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THE INTEGRAL TRANSFORM METHOD FOR DEVELOPING TURBULENT FLOW WITHIN CHANNELS: ONE-EQUATION TURBULENCE MODELS

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Abstract. *The present work deals with the hybrid numerical-analytical solution of the developing turbulent flow within parallel-plate channels through application of the Generalized Integral Transform Technique (GITT). The turbulent flow is analyzed using four turbulence models that employ the eddy viscosity as the transported variable and is adopted the stream-function formulation. The main goals are to make available a hybrid general approach for evaluating all kinds of one-equation turbulence models, generalizing previous works based on the integral transforms that made use of simple algebraic and one-equation turbulent kinetic energy-based models, as well as to progress toward future implementations of more general and more complex turbulence models, as pointed out in the literature on this field of research. Velocity and friction factor potentials are evaluated for Reynolds number ranging from 35000 to 50000, illustrating the versatility of the GITT approach in address problems where strong coupling and non-linearities are inherently present. Analyses of convergence for the evaluated potentials are shown, and critical comparisons between experimental and theoretical results are performed. Finally, it is concluded that the present methodology could be employed as an appropriate benchmarking tool in evaluating the abilities of new turbulence models, as the overshooting velocity phenomen intrinsically present in this type of flow, or in predicting the main characteristics of turbulent channel flows, due mainly to its hybrid numerical-analytical nature.*

Keywords*: Integral Transforms, Turbulent Channel Flow, One-Equation Turbulence Models*

1. INTRODUCTION

Despite the recent and progressive developments in direct simulation of turbulent flows, the concept of Reynolds averaging, and the associated turbulent modeling for closure, still remains as a practical tool in engineering simulations. Furthermore, the development of techniques for solving the transport equations governing the convective-diffusive problems is a well defined field of research in both contexts of applied mathematics and physical sciences. The literature that addresses this issue (modeling and solution) is in continuous development and several purely numerical techniques have been employed to solve the highly non-linear equations that govern this class of flows (Wilcox, 1994; Frisch, 2001; Rodi and Fueyo, 2002; Davidson, 2007). Finally, with the successful development of the hybrid method called Generalized Integral Transform Technique - GITT (Cotta, 1993; Cotta, 1998; Cotta and Mikhailov, 1997; Santos *et al*., 2001), a new approach has been firmly established. The integral transform method is a spectral-type technique based on eigenfunction expansions that blends the best ingredients of both analytical and numerical techniques, being currently employed in all fields of engineering where convective and diffusive effects are presents.

Therefore, following previous successful implementations of this methodology on turbulent channel flow analysis who made use of simple algebraic and one-equation K-L turbulence models (Pimentel, 1993; Lima, 1995; Lima, 2000; Pimentel and Lima, 2001), the present work progresses toward future application of the GITT by employing more universal and general turbulence models, following the trend of literature in this field of research. Then, four one-equation turbulence models based on a transport equation for the turbulent viscosity were tested in the present work, namely: the model developed by Sekundov (1971), the model by Baldwin and Barth (1990), the model and some of its variations due to Spalart and Allmaras (1992a , 1992b,1994) and the model developed by Menter (1997).

Although employing the eddy viscosity concept, these models are designed to be more general than the well-known one-equation K-L turbulence model, since they do not require any "a priori" additional explicit length scale, as it is required by the K-L turbulence model, but only one transport equation for the turbulent viscosity or for a turbulent variable directly related to the eddy viscosity. Therefore, since they present computational stability similar to the algebraic ones, and are of easy numerical implementation, this type of turbulence model has recently gained attention from the scientific researchers in this field of research. Besides the previously cited, others interesting eddy viscosity transport equation turbulence models can be found in Nee and Kovasznay (1968), Gulyaev *et al*. (1993), Vasiliev *et al*. (1997) and Nagano *et al*. (1997).

Within this context, the aim of the present work is three-fold: first, investigate the numerical behavior of convergence rates of the employed eigenfunction expansions in representing velocity, turbulent viscosity and related potentials. Second, as explained before, extend the application of the GITT method by using more complex turbulence models, making available a hybrid general approach for evaluating all kinds of one-equation turbulence models. Finally, verify the main capabilities of the employed turbulence models in representing, with some degree of accuracy, the main features present in a developing turbulent channel flow, based on a hybrid approach that, through its direct and automatic control of global error, is presented as one of the best methodologies for benchmarking purposes.

2. MATHEMATICAL FORMULATION

It is considered here the steady state turbulent developing flow, of an incompressible fluid, within a parallel-plates channel of height 2b. Fluid enters the channel under uniform and parallel flow conditions, and it is assumed that transition laminar-turbulent occurs straight at the inlet of the channel. Since previous works based on integral transforms had demonstrated its advantages, it is employed the streamfunction-only formulation.

So, under the previous assumptions, the governing boundary layer equations for the streamfunction and the related turbulent transport variable are written as:

$$\frac{\partial\psi}{\partial y}\frac{\partial^3\psi}{\partial x\partial y^2}-\frac{\partial\psi}{\partial x}\frac{\partial^3\psi}{\partial y^3}=\frac{\partial}{\partial y}\left[\frac{\partial}{\partial y}\left((1+\nu_t)\frac{\partial^2\psi}{\partial y^2}\right)\right]\quad ;\qquad 0<y<1,\quad x>0 \tag{1}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial R}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial R}{\partial y}=\frac{\partial}{\partial y}\left[\left(\frac{1}{\sigma_d}+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial R}{\partial y}\right]+PD\quad ;\qquad 0<y<1,\quad x>0 \tag{2}$$

These equations are submitted to the inlet and boundary conditions:

$$\left.\begin{aligned}&\psi(0,y)=y\\ &\left.\frac{\partial\psi}{\partial x}\right|_{x=0}=0\\ &R(0,y)=R_e(y)\end{aligned}\right\}\quad ;\qquad x=0,\quad 0<y<1 \tag{3-5}$$

$$\left.\begin{aligned}&\psi(x,0)=0\\ &\left.\frac{\partial^2\psi}{\partial y^2}\right|_{y=1}=0\\ &\left.\frac{\partial R}{\partial y}\right|_{y=0}=0\end{aligned}\right\},\quad y=0,\quad x>0\quad ;\qquad \left.\begin{aligned}&\psi(x,1)=1\\ &\left.\frac{\partial\psi}{\partial y}\right|_{y=1}=0\\ &R(x,1)=0\end{aligned}\right\},\quad y=1,\quad x>0 \tag{6-11}$$

2.1. Turbulence Models

The variable ***R*** used in the previous formulation was employed as a general transport turbulent variable in order to generalize the present approach for different one-equation transport turbulence models. The last term on the turbulent model equation, PD, represents the production and dissipation contributions of each specific turbulence model. Therefore, any one-equation turbulence model can be written as Eq. (2).

As previously introduced, the eddy viscosity transport models that were tested are due to Sekundov (1971) – SE^{71} model, who developed the turbulent viscosity differential equation on the basis of the kinetic-energy balance of turbulence, being mathematically described for elliptic cases in Vasiliev *et al*. (1997); the model due to Baldwin and Barth (1990) – BB^{90} Model, who developed their model from the two-equation K-ε model and a number of additional simplifying assumptions, being easily found in Wilcox (1994); three versions of the model due to Spalart-Almaras (1992a, 1992b, 1994): SA^{92a}, SA^{92b} and SA^{94}, who, by the belief that generating a one-equation model as a simplified version of the K-ε model was not the optimal, developed their model based on empiricism, arguments of dimensional analysis and Galilean invariance; and, finally, the model developed by Menter (1997): ME^{97} Model, who, by reexamining and establishing a firm connection between one- and two-equation models of turbulence, developed his one-equation transport model in a similar sense as done by Baldwin and Barth (1990).

The functions and constants characterizing each turbulence model are described as follows.

a) Sekundov Model (1971) – SE Model: $R = \nu_t$

$$\sigma_d = \sigma = 1.0 \ , \qquad \sigma_\varepsilon = \sigma_t = 0.5 , \qquad PD = R\left[0.2\,\alpha\,\Gamma - \frac{3R+50}{S_w^2} \right] ,$$

$$\Gamma = \mathrm{Re}\left| \frac{\partial^2 \psi}{\partial y^2} \right| \ , \qquad \alpha = \frac{\left(\frac{R}{8}\right)^2 + 1.4\left(\frac{R}{8}\right) + 0.2}{\left(\frac{R}{8}\right)^2 - 1.4\left(\frac{R}{8}\right) + 1.0} \ , \qquad S_w = (1-y) \tag{12}$$

b) Baldwin and Barth Model (1990) – BB Model: $R = \tilde{R}_T$

$$\nu_T = C_\mu\, R\, D_1 D_2 \ , \qquad \sigma_d = \sigma = 1.0 \ , \qquad \frac{1}{\sigma_\varepsilon} = \frac{(C_{\varepsilon 2} - C_{\varepsilon 1})\sqrt{C_\mu}}{\kappa^2} \ ,$$

$$C_\mu = 0.09 \ , \qquad \kappa = 0.41 , \qquad C_{\varepsilon 1} = 1.2 \ ,$$

$$C_{\varepsilon 2} = 2.0 , \qquad PD = (C_{\varepsilon 2}\, f_2 - C_{\varepsilon 1})\sqrt{R\,P} - 2\frac{1}{\sigma_\varepsilon}\frac{\partial \nu_T}{\partial y}\frac{\partial R}{\partial y} \ , \qquad \mathrm{P} = \mathrm{Re}^2\, \nu_{\mathrm{t}} \left(\frac{\partial^2 \psi}{\partial \mathrm{y}^2} \right)^2 \ ,$$

$$f_2 = \frac{C_{\varepsilon 1}}{C_{\varepsilon 2}} + \left(1 - \frac{C_{\varepsilon 1}}{C_{\varepsilon 2}}\right)\left(\frac{1}{\kappa Y^+} + D_1 D_2\right) \left\{ \sqrt{D_1 D_2} + \frac{Y^+}{\sqrt{D_1 D_2}}\left[\frac{D_2}{A_0^+}\exp\left(-\frac{Y^+}{A_0^+}\right) + \frac{D_1}{A_2^+}\exp\left(-\frac{Y^+}{A_2^+}\right) \right] \right\} , \tag{13}$$

$$D_1 = 1 - \exp\left(-\frac{Y^+}{A_0^+}\right), \qquad D_2 = 1 - \exp\left(-\frac{Y^+}{A_2^+}\right), \qquad A_0^+ = 26 \ ,$$

$$A_2^+ = 10 , \qquad Y^+ = \mathrm{Re}(1-y)\,u_\tau \ , \qquad u_\tau = \sqrt{\frac{1}{\mathrm{Re}}\frac{\partial^2 \psi}{\partial y^2}\bigg|_{y=0}}$$

c) Spalart-Almaras Model (1992a, 1992b, 1994) – SA Model: $R = \chi$

$$\nu_t = f_{v1}\,\chi \ , \qquad \sigma_d = \sigma_\varepsilon = \sigma = \frac{2}{3} \ , \qquad f_{v1} = \frac{\chi^3}{\chi^3 + C_{v1}^3} \ , \qquad C_{v1} = 7.3 \ ,$$

$$PD = C_{b1}(1 - f_{t2})\tilde{S}\,\chi + \frac{C_{b2}}{\sigma_d}\left(\frac{\partial \chi}{\partial y}\right)^2 - C_{w1}\, f_w \left(\frac{\chi}{d}\right)^2 \ , \qquad C_{b1} = 0.1355 \ , \qquad C_{b2} = 0.622 \ ,$$

$$d = (1-y) \ , \qquad C_{w1} = \frac{C_{b1}}{\kappa^2} + \frac{1 + C_{b2}}{\sigma_d} \ , \qquad \tilde{S} = S\, f_{v3} + \frac{\chi}{\kappa^2 d^2} f_{v2} \ , \qquad S = \left| \frac{\partial^2 \psi}{\partial y^2} \right| , \qquad \kappa = 0.41 \ , \tag{14}$$

$$f_w = g\left[\frac{1 + C_{w3}^6}{g^6 + C_{w3}^6} \right]^{1/6} , \qquad g = r + C_{w2}\left(r^6 - r\right) \ , \qquad r = \frac{\chi}{\tilde{S}\kappa^2 d^2} , \qquad C_{w2} = 0,3 \ , \qquad C_{w3} = 2.0$$

Depending on the version adopted, the following functions must be chosen:

$$f_{t2} = \begin{cases} 0 \\ C_{t3}\exp\left(-C_{t4}\chi^2\right) \\ 0 \end{cases} ; \qquad f_{v2} = \begin{cases} 1-\dfrac{\chi}{1+\chi f_{v1}} \\ 1-\dfrac{\chi}{1+\chi f_{v1}} \\ \dfrac{1}{\left(1+\chi/C_{v2}\right)^3} \end{cases} ; \qquad f_{v3} = \begin{cases} 1 \\ 1 \\ \dfrac{\left(1+f_{v1}\chi\right)\left(1-f_{v2}\right)}{\chi} \end{cases} \Rightarrow \begin{Bmatrix} \mathrm{SA}^{92a} \\ \mathrm{SA}^{92b} \\ \mathrm{SA}^{94} \end{Bmatrix} \tag{15}$$

where,

$$C_{t3} = 1.1 \ , \qquad C_{t4} = 2.0 \ , \qquad C_{v2} = 5.0$$

The two first versions can be found in the original paper of Spalart and Allmaras (1992), while the last one can be found in the work of Deck *et al*. (2002).

d) Menter Model (1997): $R = \eta$

$$\nu_t = \mathrm{D}_2\,\eta \ , \qquad \sigma_d = \sigma_\varepsilon = \sigma = 1 \ , \qquad D_2 = 1 - e^{-\left(\frac{\eta}{A^+\kappa}\right)^2} , \qquad A^+ = 13 \ , \qquad \kappa = 0.41 \ ,$$

$$PD = C_1 D_1 \eta S - C_2 E_{1e} \ , \qquad C_1 = 0.144 \ , \qquad C_2 = 1.71 \ , \qquad D_1 = \frac{D_2\eta + 1}{R+1} , \tag{16}$$

$$S = \left|\frac{\partial^2\psi}{\partial y^2}\right| , \qquad E_{1e} = C_3 E_{BB}\tan\left(\frac{E_{k\_e}\eta}{C_3 E_{BB}}\right)^2 , \qquad C_3 = 7 \ , \qquad E_{BB} = \left(\frac{\partial\eta}{\partial y}\right)^2 , \qquad E_{k\_e} = \frac{1}{S^2}\frac{\partial S}{\partial y}\frac{\partial S}{\partial y}$$

The following dimensionless groups were employed in the above problem formulation:

$$x = \frac{x^*}{b}\frac{1}{\mathrm{Re}} \ , \qquad y = \frac{y^*}{b} \ , \qquad d = \frac{d^*}{b} \ , \qquad u = \frac{u^*}{u_0} ,$$

$$v = \frac{v^*}{u_0}\mathrm{Re} \ , \qquad P = \frac{P^*}{\rho u_0^2} \ , \qquad K = \frac{K^*}{u_0^2} \ , \qquad \mathrm{Re} = \frac{u_0\, b}{\nu} \ , \tag{17}$$

$$\nu_t = \frac{\nu_t^*}{\nu} \ , \qquad K_{ec} = \frac{K_{ec}^*}{u_0^2} = \frac{3}{2}\left(\frac{\tau_{ec}}{100}\right)^2 , \qquad \chi = \frac{\tilde{\nu}}{\nu} \ , \qquad \eta = \frac{\tilde{\nu}_t}{\nu}$$

2.2. Turbulent Inlet Condition

To close the system, the turbulent variable profile at the channel inlet, $R_e(y)$, has to be specified. The ideal inlet condition would be that one experimentally obtained. However, since no experimental information is available, it is employed a procedure where the turbulent viscosity profile described by the K-L turbulence model (Wolfshtein, 1969) is made equal to the turbulent viscosity of each turbulence model used in the present work. The input key parameter in this procedure is the inlet turbulence level at the channel centerline, τ_{ec}, ranging from 0.1% to 0.8% (Lima Neto, 2006).

For the turbulence models used, the following non-linear equations are obtained, which were numerically solved through subroutine DZBREN (IMSL, 1991) with a prescribed relative error target of 10^{-11}.

$$\left.\begin{matrix} \nu_t \\ \mathrm{C}_\mu R D_1 D_2 \\ f_{v1}\chi \\ D_2\eta \end{matrix}\right\} = \nu_{tKL} \ ; \qquad \nu_{tKL} = \mathrm{Re}\, C_{\mu KL}\,\ell_\mu K_{ec}^{1/2} \ , \qquad K_{ec} = \frac{K_{ec}^*}{u_0^2} \equiv S = \frac{3}{2}\left(\frac{\tau_{ec}}{100}\right)^2 ,$$

$$\ell_\mu = (1-y)\left[1 - e^{-A_\mu R_T}\right] , \qquad R_T = \mathrm{Re}(1-y)K_{ec}^{1/2} \ , \qquad A_\mu = 0.016 , \qquad C_{\mu KL} = 0.22 \tag{18}$$

3. SOLUTION METHODOLOGY

According to GITT approach, in order to improve convergence rates, it is employed a filtering procedure for the streamfunction expansion, which homogenizes the boundary condition at the wall:

$$\psi(x,y) = \phi(x,y) + \psi_F(y) \tag{20}$$

Here, $\psi_F(y)$ is the fully developed turbulent flow profile, obtained through application of a simple algebraic turbulence model, as that one developed by Emery and Gessner (1976) and used by Lima (2000).

After that, it must be chosen auxiliar eigenvalue problems, which form the basis for the integral transformation process. Such eigenvalue problems are homogeneous versions of the original problems, and can be found in Lima Neto (2006). These eigenvalue problems permit definitions of the following inverse/integral transform pairs:

$$\phi(x,y) = \sum_{i=1}^{\infty} \tilde{Y}_i(y)\bar{\phi}_i(x) \ , \qquad \bar{\phi}_i(x) = \int_0^1 \tilde{Y}_i(y)\phi(x,y)\,dy \tag{21, 22}$$

$$R(x,y) = \sum_{i=1}^{\infty} \tilde{C}_i(y)\bar{R}_i(x) \ , \qquad \bar{R}_i(x) = \int_0^1 \tilde{C}_i(y)R(x,y)\,dy \tag{23, 24}$$

Finally, accounting for the eigenfunctions orthogonality properties, integration of Eqs. (1-5), according to integral transforms formulae, Eqs. (22) and (24), yields the following coupled infinity ordinary differential equations system:

$$\sum_{\mathrm{k}=1}^{\infty} A_{ik} \frac{\mathrm{d}\bar{\phi}_k}{\mathrm{dx}} = B_{i\phi} \ , \qquad \mathrm{i}=1, 2, 3, \ldots, \infty \tag{25}$$

$$\sum_{\mathrm{k}=1}^{\infty} G_{ik} \frac{\mathrm{d}\bar{\mathrm{R}}_k}{\mathrm{dx}} - \sum_{\mathrm{j}=1}^{\infty} H_{ij} \frac{d\bar{\phi}_j}{dx} = B_{iR} \ , \qquad \mathrm{i}=1, 2, 3, \ldots, \infty \tag{26}$$

Submitted to the initial conditions:

$$\bar{\phi}_i(0) = \int_0^1 \tilde{Y}_i \left[y - \psi_F(y) \right] dy \ , \qquad \mathrm{i} = 1, 2, 3, \ldots, \infty \tag{27}$$

$$\bar{R}_i(0) = \int_0^1 \tilde{C}_i \, R_e(y)\,dy \ , \qquad \mathrm{i} = 1, 2, 3, \ldots, \infty \tag{28}$$

The above coefficients, resulting from the integral transformation process, are defined as:

$$A_{ik} = \left[C_{ikF} - D_{ikF} \right] + \sum_{\mathrm{k}=1}^{\infty} \left[A_{ijk} - B_{ijk} \right] \bar{\phi}_j \ ; \qquad G_{ik} = \mathrm{G}_{ikF} + \sum_{\mathrm{j}=1}^{\infty} G_{ijk}\,\bar{\phi}_j \ ; \qquad H_{ij} = \sum_{\mathrm{k}=1}^{\infty} H_{ijk}\,\bar{R}_k$$

$$B_{i\phi} = \mu_i^4\,\bar{\phi}_i + E_{iF} + \mathrm{B}_{i\nu F} \ ; \qquad B_{iR} = -\frac{1}{\sigma_d}\gamma_i^2 \bar{R}_i - \frac{1}{\sigma_r} B_{i\nu R} + \overline{PD}_i \tag{29}$$

$$A_{ijk} = \int_0^1 \tilde{\mathrm{Y}}_\mathrm{i} \tilde{\mathrm{Y}}_\mathrm{j}' \tilde{\mathrm{Y}}_\mathrm{k}'' dy \ ; \qquad B_{ijk} = \int_0^1 \tilde{\mathrm{Y}}_\mathrm{i} \tilde{\mathrm{Y}}_\mathrm{j}''' \tilde{\mathrm{Y}}_\mathrm{k} dy \ ; \qquad C_{ikF} = \int_0^1 \tilde{\mathrm{Y}}_\mathrm{i} \tilde{\mathrm{Y}}_\mathrm{k}'' \psi_F' \, dy$$

$$D_{ikF} = \int_0^1 \tilde{\mathrm{Y}}_\mathrm{i} \tilde{\mathrm{Y}}_\mathrm{k} \psi_F''' \, dy \ ; \qquad E_{iF} = \int_0^1 \tilde{\mathrm{Y}}_\mathrm{i}'' \psi_F'' dy \ ; \qquad B_{i\nu F} = \int_0^1 \tilde{\mathrm{Y}}_\mathrm{i}'' \nu_t \left[\frac{\partial^2 \phi}{\partial y^2} + \psi_F'' \right] dy \tag{30}$$

$$G_{ijk} = \int_0^1 \tilde{\mathrm{C}}_\mathrm{i} \tilde{\mathrm{Y}}_\mathrm{j}' \tilde{\mathrm{C}}_\mathrm{k} dy \ ; \qquad H_{ijk} = \int_0^1 \tilde{\mathrm{C}}_\mathrm{i} \tilde{\mathrm{Y}}_\mathrm{j} \tilde{\mathrm{C}}_\mathrm{k}' dy \ ; \qquad G_{ikF} = \int_0^1 \tilde{\mathrm{C}}_\mathrm{i} \tilde{\mathrm{C}}_\mathrm{k} \psi_F' dy$$

$$B_{i\nu R} = \int_0^1 \tilde{\mathrm{C}}_\mathrm{i}' \nu_\mathrm{t} \frac{\partial R}{\partial \mathrm{y}} dy \ ; \qquad \overline{PD}_i = \int_0^1 \tilde{\mathrm{C}}_\mathrm{i}\, PD \, dy$$

4. RESULTS

To solve the coupled system given by Eqs. (25) to (28), a program was written in Fortran 90 language and implemented on a two-processor 3.0 GHz Intel Xeon computer. In order to obtain numerical results, the expansions were truncated to finite orders $N\phi$ and NR, and a relative error criterion target of 10^{-6} was imposed to subroutine DIVPAG from IMSL (1991), which is appropriate to solve stiff ordinary differential equation systems.

Results for the main potentials, as longitudinal velocity component and friction factor, are illustrated for different Reynolds number. Where not explicitly cited, all results are showed by considering $N = N\phi = NR = 225$. Also, for all but Menter turbulence model, it was considered an uniform profile for the turbulent eddy viscosity described by the K-L turbulence model at the channel inlet, that is to say, expressions for ℓ_μ and R_t in Eq. (18) are evaluated at the channel centerline and, unless specified, for turbulence level $\tau_{ec} = 0.8\%$.

The first behavior to be analyzed in a turbulence model is its ability to reproduce the universal law of the wall in the fully developed flow region. Therefore, Figs. (1a) and (1b) illustrate some comparisons for the longitudinal velocity component, in wall coordinates, among the results produced with the present approach employing the different turbulence models and the universal law of the wall described by Arpaci and Larsen (1984) for Reynolds $35\text{x}10^3$ and $48\text{x}10^3$, respectively.

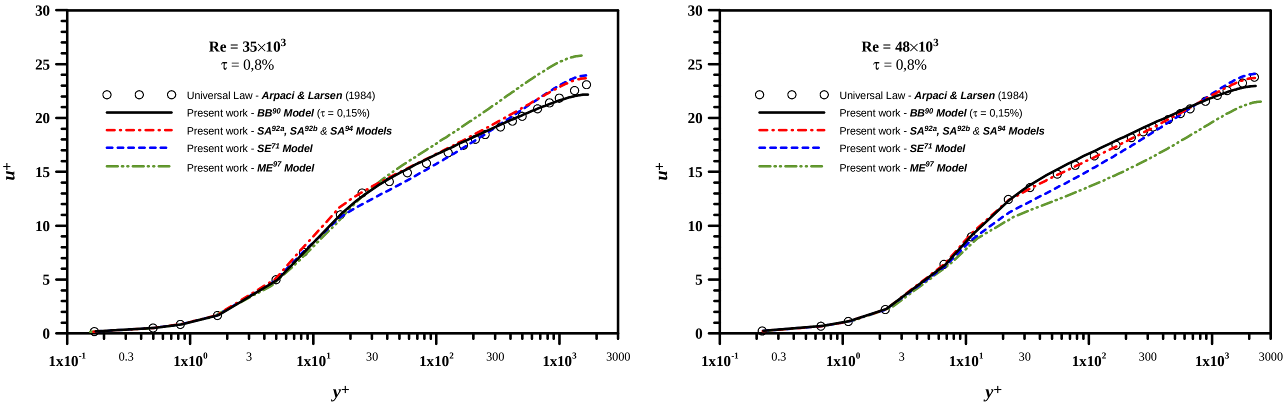

Figure 1. Longitudinal velocity profiles, in wall coordinates, for different one-equation turbulence models at the fully developed flow region: (a) $Re = 35\text{x}10^3$ and, (b) $Re = 48\text{x}10^3$

As one can see from these figures, despite the Menter model overestimates (underestimates), for low (high) Reynolds numbers, the longitudinal velocity on the central (outer) region of the channel and on the logarithm layer, all other models satisfactorily agree with the universal law of the wall. For this field, these results point out the models of Baldwin and Barth (1990) and Spalart and Allmaras (1992, 1994) as the best available choices.

The previous results suggest that a convergence behavior should be done in order to better characterize the turbulence model and better qualify the hybrid method adopted in the present work. Since the hybrid nature of the GITT approach allow for the filtering process, the fully developed region is almost automatically satisfied as the algebraic turbulence model used in that region brings most analytical features present in a turbulent channel flow (Emery and Gessner, 1976). So, unless the one-equation model adopted does not have good predictive capabilities, only a few terms will be required for a full convergence of the eigenfunction expansions in the fully developed region.

Therefore, Tab. (1) shows the convergence behavior of the longitudinal velocity component for different transversal positions, y, at axial positions $x^*/D_h = 5$. The following values were employed for the transversal coordinates, y: 0.0, 0.7, 0.97, 0.995 and 0.997, which approximately correspond to dimensionless distance to wall, or turbulent Reynolds number, y^+, of 1700, 500, 50, 8.5 and 5.0, respectively, being typical values used in a wall-bounded turbulent boundary layer analysis as can be visualized in Figs. (1a) and (1b).

From this table, one can see that, although the convergence rates are lower in the near the wall region than in the outer region, it can be considered that, at least, three significant digits are already converged for the longitudinal velocity. Then, any deficiency that would be attributed to the numerical methodology is avoided.

The great efficiency of the GITT approach is easily verified by the low order eigenfunction expansion requirement to represent the main behavior of the flow, since with just $N = 50$ this is easily attained. However, in order to produce results that could be considered benchmarks, a better convergence analysis should be done. This study will be done in future works, when more consistent results will be made available.

Table 1. Convergence behavior of the velocity component, $u(x,y)$, at different transversal coordinate for $x^*/D_h = 5$

$u(x^*/D_h = 5, y)$										
Y (y^+)	0.0 (~1700)	0.7 (~500)	0.97 (~50)	0.995 (~8.5)	0.997 (~5)	0.0 (~1700)	0.7 (~500)	0.97 (~50)	0.995 (~8.5)	0.997 (~5)
N	SE^{71} Model					BB^{90} Model (τ_{ec} = 0.15%)				
50	1.095	0.9993	0.5904	0.2940	0.1913	1.066	1.005	0.7027	0.3560	0.2330
100	1.071	1.011	0.6588	0.3458	0.2217	1.066	1.006	0.7003	0.3524	0.2303
150	1.067	1.013	0.6692	0.3678	0.2440	1.066	1.006	0.6996	0.3486	0.2267
200	1.067	1.013	0.6700	0.3669	0.2414	1.066	1.006	0.6990	0.3476	0.2257
225	1.067	1.014	0.6706	0.3663	0.2386	1.066	1.006	0.6987	0.3476	0.2260
N	SA^{92a} Model					SA^{92b} Model				
50	1.065	1.013	0.6884	0.3578	0.2344	1.065	1.013	0.6883	0.3578	0.2344
100	1.062	1.015	0.6931	0.3680	0.2417	1.062	1.015	0.6929	0.3680	0.2417
150	1.063	1.015	0.6919	0.3628	0.2359	1.063	1.015	0.6918	0.3627	0.2359
200	1.063	1.015	0.6917	0.3606	0.2305	1.063	1.015	0.6917	0.3606	0.2304
225	1.063	1.015	0.6915	0.3610	0.2293	1.063	1.015	0.6914	0.3609	0.2293
N	SA^{94} Model					ME^{97} Model				
50	1.064	1.014	0.6879	0.3576	0.2343	1.055	1.049	0.6554	0.3329	0.2224
100	1.061	1.016	0.6938	0.3682	0.2419	1.059	1.048	0.6496	0.3394	0.2227
150	1.062	1.016	0.6935	0.3627	0.2358	1.062	1.048	0.6378	0.3382	0.2232
200	1.062	1.016	0.6939	0.3603	0.2299	1.064	1.057	0.6134	0.3062	0.2042
225	1.062	1.016	0.6938	0.3607	0.2288	1.065	1.060	0.6013	0.2939	0.1978

Also, from Tab. (1), it is clearly observed that, due to the simplicity of the flow analyzed, there are not any explicit advantages in using either version of the SA models (Spalart and Allmaras, 1992a, 1992b, 1994). Certainly, the main differences could be felt in complex flows to which the modifications were initially thought. For example, the functions f_{v2} and f_{v3} in the SA^{94} version were introduced to avoid poor convergence of the residual turbulence near reattachments, and the function f_{t3} in the SA^{92b} version should be employed when making Navier-Stokes-based predictions with aim in laminar/turbulent transitions study. In general, all models predict similar results for the longitudinal velocity at this axial position. However, it seems that Menter model flattens the mean velocity profile more than the other ones.

Then, to better see this behavior and, additionally, study the predictive capability of each turbulence model, Figs. (2a) and (2b) show transversal profiles of the longitudinal velocity component at some axial positions along the channel. These positions characterize the typical entrance and interaction zones in a developing channel flow. The results are illustrated only for $Re = 35\text{x}10^3$ and are plotted against the experimental data of Byrne *et al*. (1969-1970).

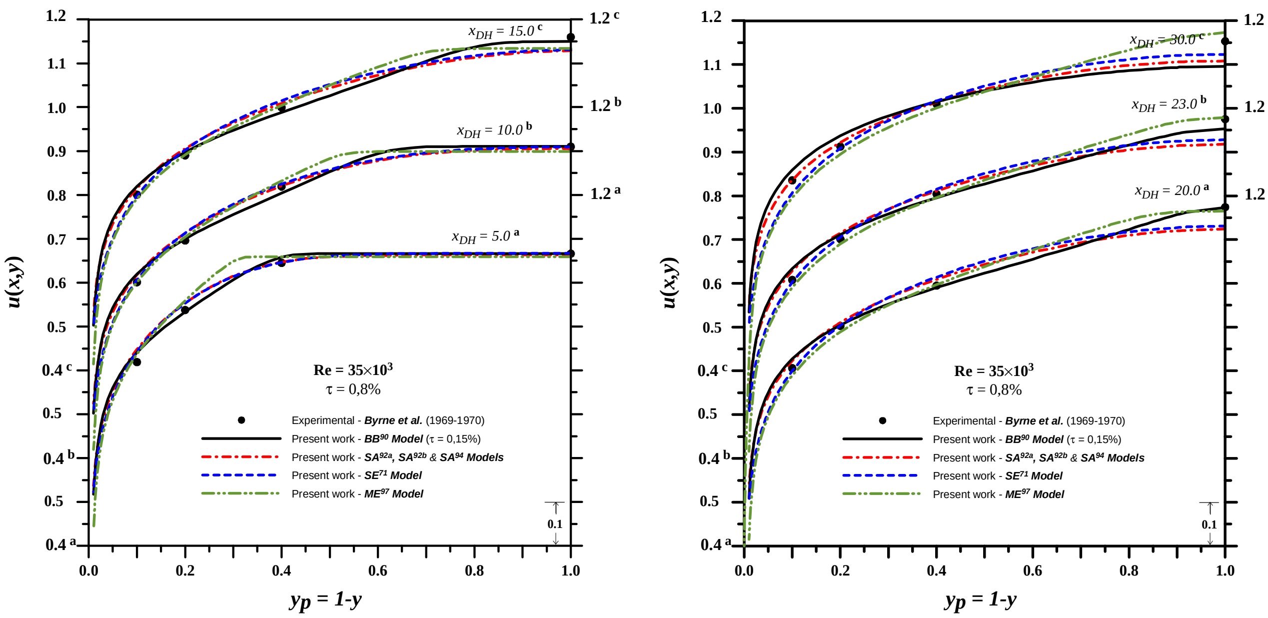

Figure 2. Transversal profiles of the longitudinal velocity component for different one-equation turbulence models and $Re = 35\text{x}10^3$ at different axial positions along the channel: (a) Entrance region and, (b) Interaction region

Figure (2a) shows the flatness of the Menter model is not so important, since this behavior is only verified in regions very near the entrance, being smeared as the flow develops along the channel. According to these figures, all turbulence models agree satisfactorily with the experimental data of Byrne *et al*. (1969-1970), on both entrance and interaction regions, indeed validating the present approach. However, as the interaction and fully developed regions are reached, it can be seen from Fig. (2b) that the centerline velocity is somewhat underestimated by all but Menter model.

Therefore, to have a better insight on this behavior, Figs. (3a) and (3b) bring the developing behavior of the centerline velocity for $Re = 35\text{x}10^3$ and $Re = 48\text{x}10^3$, respectively, making a comparison among the distinct turbulence models. Comparisons with the experimental data of Byre *et al*. (1969-1970) and Dean (1972) and with the numerical results of Stephenson (1976), who used the two-equation K-ε model and the finite difference method, are also made.

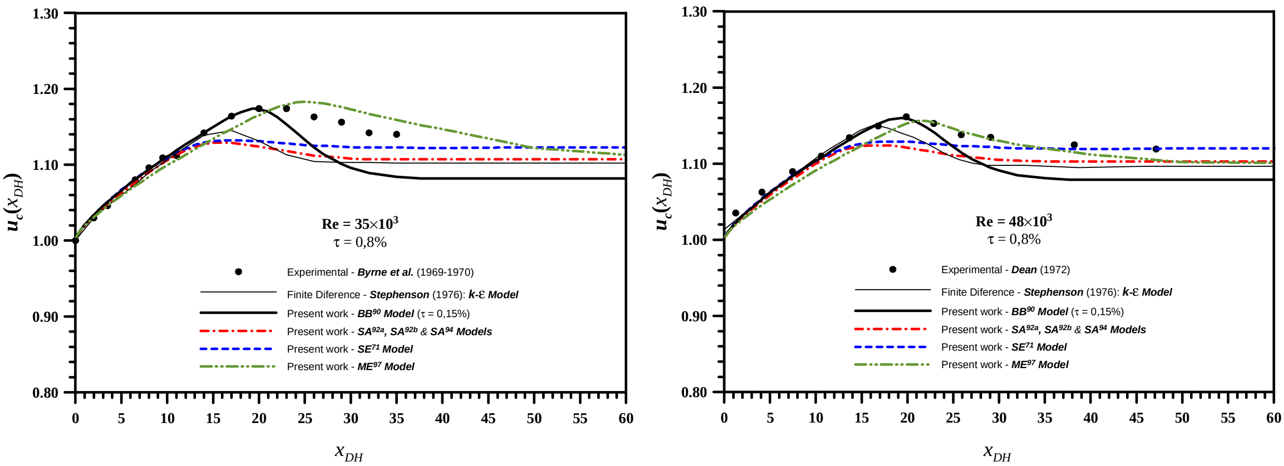

Figure 3. Non-asymptotic behavior of the centerline velocity development along the channel for different one-equation turbulence models: (a) $Re = 35\text{x}10^3$ and, (b) $Re = 48\text{x}10^3$

At the region near the entrance of the channel, all models agree well with the experimental results. On the other hand, as the interaction and fully developed flow regions are reached, the Menter model over predicts the results produced by another ones, confirming the tendency observed from Figs. (2a) and (2b). However, when making a comparison with the experimental results, the centerline velocity is well-represented by this model, being considered the best model. In general, it could be concluded that the ME^{97} model yields the best predictions for this potential, although BB^{90} and SA models offer better predictions in positions near the entrance region. The value of the peak and its position at the longitudinal axis are correctly predicted by the BB^{90} model, but the fully developed flow prediction is the worst. It seems that ME^{97} model tends to shift right the position of the centerline velocity peak.

To make a deeper investigation on the turbulence model properties, Figures (4a) and (4b) illustrate similar comparisons for the longitudinal velocity component at transversal positions near the channel wall, $y = 0.8$ and $y = 0.9$, respectively, for $Re = 35\text{x}10^3$.

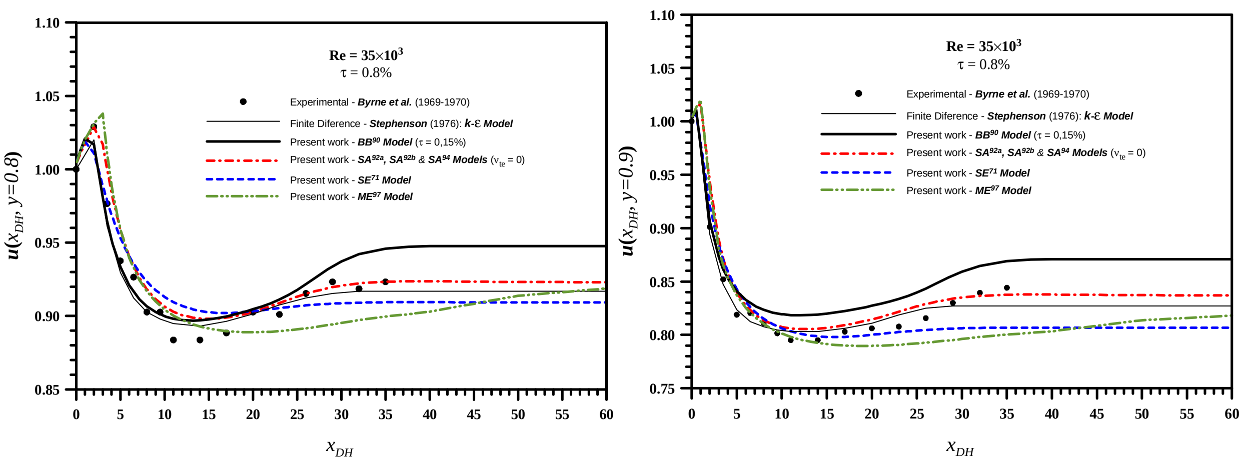

Figure 4. Non-asymptotic behavior of the longitudinal velocity component along the channel for different one-equation turbulence models and $Re = 35\text{x}10^3$: (a) $y = 0.8$ and, (b) $y = 0.9$

From these figures, near the channel wall, the ME^{97} model loses adherence to the experiments at the interactions region. The shifting behavior is still verified as the fully developed region is attained. For the BB^{90} model, the fully developed region is not so well represented, but its overall prediction characteristics is maintained. The best turbulence model seems to be the SA model. The results for the SA models were obtained imposing a null profile for the turbulence viscosity at the inlet. This is an excellent property a turbulence model would have, since experimental data is not normally available. Now, the good predictions properties of the K-ε model begin to appear.

Figures (4a) and (4b) bring a comparison between numerical and experimental results for the friction factor development along the channel. This potential is the most difficult to converge, as it has in its definition the gradient velocity at the wall. Experimental data are extracted from Marriot (1967), for Re = 35x10^3, and from Dean (1972), for Re = 48x10^3. Purely numerical results were found out from Stephenson (1976).

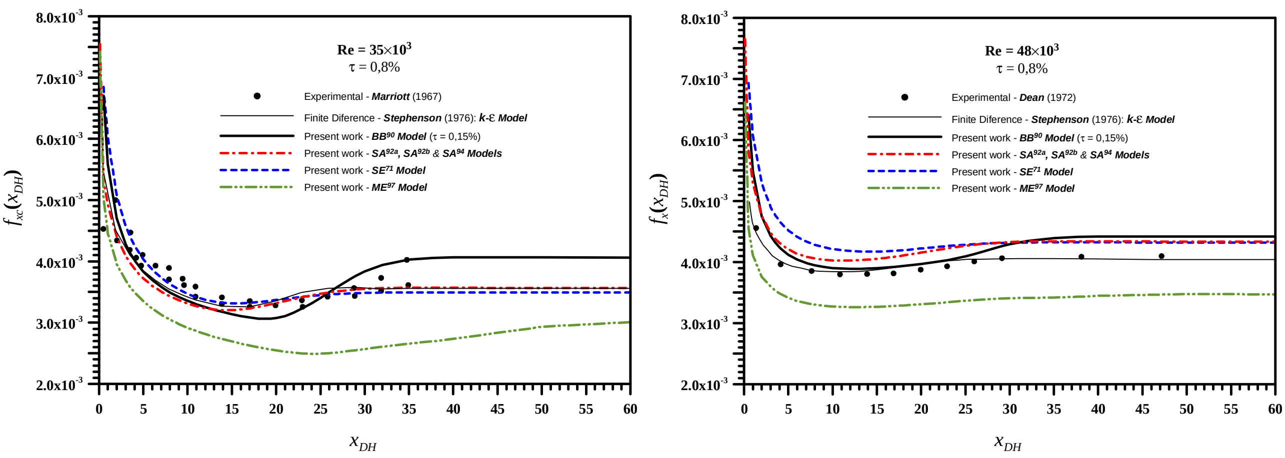

Figure 4. Non-asymptotic behavior of the friction factor development along the channel for different one-equation turbulence models: (a) Re = 35x10^3 and, (b) Re = 48x10^3

The same intrinsic non-asymptotic behavior viewed for the longitudinal velocity is also verified and studied for the friction factor. Again, differently of Figs. (2a) and (2b), and similarly as Figs. (3a) and (3b), Figs. (4a) and (4b) show that BB^{90} and SA turbulence models yield results that are in better agreement than those of ME^{97} model.

5. CONCLUSIONS

Under the previous panorama, it can be concluded that the non-asymptotic flow behavior, present in developing turbulent channel flows, is extremely difficult to be reproduced, even the two-equation K-ε model used by Stephenson (1976), which would be more general, was not able to correctly predict this phenomenon with deep accuracy. In relation to the present one-equation eddy viscosity turbulence models, overall, the Baldwin and Barth (1990) and the Spalart and Allmaras (1992a, 1992b, 1994) models present as the best choices to develop a study on numerical properties of a developing channel flow, and their properties will be used as base for future implementations of more advanced turbulence models. The best predictive properties of these models are the no requirement of any explicit length scale and the incorporation, in their formulation, of important terms that could be necessary in more complex flows. For the Sekundov (1971) model, in spite of being an one-equation turbulence model of simpler implementation (it does not use so many functions and constants), it does require a prescription of an explicit length scale, S_w, and therefore, loses universality. Related to the bad predictive capabilities of the Menter (1997) model, at its favor can be said that it was not developed to be a true turbulence model, but just to better explain the close relation between one- and two-equation turbulence models. Indeed, Tab. (1) indicates this model requires more terms on the expansions for a better convergence.

To close this theme, it should be pointed out that, in doing simulations of turbulent flows, it is a common practice by numerical codes to limit functions in production and dissipation terms of almost all turbulence models (Fluent, 2003). This practice was not adopted in the present work, since we are interested in demonstrate all numerical behaviors present in each turbulence model.

Finally, turning to the GITT approach, although a deeper study of convergence behavior has to be made, on the face of the results presented, it can be re-affirmed that it constitutes as a good methodology to be used in turbulent flow simulations due mainly to its analytical character and easy of numerical implementation.

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