RESPONSE OF AVAILABLE CRITERIA FOR VORTEX IDENTIFICATION ON SOME CHAOTIC AND TURBULENT FLOWS

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Abstract. *The performance of different criteria for vortex identification available in the literature is investigated for a number of special flows emphasizing the results obtained for chaotic laminar and turbulent flows. The so called ABC flow (u,v,w)=(A sin z + C cos y, B sin x + A cos z, C sin y + B cos x) is an interesting example of a laminar Trkalian Beltramian flow (with no Lamb surfaces) exhibiting chaotic behavior. The LES results for the 3-D cavity in the turbulent regime is another flow investigated. For every criterion (such as Q, Delta, lambda 2, etc) selected from the literature, it is created an objective (frame indifferent) version by simply substituting the vorticity by the relative (or absolute) vorticity, the vorticity as measured by an observer attached to the frame of the eigenvectors of the rate-of-strain tensor. Additionally, a new vortex definition based on the non-alignment of the rate-of-strain tensor and its covariant convected time derivative is compared with classical definitions.*

Keywords: *Vortex; Coherent vortical structure; Frame invariance*

1. INTRODUCTION

1.1 The vortex definition problem

The word *vortex* is frequently used when one wants to describe, understand, and explain flow patterns in fluid dynamic problems, but the connection of this entity to a rational definition is controversial. The main reason for this is the lack of agreement on what are the requisites a vortex should have. Is the vortex an entity that should be defined by a *kinematic* or a *dynamical* criterion? Should a vortex be defined by *Lagrangean* or *Eulerian* quantities? The definition of a vortex should be *Galilean invariant* or *objective*? Should it have subjective thresholds or must be problem-independent? Should we seek for a definition looking at the velocity gradient only or do we have to observe acceleration gradients also?

These questions do not have a consensual answer and are in the very heart of the controversy. The most used mathematical criteria for vortex identification are listed below. For a brief description, the reader is referred to an accompanied paper entitled "What is a vortex?" to be presented in this same congress.

1.2 Most usual criteria

Hunt et al. (1988) define a vortex as a connected region in space where

$$Q = \frac{1}{2}\left(|\mathbf{W}|^2 - |\mathbf{D}|^2\right) > 0 \tag{1}$$

where $\mathbf{W}$ and $\mathbf{D}$ are, respectively the skew-symmetric and symmetric parts of the velocity gradient and the operator $|\ |$ indicates the Euclidean norm of a tensor.

Chong et al. (1990) define a vortex as a connected region in space where there are a real and two complex conjugates eigenvalues of the velocity gradient. This is the so-called $\Delta - criterion$ given by

$$\Delta = \frac{Q^3}{27} + \frac{III_L^2}{4} > 0 \tag{2}$$

where III_L is the third invariant (determinant) of the velocity gradient.

Another very important criterion in the literature was proposed by Jeong and Hussain (1995). This criterion defines a vortex as a connected region where

$$\lambda_2^{\mathbf{D}^2+\mathbf{W}^2} < 0 \tag{3}$$

where $\lambda_2^{\mathbf{D}^2+\mathbf{W}^2}$ is the intermediate eigenvalue of tensor $\mathbf{D}^2 + \mathbf{W}^2$.

Tabor and Klapper (1994) presented a systematic study on the stretching and alignment dynamics in general 2D and define a region of rotation-like behavior as

$$Q_D = \frac{1}{2}\left(|\mathbf{W} - \mathbf{\Omega}|^2 - |\mathbf{D}|^2\right) > 0 \tag{4}$$

where $\mathbf{\Omega}$, measures the rate of rotation of the eigenvectors of $\mathbf{D}$, defined as

$$\mathbf{\Omega} = \dot{\mathbf{e}}_i^D \mathbf{e}_i^D. \tag{5}$$

where $\mathbf{e}_i^D$ is an eigenvectors of $\mathbf{D}$.

Zhou et al. (1999) defines a vortex as a connected region where where

$$\lambda_{ci}^2 > \delta \tag{6}$$

where λ_{ci} is the imaginary part of the two complex eigenvalues of the velocity gradient and δ is a threshold generally chosen as percentage of its maximum value.

Chakraborty et al. (2005) proposed a further step on the analysis of Zhou et al. (1999) by adding to the swirling strength criterion, $\lambda_{ci}^2 > \delta$ a condition on the *inverse spiraling compactness*, $\frac{\lambda_{cr}}{\lambda_{ci}}$ of the form

$$\frac{\lambda_{cr}}{\lambda_{ci}} > \epsilon \tag{7}$$

1.3 Creating objective criteria

The above mentioned criteria are non-objective. It is interesting to notice that these criteria can have an objective counterpart if we change the vorticity (a non-objective quantity) to the relative vorticity, that is the vorticity as seen by an observer attached to the eigenvectors of the rate-of-strain. Therefore, we can see how these new objective criteria perform in general flows.

1.4 A new criterion

A new criterion can be constructed by considering the persistence-of-straining tensor. As shown by **?** this quantity can be related to a competition between hyperbolic and elliptical types of flow. The persistence-of-straining tensor is defined as

$$\mathbf{P} = \mathbf{D}\overline{\mathbf{W}} - \overline{\mathbf{W}}\mathbf{D} \tag{8}$$

1.5 The present work

In the present work we test usual criteria of the literature as well as a new one proposed recently for two important flow problems chosen in order to identify rotation-like (elliptical) regions and stretching-like (hyperbolical) regions: the laminar ABC flow and the 3D turbulent cavity flow.

2. ABC FLOW

The ABC flow is a classical flow due to its chaotic behavior even for laminar flows (Dombre et al., 1986).

2.1 Lamb vector and helicity density

The local geometrically orthogonal decomposition of the velocity vector $\mathbf{v}$ with respect to the vorticity vector $\mathbf{w}$ introduces two quantities of crucial importance in vorticity dynamics: the vector $\mathbf{w} \times \mathbf{v}$, known as *Lamb vector*, and the scalar $\mathbf{w} \cdot \mathbf{v}$, known as helicity density. The two interesting non-trivial cases are when the helicity density or the Lamb vector vanishes. When $\mathbf{w} \cdot \mathbf{v} = 0$ and $\mathbf{w} \times \mathbf{v} \neq \mathbf{0}$, the flow is called *complex lamellar flow*. It exists if and only if

$$\mathbf{v} = \lambda \nabla \xi \tag{9}$$

where $\xi = const$ are equi-potential surfaces orthogonal to the streamlines everywhere (potential flow, also called lamellar flow, is obtained when $\lambda = 1$) . When $\mathbf{w} \times \mathbf{v} = \mathbf{0}$ and $\mathbf{w} \cdot \mathbf{v} \neq 0$ the streamlines are parallel to the vorticity lines, or

$$\mathbf{w} = \zeta \mathbf{v} \tag{10}$$

which implies that the velocity is an eigenvector of the curl operator. This kind of flow is called Beltrami (or helical) flow. If ζ is constant, the flow is specifically called Trkalian.

When the Lamb vector is a complex lamellar field or

$$(\mathbf{w} \times \mathbf{v}) \cdot [\nabla \times (\mathbf{w} \times \mathbf{v})] \Rightarrow \mathbf{w} \times \mathbf{v} = g \nabla h \tag{11}$$

there exist a set of surfaces $h = const$, called *Lamb surfaces* which are orthogonal to the Lamb vector everywhere. It can be shown that the existence of the Lamb surfaces imply the integrability of the system and therefore this kind of flow cannot be chaotic. Therefore, A Beltramian flow is a candidate of a chaotic flow. However, if $\nabla\zeta \neq 0$ the velocity is still integrable, since the velocity will be on the surfaces normal to $\nabla\zeta$. Therefore, the only possibility of an incompressible chaotic steady flow is when $\nabla\zeta = 0$, where the flow is Trkalian.

Arnold (1965), seeking steady inviscid chaotic flow, proposed a Trkalian flow where $\zeta = 1$ or $\mathbf{w} = \mathbf{v}$. The ABC flow in cartesian coordinates is given by

$$u = A\,\mathrm{sinz} + \mathrm{C}\,\mathrm{cosy}$$

$$v = B\,\mathrm{sinx} + \mathrm{A}\,\mathrm{cosz}$$

$$w = C\,\mathrm{siny} + \mathrm{B}\,\mathrm{cosx} \tag{12}$$

2.2 Results for the ABC flow

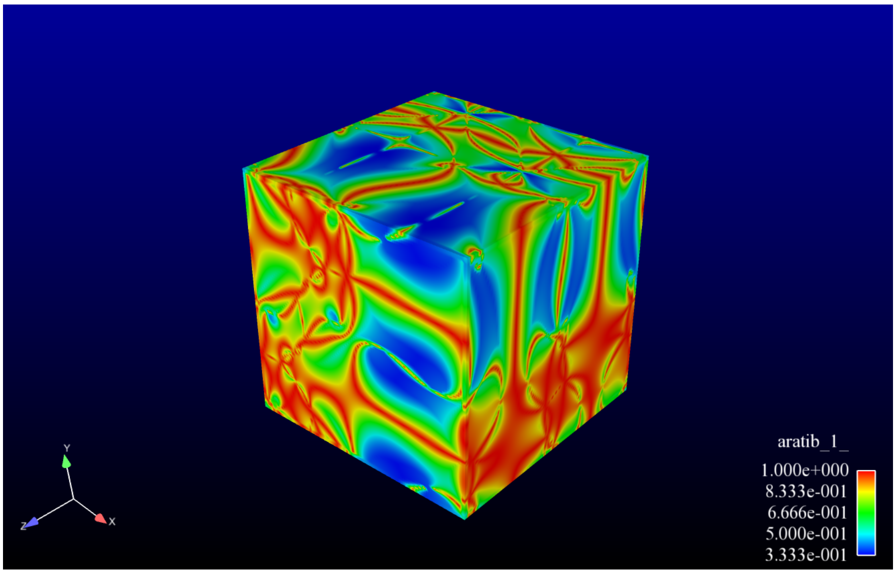

Figure 1. Anirat-b1.

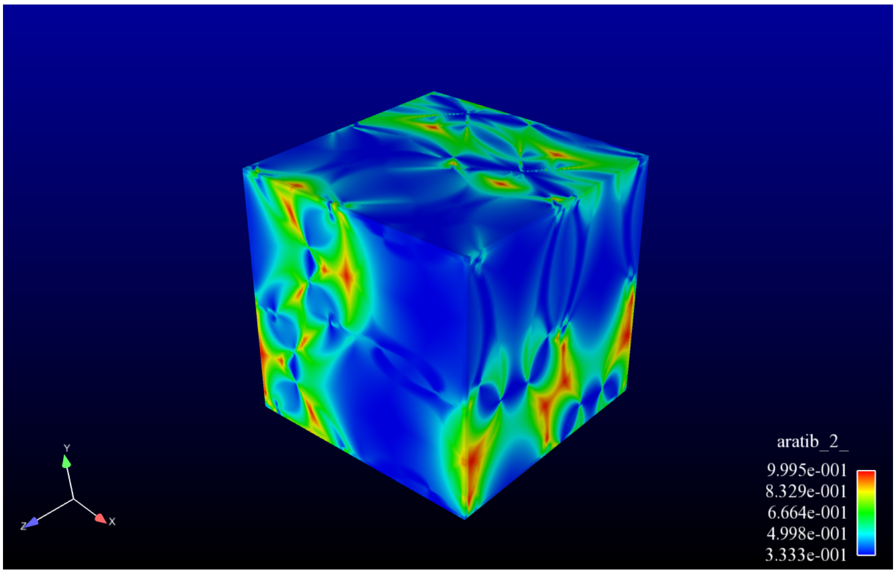

Figure 2. Anirat-b2.

Figure 6 shows the values of the isotropic normalized ratio that compares linear acceleration deformation, in the sense provided by the covariant convected time derivative, to angular acceleration gradient (in the same sense). Higher values correspond to hyperbolic-like behavior.

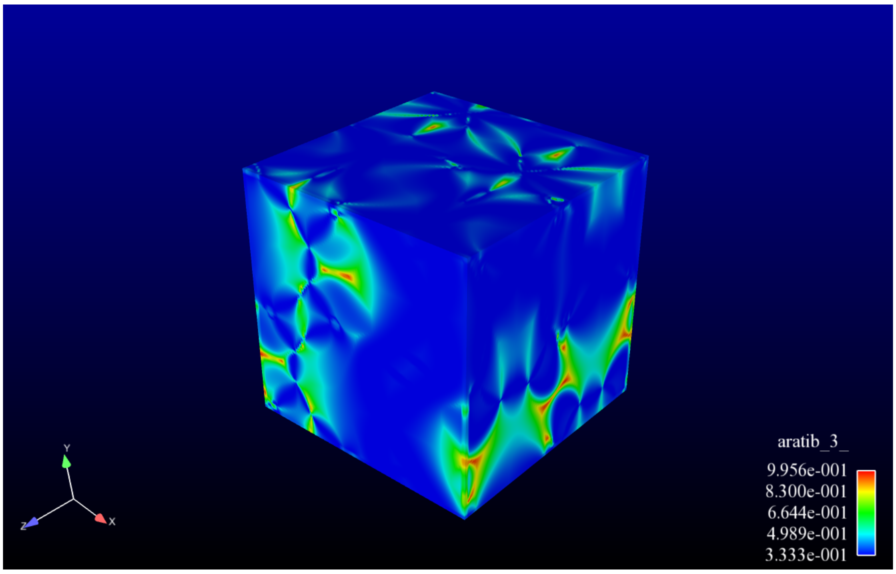

Figure 3. Anirat-b3.

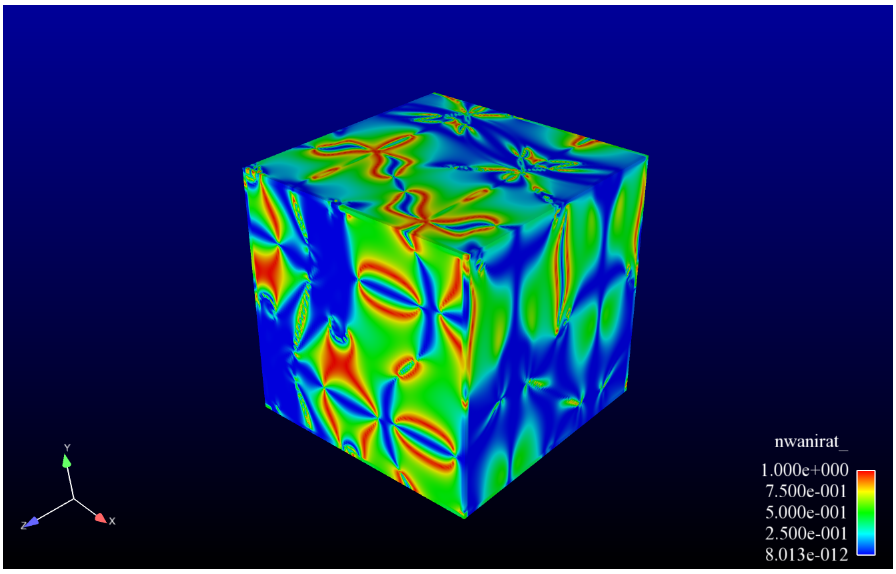

Figure 4. Anisotropic ratio at the plane defined by the relative rate of rotation vector.

One of the three fields of anisotropic index associated to a line-method is shown in Fig. 7. Since the orientation of the index is different depending on the point considered, we have decided to produce indexes based on the comparison between the three anisotropic indexes of each method. What is shown in Fig. 7 is related to the highest (among three) value of the tendency to evolve persistently the same material line.

3. THE 3D CAVITY

3.1 LES

Large Eddy Simulation methodology is about filtering of the equations of movement and decomposition of the flow variables into a large scale (resolved) and a small scale (unresolved) parts. The filtering process is applied on the governing equations for separate the fields that contains the large and sub-grid scales. After performing the volume averaging, the filtered Navier-Stokes equations become

$$\frac{\partial \left(\rho \bar{\mathbf{U}}\right)}{\partial t} + \nabla \cdot \left(\rho \overline{\mathbf{U}\mathbf{U}}\right) = -\nabla p + \mu \nabla^2 \bar{\mathbf{U}} + \mathbf{f}_B \quad (13)$$

Developing the non-linear transport term and introducing the sub-grid scale (SGS) stresses $\boldsymbol{\tau} = \bar{\mathbf{uu}} - \bar{\mathbf{U}}\bar{\mathbf{U}}$, the filtered Navier-Stokes equations can be rewritten as

$$\frac{\partial\left(\rho\bar{\mathbf{U}}\right)}{\partial t} + \nabla\cdot\left(\rho\bar{\mathbf{U}}\bar{\mathbf{U}}\right) = -\nabla p + \mu\nabla^2\bar{\mathbf{U}} - \nabla\cdot(\rho\boldsymbol{\tau}) + \mathbf{f}_B \tag{14}$$

The dynamic sub-grid scale model was used with the Large Eddy Simulation to obtain the sub-grid scales. In this sub-grid model the proportionality coefficient is computed as a function of time and space. As a consequence, some difficulties on finding a correct constant value in heterogeneous meshes, as in the Smagorinskyńs model are avoided.

The expression that defines the turbulent viscosity, μ_T can be written as

$$\mu_T = C\Delta^2\,||\mathbf{D}|| \tag{15}$$

where C is the proportionality coefficient, calculated in CFX along time and space as a function of the velocity fluctuations and the rate of strain tensor and is the length scale of the grid filter.

3.2 Description

The first experimental results for lid-driven cavity flows was published in the work of Koseff and Street (1984), showing the three-dimensionality of the problem. The main characteristic of this kind of flow is the secondary vortices observed in the upper corners and a primary one along the complete space. Results from Migeon el al. (2003) considered parallel unsteady, three dimensions lid-driven cavity and have shown the development of Taylor-like vortices. Recent works from Ku et al. (1987) and Babu and Korpela (1994) show the comparison between two and three-dimensional simulations and, agreeing with the work of Koseff and Street (1984), their results shown a great difference in the development of vortical structures in both cases. Direct numerical simulations approach were performed in the work of Leriche and Gavrilakis (2000) and large-eddy simulation by Zang et al. (1993), Deshpande and Milton (1998), Hassan Barsamian (2001). These works compared the results with the experimental data from Koseff and Street (1984) and show statiscal characteristics and the evolution of coherent structures. Recent work from Padilla (2008) showed the power spectra and the streamlines for parallel and non-parallel cavities for Reynolds up to 3000. In this paper will be present a comparison between many vortical structures identification criteria in a parallel lid-vortex cavity for Reynolds number equal to 10000.

3.3 Results for the 3D cavity

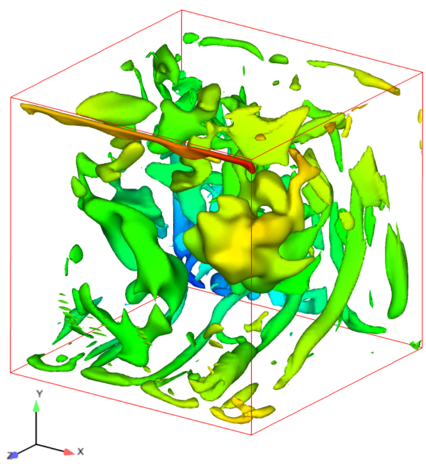

Figure 5. Isosurfaces of Δ.

4. FINAL REMARKS

We have presented two application of the theory developed in an accompanied paper concerning flow classification. The general results are complex in nature and the full interpretation are in order. We would like to observe that there a huge quantity of results that were impossible to add to the present article due to the limiting memory space.

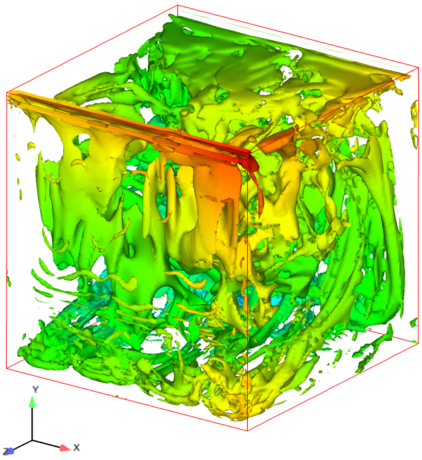

Figure 6. Isosurfaces of isoratio.

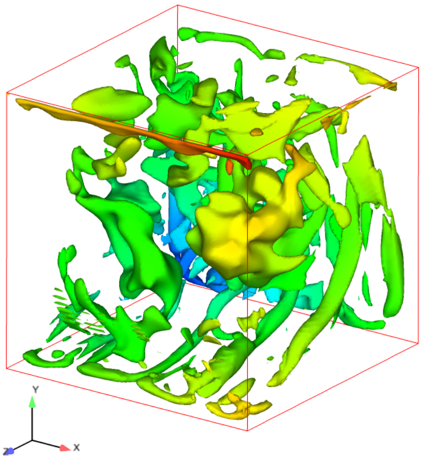

Figure 7. Isosurface of λ_2.

5. ACKNOWLEDGEMENTS

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