \theta) = \begin{bmatrix} \cos(\Delta \theta) & -\sin(\Delta \theta) \\ \sin(\Delta \theta) & \cos(\Delta \theta) \end{bmatrix}$$

is adopted as a basic gate of QEA, where $\Delta \theta, i=1,\ldots,m$ is the rotation angle of each Q-bit toward either 0 or 1 state depending on its sign. The values of $\Delta \theta, i=1,\ldots,m$ should be designed in compliance with the application problem. Then let us briefly review the procedure of QEA in Figure 1. For more details the reader is referred to Han and Kim (2002).

### Figure 1. Pseudocode of QEA for continuous optimization.

In Figure 1, $Q(t) = \{q_1^t, q_2^t, \ldots, q_n^t\}$ is a population of $n$ Q-bit individuals at generation $t$; $q_j^t$ is the $j$th ($j=1,2,\ldots,n$) individual defined as

$$q_j^t = \begin{bmatrix} \alpha_{j1}^t & \alpha_{j2}^t & \cdots & \alpha_{jm}^t \\ \beta_{j1}^t & \beta_{j2}^t & \cdots & \beta_{jm}^t \end{bmatrix}$$

and $P(t) = \{x_1^t, x_2^t, \ldots, x_n^t\}$ is a set of binary solutions from observing the states of $Q(t)$, where $x_j^t$ is the binary solution by observing $q_j^t$ ($j=1,2,\ldots,n$). A set of $n$ binary solutions $B(t) = \{b_1^t, b_2^t, \ldots, b_n^t\}$ is maintained at the generation $t$, where $b_j^t$ is the best $j$th binary solution $x_j$ between the generations 0 and $t$. $b$ is the best binary solution among $B(t)$.

In the ‘initialize $Q(0)$’ step, each pair of Q-bit probability amplitudes, $\alpha_{ji}$ and $\beta_{ji}$, $i=1,2,\ldots,m$, are initialized with $1/\sqrt{2}$, $\forall q_j^t \in Q(t)$. The next step generates a set of binary solutions $P(t) = \{x_1^t, x_2^t, \ldots, x_n^t\}$ where each bit of $x_j^t$, $j=1,2,\ldots,n$, is formed by determining the explicit state of each Q-bit of $q_j^t$, $|0\rangle$ state or $|1\rangle$ state, according to either $|\alpha_{ji}|^2$ or $|\beta_{ji}|^2$ of $q_j^t$, $i=1,2,\ldots,m$. For example, to form a explicit state of $i$th bit of $x_j^t$ ($i=1,2,\ldots,m$) a number $h$
in range \([0,1]\) is generated randomly with uniform distribution. Then, if \(h < |\beta_j'|\) the \(i\)th bit of \(x_j'\) is set to be 1, otherwise, it is set to be 0. Each solution \(x_j' \in P(t)\), \(j = 1,2,\ldots,n\), is a binary string of length \(m\), and is evaluated to give some measure of its fitness. However, before evaluating the solutions, \(P(t)\) is converted from binary to floating point representation \(\mathcal{P}(t)\). The initial binary solutions \(P(t)\) are stored in \(B(t)\), and the best binary solution \(b\) among \(B(t)\) is then selected and stored. In the while loop, the quantum gate \(U(\Delta \theta)\) is used to update \(Q(t-1)\) so that fitter states of the Q-bit individuals are generated. The \(i\)th Q-bit value \((\alpha_i', \beta_i')\) of \(q_j'\) is updated as

\[
\begin{bmatrix}
\alpha_i'^{t+1} \\
\beta_i'^{t+1}
\end{bmatrix} = U(\Delta \theta) \begin{bmatrix}
\alpha_i' \\
\beta_i'
\end{bmatrix} = \begin{bmatrix}
\cos(\Delta \theta') & -\sin(\Delta \theta') \\
\sin(\Delta \theta') & \cos(\Delta \theta')
\end{bmatrix} \begin{bmatrix}
\alpha_i' \\
\beta_i'
\end{bmatrix}
\]

The best solutions among \(P(t)\) and \(B(t-1)\) are then selected in the next step, and if the best current solution is fitter than the best stored solution, the best stored solution will be replaced by this current solution.

Figure 2 depicts the polar plot of the rotation gate for Q-bit individuals. In the economic dispatch problem evaluated in this work, the angle parameters and lookup table (see Table 1) used for the rotation gate of classical QEA are the same adopted in Han and Kim (2002). In this work, \(\theta_3 = 0.05\pi\), \(\theta_5 = -0.05\pi\), and 0 for the rest were used. The magnitude of \(\Delta \theta\) has an effect on the speed of convergence, but if it is too big, the solutions may diverge or converge prematurely to a local optimum. The sign of \(\Delta \theta\) determines the direction of convergence (Han and Kim, 2002).

![Polar plot of the rotation gate for Q-bit individuals](image)

**Figure 2.** Plot polar of the rotation gate for Q-bit individuals.

<table>
<thead>
<tr>
<th>(x_i')</th>
<th>(b_i')</th>
<th>(f(x'_i) \geq f(b'_i))</th>
<th>(\Delta \theta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>False</td>
<td>(\theta_1)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>True</td>
<td>(\theta_2)</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>False</td>
<td>(\theta_3)</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>True</td>
<td>(\theta_4)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>False</td>
<td>(\theta_5)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>True</td>
<td>(\theta_6)</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>False</td>
<td>(\theta_7)</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>True</td>
<td>(\theta_8)</td>
</tr>
</tbody>
</table>
If the global migration condition is satisfied, the best solution is migrated to $B(t)$ globally. If the local migration condition is satisfied, the best one in a local group in $B(t)$ is migrated to others in the same local group. The migration process can induce a variation of the probabilities of a Q-bit individual. A local group in QEA is defined as the subpopulation affected mutually by a local migration, and its size is the number of individuals in the local group (Han and Kim, 2006).

In terms of stopping criterion, a limit of generation counter $t_{max}$ is adopted.

### 3.2. Proposed IQEA

The proposed IQEA adopted a NOT gate to tune the Q-bits if the best fitness did not improve while the following relation is satisfied: $(t/t_{max}) > 0.01$. In this case, a position $i$ of Q-bit individual at generation $t$ is sorted and the $i$th Q-bit value $(\alpha_i, \beta_i)$ is updated using the NOT gate. The NOT gate has 50% of application probability. This procedure has inspiration in mutation operation used in several evolutionary algorithms.

Choosing suitable control parameter values in QEA is, frequently, a problem dependent task and requires previous experience of the user. Despite its crucial importance, there is no consistent methodology for determining the control parameters of evolutionary algorithms, which are, most of the time, arbitrarily set within some predefined ranges (Maruo et al., 2005).

In context of population-based algorithms, an attractive and repulsive approach was introduced by Ursem (2002) and Ursem (2003), in particle swarm optimization design, and other modified diversity measure was introduced in Coelho et al. (2009), in a differential evolution approach to escape from the current local optimum. It uses a diversity measure to control the population. The result is a powerful algorithm that alternates between phases of attraction and repulsion. The application of diversity measures can be an alternative strategy to improve the convergence performance of QEA with adaptive evolution mechanism.

The trade-off between the exploration (i.e. the global search) and the exploitation (i.e. the local search) of the search space is critical to the success of a QEA approach. The tuning of rotation angle $\Delta \theta_k$ given by Table 1 is a key factor affecting the QEA’s convergence proposed in Han and Kim (2002).

In this paper, it is proposed the following rule for the tuning of $\theta_3$ and $\theta_5$ (zero for the rest was used) using a diversity measure based on the number of 1’s in $P(t) = \{x'_1, x'_2, \ldots, x'_n\}$. In context of IQEA, the adopted approach to tune the rotation angle $\theta$ is given by pseudocode of Figure 3.

```
If f > f_{best} and P_j(t) = 0 and B_j(t) = 0
  \theta_3 = (1 - \text{sum}(P_j(t))) \cdot \pi ;
End
If f > f_{best} and x'_j = 1 and B_j(t) = 0
  \theta_5 = -(1 - \text{sum}(P_j(t))) \cdot \pi ;
End
```

Figure 3. Pseudocode of IQEA.

### 4. SIMULATION RESULTS

This case study consisted of 13 thermal units of generation with the valve-point effects, as given in Table 2. The system data shown in Table 2 is also available in Sinha et al. (2003) and Wong and Wong (1994). In this case, the load demand expected to be determined was $P_D = 1800$ MW.

Each optimization method was implemented in Matlab (MathWorks). All the programs were run on a 3.2 GHz Pentium IV processor with 2 GB of random access memory. In each case study, 30 independent runs were made for each of the optimization methods involving 30 different initial trial solutions for each optimization method.

The total number of Q-bits adopted was the population size ($n$) times the number of bits per solution ($m$), i.e., 20 x 416 = 8320. The criterion stopping was 1000 generations. A key factor in the application of optimization methods is how the algorithm handles the constraints relating to the problem. In this work, a penalty-based method inspired in Noman and Iba (2008) was used. In this context, to avoid the violation of equality constraint given by equation (1) of the power balance criterion, a repair process is applied to each solution in order to guarantee that a generated solution by QEA or IQEA will be feasible.

Numerical results obtained for this case study are given in Table 3 and Figure 4, which shows that the IQEA has both a better economic cost and lower mean cost than the classical QEA. The best results obtained for solution vector $P_n$, $i=1,\ldots,13$ with HIS with minimum cost of 17961.2170 $/h$ is given in Table 4. Table 5 compares the results obtained
in this paper with those of other studies reported in the literature. Note that in the studied case, the best result reported here using IQEA is comparatively lower than recent studies presented in the literature.

Table 2. Data for the benchmark of 13 thermal units.

<table>
<thead>
<tr>
<th>Thermal unit</th>
<th>$p_{i}^{min}$</th>
<th>$p_{i}^{max}$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$e$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>680</td>
<td>0.00028</td>
<td>8.10</td>
<td>550</td>
<td>300</td>
<td>0.035</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>360</td>
<td>0.00056</td>
<td>8.10</td>
<td>309</td>
<td>200</td>
<td>0.042</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>360</td>
<td>0.00056</td>
<td>8.10</td>
<td>307</td>
<td>150</td>
<td>0.042</td>
</tr>
<tr>
<td>4</td>
<td>60</td>
<td>180</td>
<td>0.00324</td>
<td>7.74</td>
<td>240</td>
<td>150</td>
<td>0.063</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
<td>180</td>
<td>0.00324</td>
<td>7.74</td>
<td>240</td>
<td>150</td>
<td>0.063</td>
</tr>
<tr>
<td>6</td>
<td>60</td>
<td>180</td>
<td>0.00324</td>
<td>7.74</td>
<td>240</td>
<td>150</td>
<td>0.063</td>
</tr>
<tr>
<td>7</td>
<td>60</td>
<td>180</td>
<td>0.00324</td>
<td>7.74</td>
<td>240</td>
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<tr>
<td>8</td>
<td>60</td>
<td>180</td>
<td>0.00324</td>
<td>7.74</td>
<td>240</td>
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</tr>
<tr>
<td>9</td>
<td>60</td>
<td>180</td>
<td>0.00324</td>
<td>7.74</td>
<td>240</td>
<td>150</td>
<td>0.063</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
<td>120</td>
<td>0.00284</td>
<td>8.60</td>
<td>126</td>
<td>100</td>
<td>0.084</td>
</tr>
<tr>
<td>11</td>
<td>40</td>
<td>120</td>
<td>0.00284</td>
<td>8.60</td>
<td>126</td>
<td>100</td>
<td>0.084</td>
</tr>
<tr>
<td>12</td>
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<td>8.60</td>
<td>126</td>
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<tr>
<td>13</td>
<td>55</td>
<td>120</td>
<td>0.00284</td>
<td>8.60</td>
<td>126</td>
<td>100</td>
<td>0.084</td>
</tr>
</tbody>
</table>

Table 3. Convergence results (50 runs) of a case study of 13 thermal units with valve point and $P_D = 1800$ MW.

<table>
<thead>
<tr>
<th>Optimization Method</th>
<th>Maximum Cost ($/h$)</th>
<th>Minimum Cost ($/h$)</th>
<th>Mean Cost ($/h$)</th>
<th>Standard Deviation ($/h$)</th>
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<tr>
<td>QEA</td>
<td>18555.3135</td>
<td>18198.4452</td>
<td>18336.8580</td>
<td>128.4785</td>
</tr>
<tr>
<td>IQEA</td>
<td>18416.2340</td>
<td>17961.2170</td>
<td>18268.1944</td>
<td>119.5887</td>
</tr>
</tbody>
</table>

Figure 4. Convergence of mean of best $f$ value for QEA and IQEA approaches in 30 runs.
Table 4. Best result (50 runs) obtained for the case study using IQEA.

<table>
<thead>
<tr>
<th>Power</th>
<th>Generation (MW)</th>
<th>Power</th>
<th>Generation (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>628.3187</td>
<td>( P_8 )</td>
<td>109.8666</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>224.4007</td>
<td>( P_9 )</td>
<td>109.8667</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>147.9028</td>
<td>( P_{10} )</td>
<td>40.0201</td>
</tr>
<tr>
<td>( P_4 )</td>
<td>109.8666</td>
<td>( P_{11} )</td>
<td>40.0015</td>
</tr>
<tr>
<td>( P_5 )</td>
<td>109.8666</td>
<td>( P_{12} )</td>
<td>55.0115</td>
</tr>
<tr>
<td>( P_6 )</td>
<td>109.8667</td>
<td>( P_{13} )</td>
<td>55.0025</td>
</tr>
<tr>
<td>( P_7 )</td>
<td>60.0090</td>
<td>( \sum_{i=1}^{13} P_i )</td>
<td>1800.0000</td>
</tr>
</tbody>
</table>

Table 5. Comparison of results for the economic dispatch optimization problem with 13 thermal units.

<table>
<thead>
<tr>
<th>Optimization Technique</th>
<th>Objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cultural differential evolution (Coelho et al., 2008)</td>
<td>17963.94</td>
</tr>
<tr>
<td>Differential evolution (Noman and Iba, 2008)</td>
<td>17963.83</td>
</tr>
<tr>
<td>Genetic algorithm based on differential evolution (He et al., 2008)</td>
<td>17963.83</td>
</tr>
<tr>
<td>Hybrid differential evolution (Wang et al., 2007)</td>
<td>17975.73</td>
</tr>
<tr>
<td>Improved evolutionary programming (Sinha et al., 2003)</td>
<td>17994.07</td>
</tr>
<tr>
<td>Particle swarm optimization (Victoire and Jeyakumar, 2004)</td>
<td>18030.72</td>
</tr>
<tr>
<td>Self-tuning hybrid differential evolution (Wang et al., 2007)</td>
<td>17963.79</td>
</tr>
<tr>
<td>Best result of this paper using IQEA</td>
<td>17961.2170</td>
</tr>
</tbody>
</table>

5. CONCLUSION

Recently, Han and Kim (2002) proposed QEA, for minimization problems, where the Q-bit representation was adopted based on the concepts and principles of quantum computing. The characteristic of the representation is that any linear superposition can be represented. The smallest unit of information stored in a two-state quantum computer is called Q-bit, which may be in the “1” state, in the “0” state or in any superposition of the two.

In this paper, inspired on the principles of quantum computation and its superposition of states, based on Q-bits and diversity measure, IQEA was proposed. The performance of the IQEA was compared with classical QEA. It was found that IQEA approach handles the problem of premature convergence found in classical QEA effectively by tuning of rotation angle \( \Delta \theta \) using an adaptive approach based on diversity measure. Furthermore, the proposed IQEA outperformed other methods reported in literature in terms of best solution for the thirteen-unit benchmark system of economic dispatch problem.

6. ACKNOWLEDGEMENTS

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7. REFERENCES


8. RESPONSIBILITY NOTICE

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