PLASTIC COLLAPSE OF PRESSURIZED PIPES UNDER BENDING

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Abstract. *Limit analysis theory is used to predict plastic collapse of pressurized pipes submitted to axial and bending loads. A non-linear plastic yield function for internal pressure, bending moment and axial loads is presented. Additionally, a finite element software, specially designed to solve pipes limit analysis problems is developed. Straight and curved beam finite elements are considered for describing pipes geometrical and mechanical behavior. The comparison of proposed model with well established models is presented, including behavior differences for curved pipes when loaded at opening or closing in-plane bending.*

Keywords: *plastic collapse, pipes, limit analysis*

1. INTRODUCTION

Limit Analysis is a subject extensively studied by several authors such as (Borges et al., 1996), (Lubliner, 1990) and (Hodge, 1959). It deals with the determination of loads that produce plastic collapse, that is, a state where pure plastic strain rates take place under constant stress distributions. In this work limit analysis is used to estimate the load which a pressurized pipe under bending fails by plastic collapse, supposing that premature failure due to geometric instability does not occur. Tubular structures are modeled by one-dimensional elastic ideally plastic beam elements with nonlinear yielding surfaces defining the domain of admissible stresses.

In last decade many papers were published concerning plastic collapse of pipes submitted to bending and internal pressure loads. Kim and Oh (2006) adopted 3D finite element models and analytic solutions to assess the effects on plastic collapse for opening and closing bending pipes. Robertson et al (2005) considered proportional and sequential loadings - first bending and then internal pressure or first internal pressure and then bending – to verify the influence of loading sequence in plastic collapse and in geometric instabilities. Chattopadhyay (2002) analyzed the effect of internal pressure on the strength of bent pipes submitted to opening and closing in-plane bending. Bai and Hauch (1999) analyzed the interaction between pressure, longitudinal force and bending moment in capacity of pipes to resist plastic collapse and/or geometric instabilities. A comparison of failures due to internal or external pressure is discussed, as well as the differences between load and displacement control to access geometric instability failures.

In this work tubular structures are modeled as uniaxial elements with nonlinear yielding surfaces. Several hypotheses have to be stated such as: the material is considered elastic-perfectly plastic, the structure geometry is referred to its initial configuration and it is valid only for small displacements and rotations. Also, to solve limit analysis problems a static formulation approach based in equilibrium requirements and plastic admissibility is utilized. To manage the extensive calculations demanded by the limit analysis algorithm a Visual Basic (V.B.) software was implemented by Kenedi (2008).

A review of most representative yield locus models for pipes submitted to combined loadings are done, as in Bai and Hauch (1999) where the interaction between pressure, longitudinal force and bending moment in capacity of pipes to resist plastic collapse was analyzed, generating the yield locus:

$$M_c = 4t\,r^2\sigma_{II}\sqrt{1-\left(1-\alpha^2\right)\left(\frac{\sigma_h}{\sigma_{hl}}\right)^2}\,\cos\left(\frac{\pi}{2}\frac{\dfrac{\sigma_I}{\sigma_{II}}-\alpha\dfrac{\sigma_h}{\sigma_{hl}}}{\sqrt{1-\left(1-\alpha^2\right)\left(\dfrac{\sigma_h}{\sigma_{hl}}\right)^2}}\right) \tag{01}$$

where, σ_I is the longitudinal stress, σ_{II} is the yield stress of longitudinal direction, σ_h is the hoop stress, σ_{hl} is the yield stress at hoop direction, $\alpha = f(\sigma_{II}/\sigma_{hl})$ is a correction factor, t is the wall thickness, r is the average pipe wall radius.

Kim and Oh (2006) presented a yield locus expression for a pipe under combined loading of internal pressure and in-plane bending:

$$M = 4t\,r^2\sigma_0\left(1.04\lambda^{2/3}\right)\left(1 - P\frac{r}{t\sigma_0}\right)^{1/3} \tag{02}$$

where, σ_0 is the limit stress of an elastic-perfectly plastic material, $\lambda = \frac{R\,t}{r^2}$ is the bend characteristic, P is the internal pressure and R is bend radius. The authors, based on Finite Element (F.E.) results, proposed a yield locus for pipe bends under pressure and in-plane bending loading:

$$\frac{M}{M_0} = 1 - \left(\frac{1 - r/R}{1 - r/(2R)}\frac{P}{P_0}\right)^3 \tag{03}$$

$$M_0 = \left(4r^2 t\sigma_0\right)\left(0.935\lambda^{2/3}\right) \quad \text{and} \quad P_0 = \left(\frac{2}{\sqrt{3}}\sigma_0\frac{t}{r}\right)\left[\frac{1 - r/R}{1 - r/(2R)}\right] \tag{04}$$

where, M_0 is the limit in-plane moment of a pipe bend and P_0 is the limit pressure of a pipe bend.

Goodall (1978) proposed, using a small displacement analysis, a yield locus expression for elbows under combined loading of internal pressure and in-plane bending:

$$M_L = 1.04h^{2/3}\left(1 - \frac{P\,r_m}{2t\sigma_y}\right)^{1/3}\left(D^2 t\sigma_y\right) \tag{05}$$

where, M_L is the limit in-plane moment of a pipe bend, $h = \frac{tR_b}{r_m^2}$ is the elbow factor, r_m is the mean radius of elbow cross section, D is the outer diameter of elbow cross section and σ_y is the material yield stress.

Note that these models consider combined loading of internal pressure and in-plane bending, but only the Bai and Hauch's model also considerers the axial load.

2. PROPOSED MODEL

A model is proposed to estimate plastic collapse of pipes submitted to a combination of axial load *N*, bending moment *M* and internal pressure *P*, as shown schematically at Fig. 1.

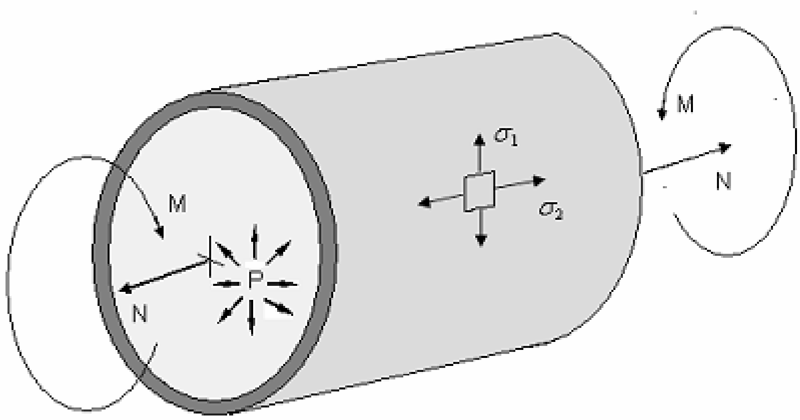


Figure 1. Segment of a pipe submitted to combination of axial load *N*, bending moment *M* and internal pressure *P*.

To solve limit analysis problems a formulation, based in equilibrium requirements and plastic admissibility, was developed with the utilization of mathematical programming (Borges et al., 1996). Also a yield function for open-ended and close-ended pipes, in conjunction with the limit analysis algorithm, was implemented in V.B. software to handle the massive quantity of calculations and provide a professional data input and results output (Kenedi, 2008).

The transversal section geometrical variables of a thin-walled pipe are shown at Fig. 2.

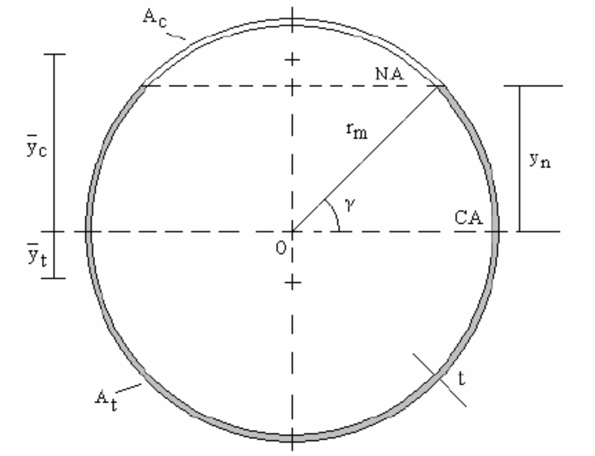


Figure 2. Transversal section of a thin-walled pipe.

Where, CA and NA are respectively centroidal and neutral axes, r_m is the average pipe wall radius, t is the wall thickness, y_n is the distance from CA to NA. The transversal area A is divided in two areas by NA, the area submitted to tensile stress A_t (shaded area) and the area submitted to compressive stress A_c (assuming applied positive bending moment). $\bar{y}_t$ is the distance between centroid of area A (shown with a 0) and the centroid of A_t and $\bar{y}_c$ is the distance between centroid of area A and the centroid of A_c. γ is an angle ranging from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ rad.

The application of equilibrium to a pipe section, submitted to tensile axial force, positive bending moment and internal pressure, shown at Fig.1, in conjunction with the utilization of Mises criterion, were utilized to obtain the yielding function expressions. The inclusion of internal pressure, as dead load, significantly altered the prediction of plastic collapse of a pipe. At a previous work Kenedi et al. (2008) showed, in details, how to obtain a yield locus for pressurized pipes, submitted to combined loadings, with open-ended and close-ended extremities shown, respectively, at expressions (06) and (07).

$$m = \pm\sqrt{1-p^2}\cos\left[\frac{\pi}{2}\frac{n}{\sqrt{1-p^2}}\left(n-\frac{p}{\sqrt{3}}\right)\right] \quad \text{(open-ended)} \tag{06}$$

$$m = \pm\sqrt{1-p^2}\cos\left(\frac{\pi}{2}\frac{n}{\sqrt{1-p^2}}\right) \quad \text{(close-ended)} \tag{07}$$

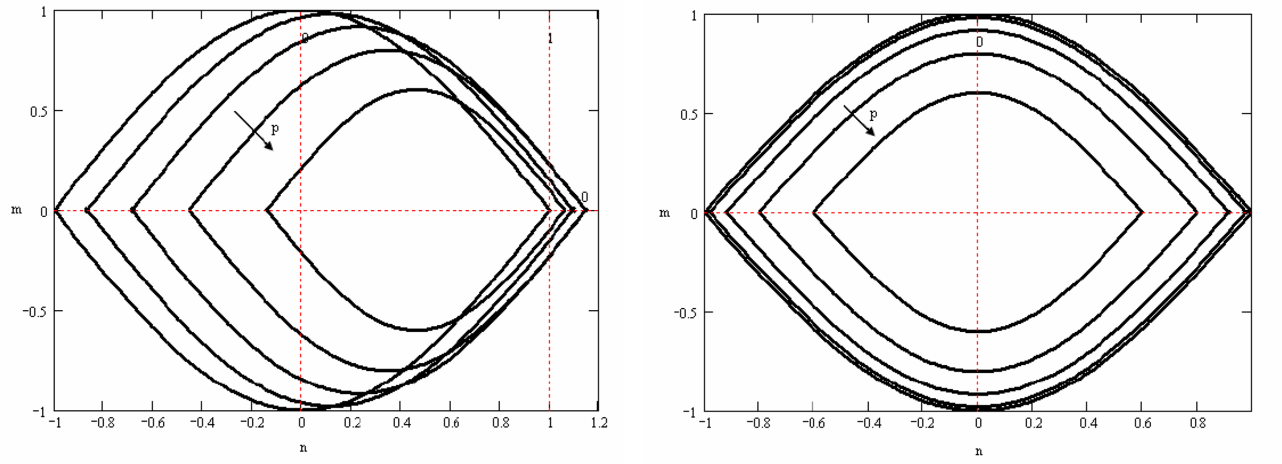
For a particular case of null axial load, $n = 0$, expressions (06) and (07) converge to:

$$m = \pm\sqrt{1-p^2} \tag{08}$$

$$n = \frac{N}{N_0} \text{ and } N_0 = 2\pi r_m t\sigma_y \text{ , } m = \frac{M}{M_0} \text{ and } M_0 = 4r_m^2 t\sigma_y \text{ , } p = \frac{P}{P_0} \text{ and } P_0 = \frac{2}{\sqrt{3}}\frac{t}{r_m}\sigma_y \tag{09}$$

where, σ_y is the material yield stress, n, m and p are, respectively, the normalized axial load, bending moment and internal pressure, N_0 , M_0 *and* P_0 are, respectively, the axial load, the bending moment and the internal pressure that yields the cross section entirely.

Figure 3 shows the limiting yielding surfaces, respectively for open-ended and close-ended pipes, calculated from the application of expressions (06) and (07), submitted to a combination of axial load *N*, bending moment *M* and internal pressure *P*:



(a)

(b)

Figure 3. Limiting yielding surfaces for: (a) open-ended and (b) close-ended pipes.

Figure 3 presents two graphics for values of *n* versus *m*, for normalized pressures $0 \leq p \leq 0.8$, with increments of $0.2p$. Note that the arrows show *p* increse direction. At Figs. 3.a and 3.b, for $p = 0$ the limiting yielding surfaces are identical and the large ones. As *p* increases the limiting yielding surfaces becomes smaller. For open-ended pipes the limiting yielding surfaces dislocates to the rigth side of the graphic and gets smaller as *p* increses, while for close-ended pipes the limiting yielding surfaces maintain concentric and becomes smaller as *p* increses.

The models presented for yield locus for combined loading of internal pressure and in-plane bending of pipes can be compared with expression (08) of the proposed model, with results shown graphically at figure 4:

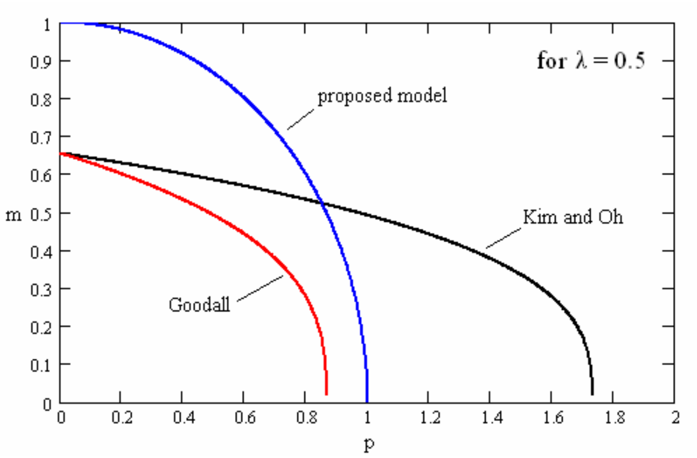


Figure 4. Comparative graphic *p* versus *m* of yield locus of analyzed models.

Figure 4 shows, for $\lambda = 0.5$, the performance of the models of yield locus for pipes submitted to combined loading of internal pressure and in-plane bending.

3. RESULTS

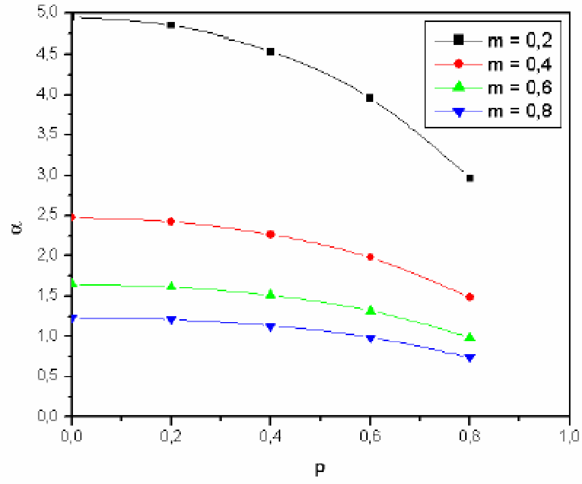
The yielding functions (06) and (07) were used by the algorithm, based in nonlinear optimization procedure for limit analysis proposed by Borges et al. (1996), and implemented with V.B. software by Kenedi (2008). This was used to do the necessary calculations to estimate the plastic collapse factor α for one-dimensional structures. Two examples of limit analysis application to estimate plastic collapse failures are shown, one for straight pipes and other for elbows. First example shows, at Fig. 5, straight pipes, with close-ended and open-ended extremities, submitted to combinations of axial load *N*, bending moment *M* and internal pressure *P*.

(a)

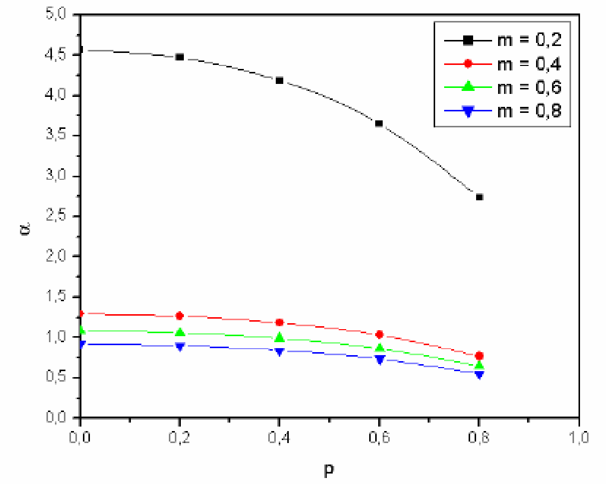
(b)

Figure 5. Straight pipe submitted to loads combinations: (a) close-ended and (b) open-ended.

Figure 6 shows plastic collapse factor α versus internal pressure p for several normalized moments m and axial loads n for a close-ended straight pipe, as at Fig. 5.a.



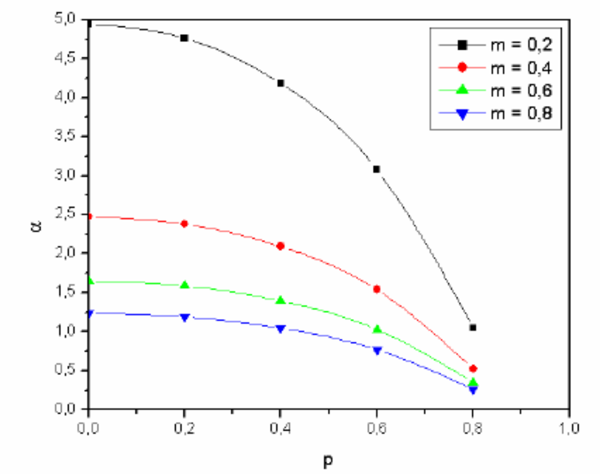
(a)



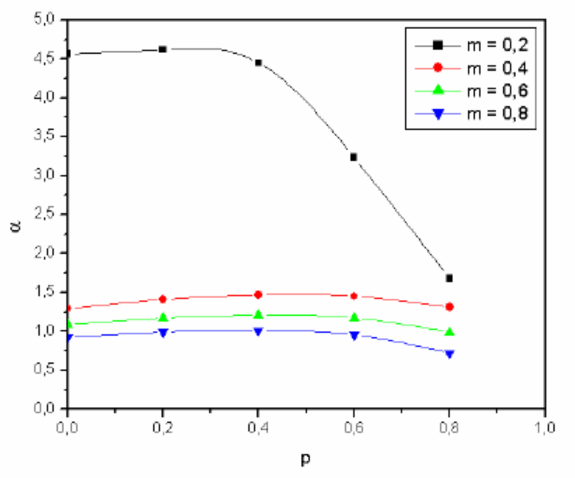
(b)

Figure 6. Plastic collapse factor α versus internal pressure p for several normalized moments m for close-ended straight pipes: (a) for $n = 0$ and (b) for $n = 0.5$.

Figures 6.a and 6.b shows that plastic collapse factor α decreases as m, n or p increases. Also, the addition of axial load pushes the curves downwards decreasing the capacity of close-ended pipes to resist to plastic collapse. Figure 7 shows α versus p for several normalized moments m and axial loads n for an open-ended pipe, as at Fig. 5.b.



(a)

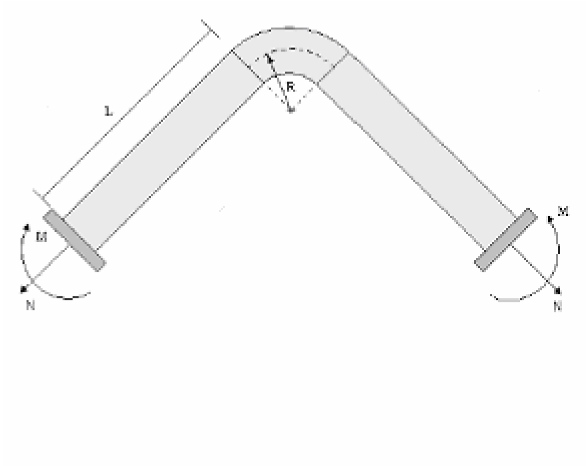


(b)

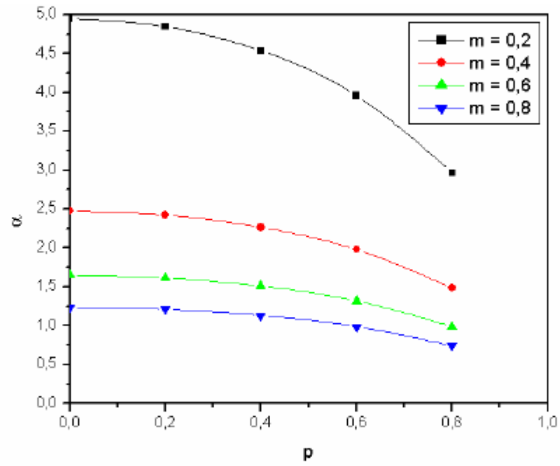
Figure 7. Plastic collapse factor α versus internal pressure p for several normalized moments m for open-ended straight pipes: (a) for $n = 0$ and (b) for $n = 0.5$.

Figures 7.a shows that plastic collapse factor α decreases as m or p increases. The addition of axial load, at Fig.7.b induces a singular behavior that the plastic collapse discretely increases as internal pressure increases until half scale and then begins to decrease with further increase of pressure.

A second example (Figs. 8 and 9) presents the plastic collapse factor α versus internal pressure p respectively for close-ended and open-ended elbows submitted to a combination of axial load N, bending moment M and internal pressure P.



(a)



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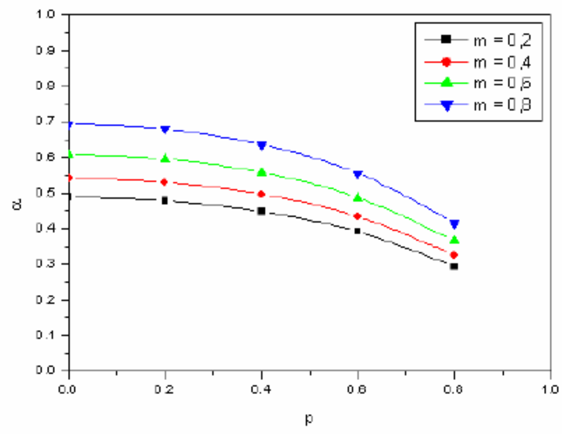
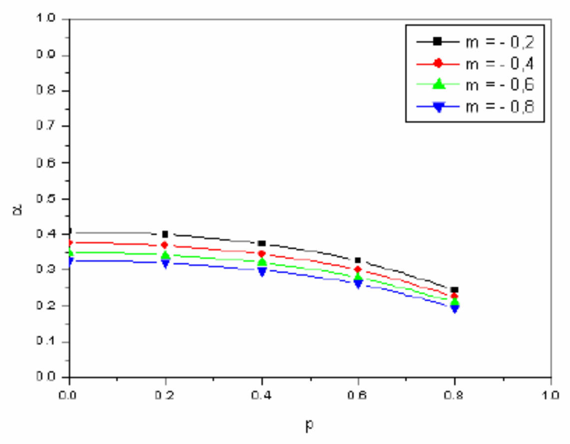


(c)

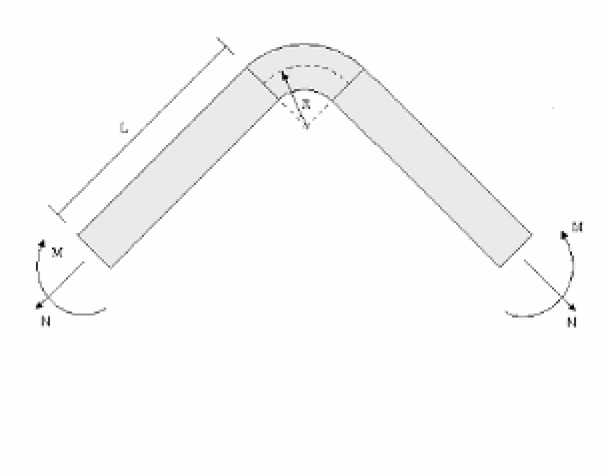


(d)

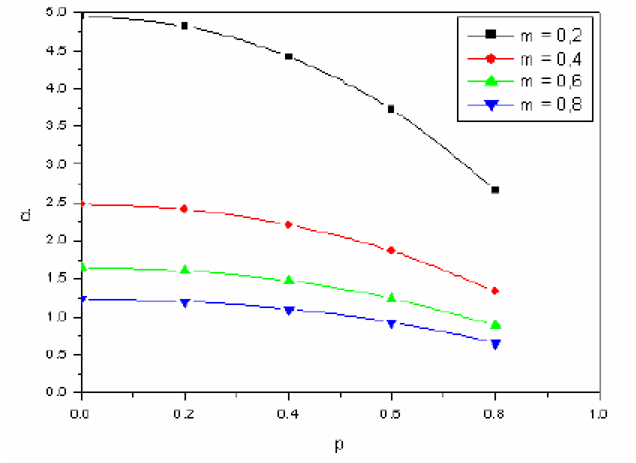
Figure 8. Plastic collapse factor α versus internal pressure p for several normalized moments m for close-ended elbows: (a) close-ended elbow, (b) for $n = 0$ and positive m, (c) for $n = 0.1$ and positive m and (d) for $n = 0.1$ and negative m.

Figure 8.a shows an elbow with close-ended submitted to a combination of axial load N, bending moment M and internal pressure P. Figure 8.b, for $n = 0$, shows similar appearance as Fig. 6.a of straight pipe submitted to same loadings. Figures 8.c and 8.d, both with $n = 0.1$, show differences in plastic collapse performance as the elbow is submitted to a positive m (at Fig. 8c) or a negative m (at Fig. 8d). Comparing results of positive m (opening bending) with results of negative m (closing bending), it is clear that the pressurized close-ended elbow submitted to opening bending have a better performance in plastic collapse than those submitted to closing bending.

Figure 9.a shows an elbow with open-ended submitted to combination of axial load N, bending moment M and internal pressure P.



(a)



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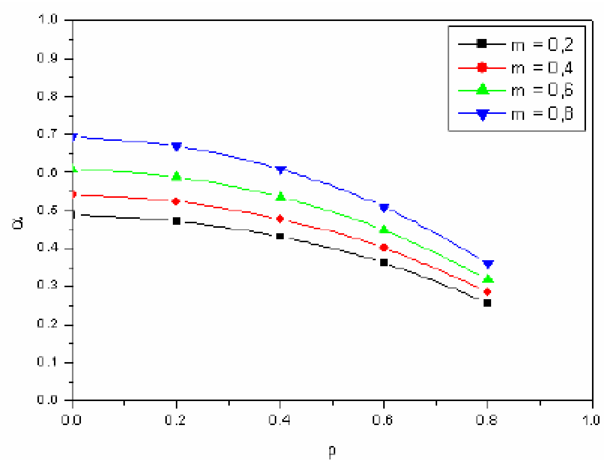

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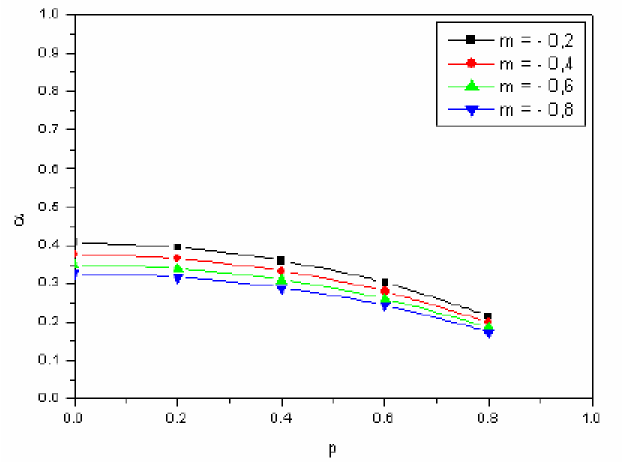

(g)

Figure 9. Plastic collapse factor α versus internal pressure p for several normalized moments m for open-ended elbows: (a) open-ended elbow, (b) for $n = 0$ and positive m, (c) for $n = 0.1$ and positive m and (d) for $n = 0.1$ and negative m.

Fig. 9.b, for $n = 0$ shows the similar appearance of Fig. 7.a for straight pipe submitted by same loadings. Figures 9.c and 9.d, both with same $n = 0.1$, show differences in plastic collapse performance as the elbow is submitted to opening bending in comparison with results of closing bending. The better performance in plastic collapse for opening bending is maintained for open-ended extremities elbows.

4. CONCLUSIONS

A limit analysis model for pressurized pipes was proposed. Additionally a software under Visual Basic® platform, specially projected to solve the resulting finite element problem, was developed. The plastic collapse of straight and curved pressurized pipes under axial and in-plane bending loading was analyzed. The initial results have shown that the internal pressure tends to reduce the pipe's capacity to bear additional bending and axial loading. For straight pipes the open-ended and closed-ended extremities produced appreciable effects while for elbows there were only little effects. The opening or closing bending altered significantly the performance of elbows, having the opening bending a better performance.

The main goal of the present approach is the possibility of carrying out collapse analysis of pressurized pipes without the need of expensive three-dimensional models. Consequently, it allows a rapid and efficient collapse analysis of pipelines, or any kind of long tubular structures, when , for example, it is necessary to assess line integrity due to off-design or unexpected loading.

In order to have a more realistic model, useful for a wide range of applications, it is mandatory the inclusion of other geometrical and mechanical effects to the proposed one-dimensional model as, for example, ovalization, external pressure and buckling constrains.

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