FRACTION OF MASS ATTACHED TO THE WALL IN THE LIQUID-LIQUID DISPLACEMENT IN CAPILLARY TUBES WHEN THE DISPLACING LIQUID IS A POWER-LAW NON-NEWTONIAN FLUID

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Abstract. *The motion of two immiscible liquids in a capillary tube is analyzed, theoretically and numerically, for the case where a residual film confines the displacing liquid to the core of this tube and the displacing fluid can be modeled with the power-law constitutive equation. An elliptic mesh generation technique, coupled with the Galerkin Finite Element Method is used to compute the velocity field and the configuration of the interface between the two fluids. The mesh architecture was conceived to capture the secondary recirculation that is formed in the injected fluid when the full-recirculation flow regime is achieved. A comparison with the liquid-liquid displacement of two Newtonian fluids is given. The results show that the fraction of mass attached to the wall increases with the Capillary number and the power-law index of the displacing fluid and decreases with the viscosity ratio.*

Keywords: *Liquid-liquid displacement, power-law fluids, finite element method*

1. INTRODUCTION

The motion of two liquids in internal passages is a phenomenon that occurs in many industrial processes as enhanced oil recovery, primary cementing of oil wells and others chemical and cleaning operations.

In particular, for the enhanced oil recovery, it is common to inject polymeric solutions so as to take advantage of the complexity of the displacing fluid to optimize the extraction of oil. The present work is a step on this direction. We investigate how the rheological properties of the injected fluid influence the fraction of mass that remains attached to the wall. An important application is in oil recovery from porous media, where the liquid film of the displaced oil that remains attached to the rocks has a significance on the efficiency of the recovery during water flooding operation. Typically, because of the high viscosity levels and slow displacement velocities involved, these kind of processes occur with negligible inertial effects. Besides that, as a consequence of the small length scale, the capillary forces play a fundamental role on the physics of the phenomena.

Figure 1 shows a scheme of the problem analyzed. The tube is initially occupied by a phase 2 when phase 1 is injected, forming a two phase flow. As an idealized model, phase 1 forms a long drop that displaces phase 2 leaving behind a residual layer of fluid attached to the wall. When inertia is negligible, this front moves with a constant velocity, U, and the configuration of the interface achieves a fixed shape with a constant width of residual layer far enough from the tip of the drop. The fraction of mass that remains attached to the wall, m, is defined as

$$m = 1 - \frac{R_b^2}{R_0^2}, \tag{1}$$

where R_0 is the radius of the tube and R_b is the radius of the drop when it has reached its constant value, as seen in Fig. 1.

The simplest form of fluid-fluid displacement in capillary tubes is when a gas (in fact an inviscid fluid) is displacing a Newtonian liquid. This was first analyzed by Fairbrother and Stubbs (1935), Bretherton (1961), Taylor (1961), Cox (1962), and others. In this case, the dimensionless parameter that governs the problem is the capillary number, $Ca \equiv \frac{\mu_2 U}{\sigma}$, that signifies the relative importance of viscous stresses to interfacial-tension stresses. The next step in complexity found in the literature comes from one of the following problems: a) gas-displacement of a non-Newtonian material or b) Newtonian-Newtonian liquid displacement.

When gas is displacing a non-Newtonian material, besides Ca, there are other dimensionless numbers, related to the rheology of the non-Newtonian fluid, that govern the problem. These dimensionless non-Newtonian numbers can be: the power-law index (Sousa et al. (2007)) to explore the dependence of the viscosity on the deformation rate, the Bingham number (Poslinski et al. (1995), Allouche et al. (2000), Dimakopoulos and Tsamopoulos (2003), de Souza Mendes (2007), Sousa et al. (2007)), when investigating the effects of an yield stress, the Deborah number (Huzyak and Koelling (1997), Lee et al. (2002), Dimakopoulos and Tsamopoulos (2004)), to study the influence of the elasticity. Depending on the non-Newtonian fluid considered, other parameters are needed to describe specific features of the constitutive model. The complexity comes from the non-Newtonian material considered due to non-linearities that rises from the interaction

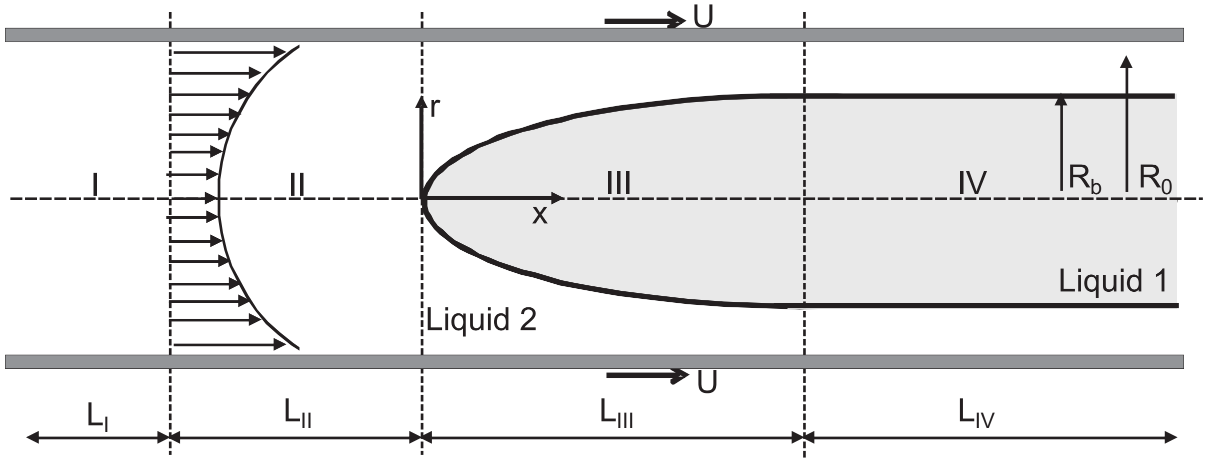

Figure 1. Schematics of the analyzed problem.

between rheology and the forces that come from interfacial stresses.

The investigation of Newtonian-Newtonian liquid displacement in capillary tubes also requires another dimensionless number (Goldsmith and Mason, 1963), generally taking the form of a viscosity ratio (another equivalent dimensionless number could be a second capillary number based on the viscous stress of the displacing fluid). Goldsmith and Mason (1963) and Soares et al. (2005) analyzed experimentally this problem, Hodges et al. (2004) did a theoretical analysis, while Westborg and Hassager (1989), Martinez and Udell (1990), Soares et al. (2005), and **?** employed numerical simulations. In the latter approach, the problem is more complex, not only for physical reasons, but also for computational and numerical ones. Differently from the gas-displacement, the liquid-liquid displacement problem requires the solution of the conservation equations in the displacing fluid domain also. Besides that, in the force balance at the interface, the normal stress at the displacing fluid side is not solely given by the pressure.

The present work can be considered a further step on the direction of increasing the complexity of this problem. Here we numerically investigate the displacement in capillary tubes when a non-Newtonian fluid displaces a Newtonian one. In our case, the non-Newtonian material is a simple power-law fluid. Therefore, besides the capillary number and the viscosity ratio, the power-law index is a dimensionless parameter relevant to the problem. Although the fraction of mass is very important parameter, related to the displacement efficiency, we are also interested on the flow patterns the problem exhibits.

2. PROBLEM FORMULATION AND NUMERICAL IMPLEMENTATION

As shown in Fig. 1, the frame of reference is attached to the tip of the drop and, hence, the capillary tube wall moves with the interface velocity U. We assume that the configuration of the interface converges to a fixed shape. The flow is assumed to be isothermal, with no inertia and incompressible. The conservation of mass is given by

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_k) + \frac{\partial u_k}{\partial x} = 0, \tag{2}$$

while the equations for conservation of momentum are given by

$$\frac{1}{r}\frac{\partial}{\partial r}(rT_{(xr)_k}) + \frac{\partial}{\partial x}(T_{(xx)_k}) = 0, \tag{3}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rT_{(rr)_k}) - \frac{T_{(\theta\theta)_k}}{r} + \frac{\partial}{\partial x}(T_{(rx)_k}) = 0, \tag{4}$$

where the subscripts $k = 1, 2$ labels the two liquids considered and u and v are, respectively, the axial and radial components of the velocity field $\mathbf{u}$. $T_{xx}, T_{xr}, T_{rx}, T_{rr}$ and $T_{\theta\theta}$ are the components of the stress tensor $\mathbf{T}$.

The boundaries are labelled from 1 to 5 (see Fig. 2). At Boundary 4, the flow is taken to be fully developed and the pressure is assumed to be uniform. Hence,

$$\mathbf{n}\cdot\nabla\mathbf{u_2} = 0 \quad , \quad p_2 = P_{in}, \tag{5}$$

where $\mathbf{n}$ is the unit vector normal to the boundary surface and p_2 is the constant pressure field on phase 2 at the inlet.

At Boundary 1, the flow is assumed to be fully developed, but the pressure is free. This condition is given by

$$\mathbf{n}\cdot\boldsymbol{\tau_k} = \mathbf{0}. \tag{6}$$

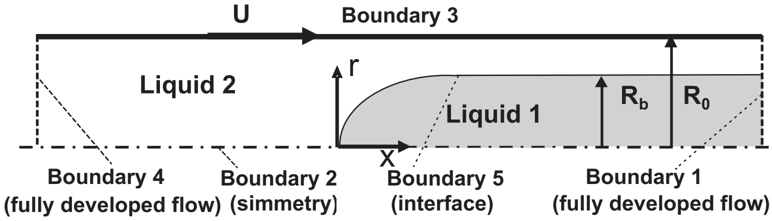

Figure 2. Boundary conditions.

where $\boldsymbol{\tau_k}$ is the extra-stress tensor ($\boldsymbol{\tau}_k = \mathbf{T}_k + p_k\mathbf{I}$). Along the symmetry axis, Boundary 2, both the shear stress and the radial velocity vanish. Hence,

$$\mathbf{n} \cdot [\mathbf{T_k} \cdot \mathbf{t}] = 0 \quad , \quad \mathbf{n} \cdot \mathbf{u_k} = 0, \tag{7}$$

where $\mathbf{t}$ is a unit vector tangent to the boundary surface.

No-slip and impermeability conditions are imposed along the tube wall, Boundary 3. Therefore,

$$\mathbf{u_2} = U\mathbf{e}_x, \tag{8}$$

where $\mathbf{e}_x$ is the unit vector in the x-direction.

Along the interface, Boundary 5, the traction balances the capillary pressure,

$$\mathbf{n} \cdot (\boldsymbol{T}_2 - \boldsymbol{T}_1) = \frac{\sigma}{R_m}\mathbf{n}, \tag{9}$$

where σ is the liquid-liquid interfacial tension, p_1 and p_2 are the pressures on phase 1 and phase 2, respectively and $1/R_m$ is the local mean curvature of the interface and $\frac{\sigma}{R_m}$ is the normal stress jump due to the interfacial tension, while the tangential stress at the interface is continuous. The other condition along the interface is the impermeable condition stated by

$$\mathbf{u_1} = \mathbf{u_2} = (\mathbf{u_k} \cdot \mathbf{t})\,\mathbf{t}. \tag{10}$$

2.1 Dimensionless numbers

The capillary number Ca is based on the characteristic viscous stress of the Newtonian liquid

$$Ca = \frac{\mu_2 U}{\sigma} \tag{11}$$

The viscosity ratio, N_η, however, needs the rheology of the displacing fluid to be represented. We chose as a characteristic viscosity, the one calculated at a characteristic shear rate, $\dot{\gamma} = \frac{U}{R_0}$. Hence,

$$N_\eta = \frac{\mu_2}{K\left(\frac{U}{R_0}\right)^n} \tag{12}$$

The third dimensionless number is a purely rheological one. Is the power-law exponent, n, of the displaced (power-law) fluid.

2.2 Solution of the equation system by Galerkin / Finite Element Methods

The differential equations that govern the problem are solved in a coupled manner by the Galerkin/Finite Element Method. Biquadratic basis functions ϕ_j are used to represent the velocity and nodal coordinates, while linear discontinuous functions χ_j are employed to expand the pressure field. The velocity and pressure are represented in terms of appropriate basis functions

$$u = \sum_{j=1}^{n} U_j\phi_j \quad ; \quad v = \sum_{j=1}^{n} V_j\phi_j \quad ; \quad p = \sum_{j=1}^{m} P_j\chi_j \quad ; \tag{13}$$

The coefficients of the expansions are the unknown of the problem

$$\underline{c} = \left[\begin{array}{ccc} U_j & V_j & P_j \end{array} \right]^T$$

The corresponding weighted residuals of the Galerkin method related to conservation of momentum, mass and mesh generation are:

$$R_c^i = \int_{\bar{\Omega}} \left[\frac{1}{r} \frac{\partial}{\partial r}(rv) + \frac{\partial u}{\partial x} \right] \chi_i r ||J|| d\bar{\Omega} \tag{14}$$

$$\begin{aligned} R_{mx}^i &= \int_{\bar{\Omega}} \left[\frac{\partial \phi_i}{\partial x} T_{(xx)} + \frac{\partial \phi_i}{\partial r} T_{(xr)} \right] r ||J|| d\bar{\Omega} \\ &\qquad - \int_{\bar{\Gamma}} \mathbf{e}_x \cdot (\mathbf{n} \cdot \mathbf{T}) \phi_i r \frac{d\Gamma}{d\bar{\Gamma}} d\bar{\Gamma} \end{aligned} \tag{15}$$

$$\begin{aligned} R_{mr}^i &= \int_{\bar{\Omega}} \left[\frac{\partial \phi_i}{\partial x} T_{(xr)} + \frac{\partial \phi_i}{\partial r} T_{(rr)} + \frac{\phi}{r} T_{(\theta\theta)} \right] r ||J|| d\bar{\Omega} \\ &\qquad - \int_{\bar{\Gamma}} \mathbf{e}_r \cdot (\mathbf{n} \cdot \mathbf{T}) \phi_i r \frac{d\Gamma}{d\bar{\Gamma}} d\bar{\Gamma} \end{aligned} \tag{16}$$

2.3 Solution of the non-linear system of algebraic equation by Newton's Method

As indicated above, the system of partial differential equations, and boundary conditions is reduced to a set of simultaneous algebraic equations for the coefficients of the basis functions of all the fields. This set is non-linear and sparse. It is solved by Newton's method. The linear system of equations at each Newton iteration was solved using a frontal solver.

3. RESULTS

Figures 3, 4, and 5 show the displacement efficiency as a function of the Capillary number for viscosity ratios of $N_\eta = 8$, $N_\eta = 4$, and $N_\eta = 2$ respectively. These figures show how the fraction of mass, m, varies with the power-law index, n. It can be seen that once viscosity ratio and Capillary number are fixed, m increases with the power-law exponent of the displacing fluid.

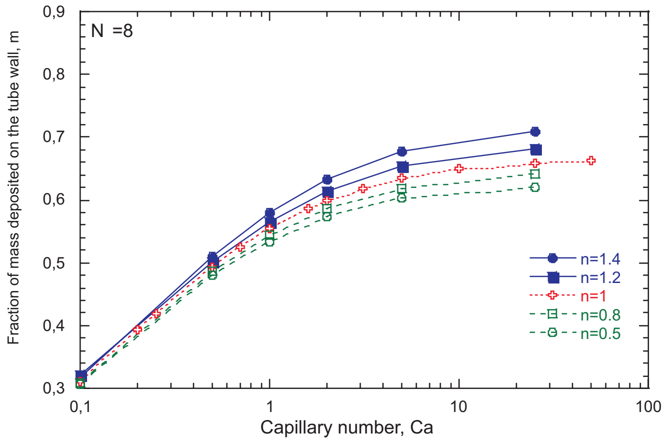

Figure 3. Fraction of mass attached to the wall as a function of the Capillary number with a fixed viscosity ratio, $N_\eta = 8$, for different values of the power-law index, $n = 0.5$, 0.8, 1, 1.2, and 1.4.

In the low Ca limit, the curves tend to converge to a single master curve, indicating that in this limit the problem is govern by the stresses induced by the interfacial tension between the liquids. We can see also that, the fraction of mass, and the dispersion concerning different values of n, are higher for lower values of viscosity ratio.

Figures 3, 3, and 3 show also that when n increases, the asymptote Capillary number above which the fraction of mass does not change is higher for higher values of n and for lower values of the viscosity ratio.

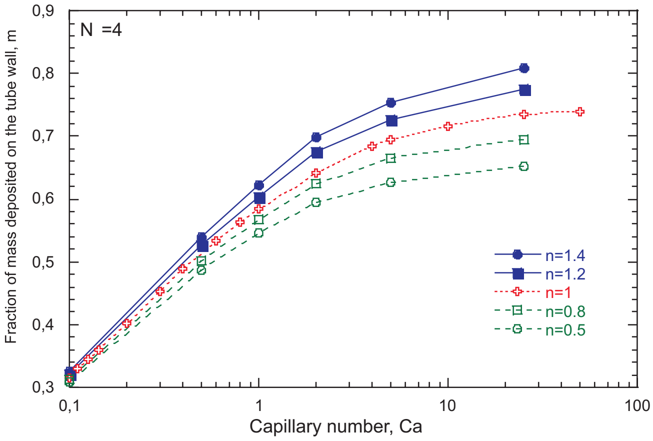

Figure 4. Fraction of mass attached to the wall as a function of the Capillary number with a fixed viscosity ratio, $N_\eta = 4$, for different values of the power-law index, $n = 0.5$, 0.8, 1, 1.2, and 1.4.

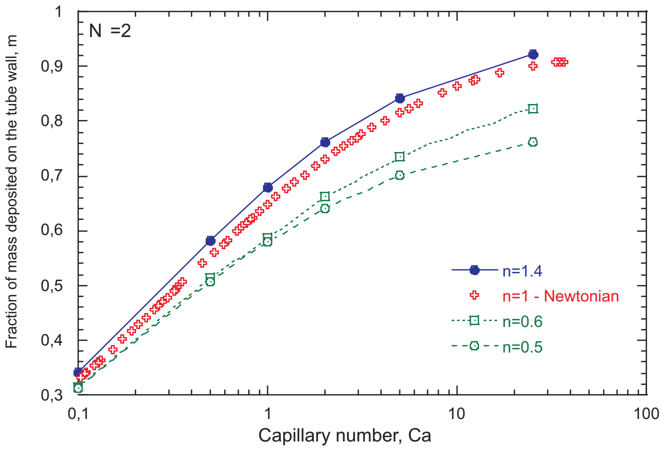

Figure 5. Fraction of mass attached to the wall as a function of the Capillary number with a fixed viscosity ratio, $N_\eta = 2$, for different values of the power-law index, $n = 0.5$, 0.8, 1, 1.2, and 1.4.

4. FINAL REMARKS

The fraction of mass that remains attached to the wall, m, in the liquid-liquid displacement in capillary tubes is analyzed for the case where the displacing liquid is a power-law fluid. The main conclusion of the investigation is that the more shear-thinning is the fluid, less mass of the displaced fluid is not recovered, increasing the displacing efficiency of the process.

5. ACKNOWLEDGEMENTS

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