EXPERIMENTAL CHARACTERIZATION OF NONLINEAR SYSTEMS USING LabVIEW

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Abstract. *This paper presents an experimental characterization of the behavior of a nonlinear system. The signals produced by nonlinear system are captured using a data acquisition board (DAQ) and processed using LabVIEW environment. The following aspects of the time series analysis are analyzed: time waveforms, phase portraits, frequency spectra, Poincaré sections and bifurcation diagram. The system behavior is experimentally mapped with the parameter variations, where are identified equilibrium points, periodic and chaotic attractors, and bifurcations.*

Keywords: *Chaos, Nonlinear dynamics, Data acquisition, Signal processing*

1. INTRODUCTION

The chaotic systems present an unpredictability behavior extremely sensitive to parameters variations and initial conditions, although they are completely described by deterministic laws and non-linear differential equations without stochastic components. This dynamical behavior has been extensively studied by mathematicians, physicians, engineers and more recently, specialists in information and social sciences, due to its great potential for commercial and industrial applications in areas such as Engineering, Informatics, Electronics, Communication, Robotics, Chemistry, Medicine, Biology, Epidemiology, Management, Finance, Information Processing, etc (Ditto and Munakata, 1995; Yang *et al.*, 2002; Cuomo *et al.*, 1993). One of the most important chaotic systems was created by Leon O. Chua in 1983, an electrical circuit constituted by a network of linear passive elements connected to a non-linear active component, known as Chua's diode, which standard form is shown in Fig. 1. Since its initial proposal, the Chua's circuit is intensely investigated and has been accepted as paradigm for study of important features of nonlinear systems, once it exhibits a very complex dynamical behavior in spite of its simplicity, presenting a rich scenario formed by a large variety of bifurcations, homoclinic orbits, and distinct periodic and chaotic attractors (Madan, 1993).

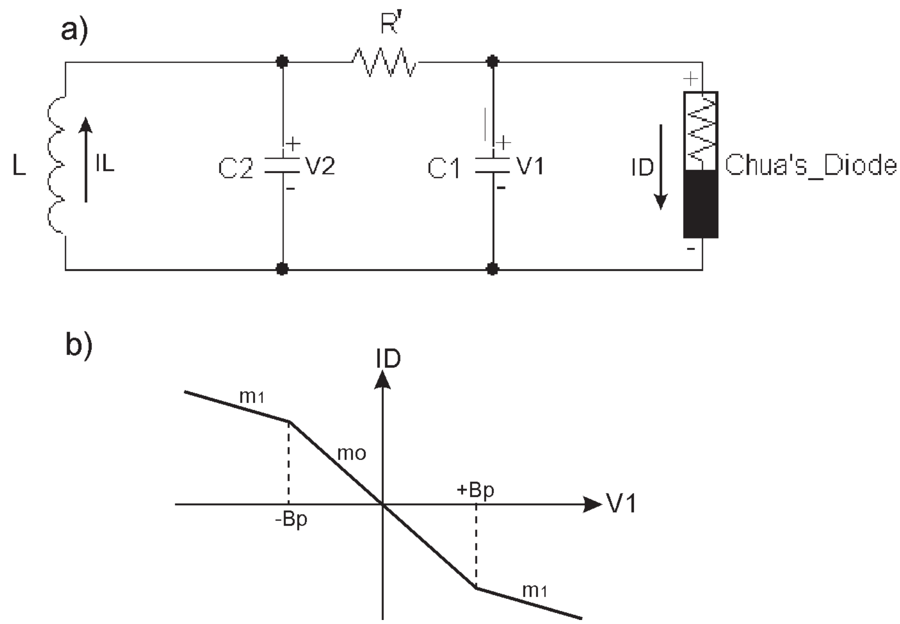

Figure 1. Schematic Chua's circuit: a) Standard form of the Chua's circuit. b) Characteristic of Chua's diode.

A characterization of the dynamical behavior of a nonlinear system only based in computer simulations is not totally reliable, and can produce few rigorous results and unsolved problems since the simulated model may not fully describe the system and numerical procedures are always subject to round-off errors. For example, chaotic orbits can be converted into periodic orbits in computer simulations due to finite precision of the machines and successive approximations at each step. Furthermore, computer simulations are completely deterministic and do not depend on natural variation of conditions, generating identical solutions when provided with identical initial conditions (Suneel, 2006). An experimental characterization of a nonlinear system plays a very important role to understand theoretical concepts and it is indispensable for a more rigorous analysis. Any unusual behavior of a dynamical system that is verified during the development of experimental procedures, intentionally or not, can motivate in-depth analysis of underlying mechanisms, generating more appropriate models to explain and predict similar events. Moreover, the experimentation can sometimes be well ahead of any theoretical analysis or simulation, particularly in an experimental system such as the Chua's circuit, which popularity in practical use often precedes any detailed analysis (Tse, 2002).

Usually, the analysis of the time series produced by the nonlinear system is an important issue to realize its experimental characterization, providing useful information and interpretations concerning to dynamical behaviors (Tse, 2002). Time-domain waveforms and phase portraits are familiar to most researchers, which can be enough in several situations to identify various dynamical properties of a system. Another technique for experimental analysis of time series is the harmonic decomposition of the measured signal using the Fast Fourier Transform (FFT), obtaining the power spectrum with the harmonics, sub-harmonics, and ultra-harmonics which provides an easy way to identify and to capture the signature of nonlinear phenomena. One approach to study nonlinear systems is the exam of a two-dimensional plane that intersects the system trajectories, known as Poincaré section, where it is possible to identify periodic, quasi-periodic or chaotic motions. An usual perspective for a qualitative analysis of the system behavior is the bifurcation diagram, a graphic representation of sampled attractors in a Poincaré section with respect to each value of a chosen system parameter, which provides a nice summary for the transition between different types of motion that can occur when one parameter of the system is varied.

This paper describes an experimental characterization of the behavior of the analogous version of the Chua's circuit proposed by Rocha and Medrano (Rocha and Medrano-T, 2008). Although several possibilities can be used to investigate nonlinear phenomena, it is considered methods based in the analysis of the times series produced by the circuit, which is captured using a low expensive data acquisition system and the LabVIEW, an integrated environment for development of applications related to measurements, tests and control. Applications are developed for the simultaneous observation of time waveforms, phase portraits, frequency spectra, Poincaré sections, and bifurcation diagram from the LabVIEW environment, which allows a real-time analysis of the acquired data for experimental identification of equilibrium points, periodic and chaotic attractors, and bifurcations. The experimental mapping of the circuit behavior is realized with the variation of a parameter bifurcation, where is observed its route to chaos.

2. ANALOGOUS CHUA's CIRCUIT

There are several alternatives for the realization of Chua's circuit, which are generally based in the combination of an electronic version of the Chua's diode with an inductor or an inductance emulator (Kiliç, 2003). In this paper, the experimental characterization is realized using the analogous Chua's circuit (Rocha and Medrano-T, 2008), which is shown in Fig. 2. This circuit is based in the dimensionless approach of the Chua's system, which eliminates equivalent parameter configurations grouping the seven original parameters R', C_1, C_2, L, m_0, m_1 and B_p into only four dimensionless groups α, β, a_0 and a_1. The experimental analogous Chua's circuit is implemented using the IC's AD633 (analog multiplier), IC TL071 (single op-amp) and IC TL074 (quad op-amp). All capacitors are $C = 4.7nF$ and the base resistance is $R = 10k\Omega$. A blue LED is used in an anti-parallel diodes configuration of the analogous Chua's diode. The normalized dimensionless parameters $a = \alpha/35$ and $b = \beta/35$ are represented by external dc voltage levels.

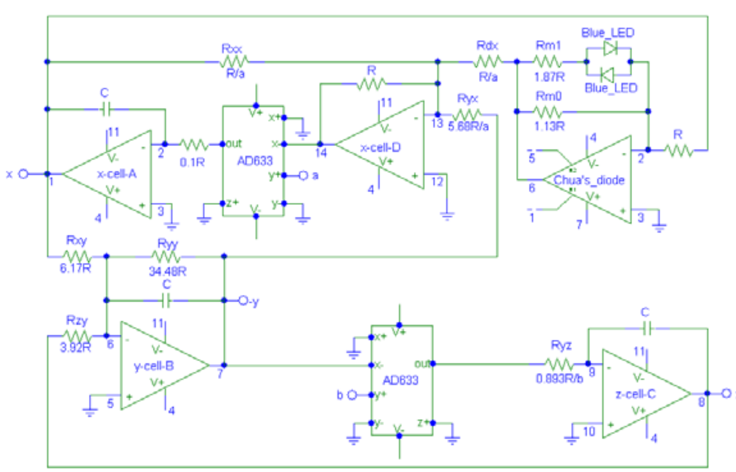

Figure 2. Analogous Chua's circuit.

3. DATA ACQUISITION

The experimental apparatus for data acquisition and signal processing is presented in the Fig. 3. The electrical signals generated by the analogous Chua's circuit are acquired at a rate of 16kSamples/s by three single ended analog inputs of a NI USB-6009, a USB based data acquisition (DAQ) and control device manufactured by National Instruments. This DAQ has 8 referenced single ended signal coupling or 4 differential signal coupling analog inputs (14-bit resolution, 48kSamples/s), and 2 analog outputs (12-bit resolution, 150 samples/second), which are used to generate the parameters a and b for the analogous Chua's circuit. It has still 12 configurable digital input/output (5V TTL/CMOS) and a 32-bit counter. The real-time data analysis is realized in the LabVIEW, a National Instruments software widely used as environment for data acquisition, prototyping and testing. The LabVIEW is a language that uses graphic representations of the functions (block diagram) for visual programming of an application, including a library of mathematical subroutines, graphics displays and other utilities extremely useful for data analysis and signal processing.

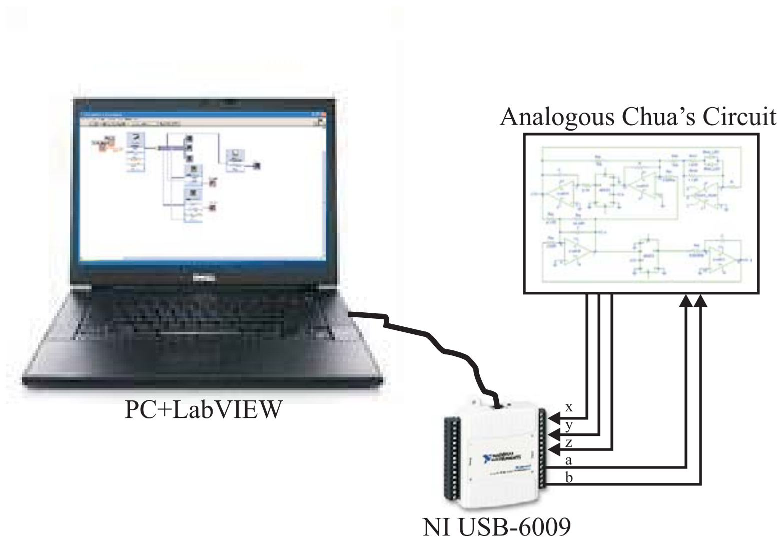

Figure 3. Experimental apparatus for data acquisition.

4. EXPERIMENTAL INVESTIGATION

4.1 Time waveforms

The visualization of time waveforms is normally a straightforward, direct and trivial method for analysis of a system. The time waveforms can be easily observed using oscilloscopes as a two-dimensional graph of the studied variable (vertical axis), plotted as function of time (horizontal axis). While periodic waveforms present a pattern, aperiodic waveforms such as those of quasi-periodicity or chaos appear to be shaking, which is generally a signature of these nonlinear phenomena. If a DAQ or a digital storage oscilloscope (DSO) is used, the waveform can be "freezed" at a certain instant and the irregular behavior of a chaotic waveform becomes apparent. The LabVIEW programming to visualize the waveforms is shown in the Fig. 4, and the Fig. 5 presents the waveforms generated by analogous Chua's circuit for $b = 3V(\beta = 105)$ and different values of a. A periodic behavior is observed for $a = 0.867V(\alpha = 30.35)$ and $a = 0.925V(\alpha = 32.40)$, since all waveforms are evidently regulars. For $a = 1,003V(\alpha = 35.10)$ and $a = 1.136V(\alpha = 39.75)$, none pattern is verified in the time evolution of the variables, which can characterize the chaotic behavior.

4.2 Phase portraits

The phase portrait is the geometric representation of the trajectories of a dynamical system in the phase plane, a coordinate frame defined by the independent variables that describe the system dynamics where all possible states of a system can be represented. The phase portraits consists in an important tool for the study of dynamical systems, since its dynamics is evidenced and identified, revealing information such as whether an attractor, a repellor or a limit cycle is present for a chosen parameter value. While a stationary system is represented by a fixed point in the phase plane, a periodic system presents a closed orbit (limit cycle). A chaotic behavior is characterized by phase portraits that confined to a well defined bounded region in the phase plane, known as stranger attractor, where the orbits never repeat the same trajectory. It can be easily distinguished from random noise which shows fuzzy edges on phase portraits. The phase portraits can be displayed in an oscilloscope, simply using the X-Y mode instead of a sweeping time base. For the visualization of the phase portraits, the block "Build XY Graph" is included in the LabVIEW programming as shown in the

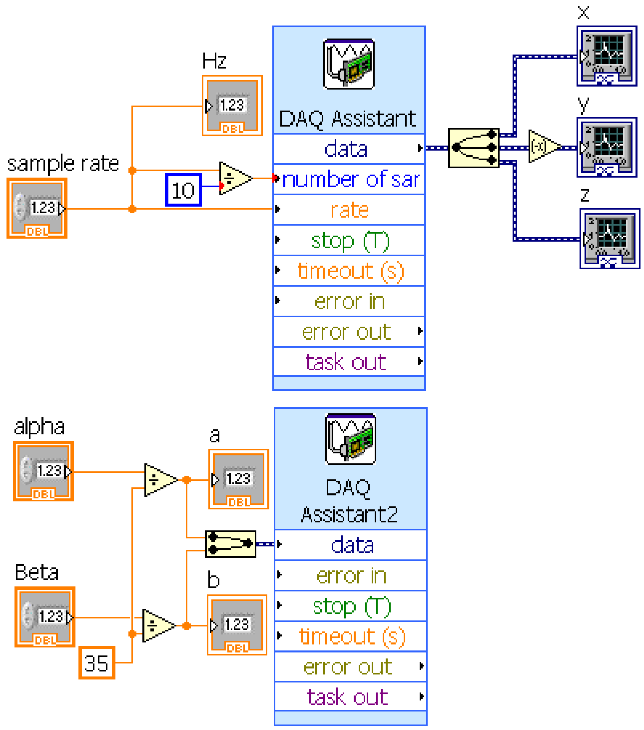

Figure 4. LabVIEW block diagram for visualization of the waveforms X vs t, Y vs t and Z vs t.

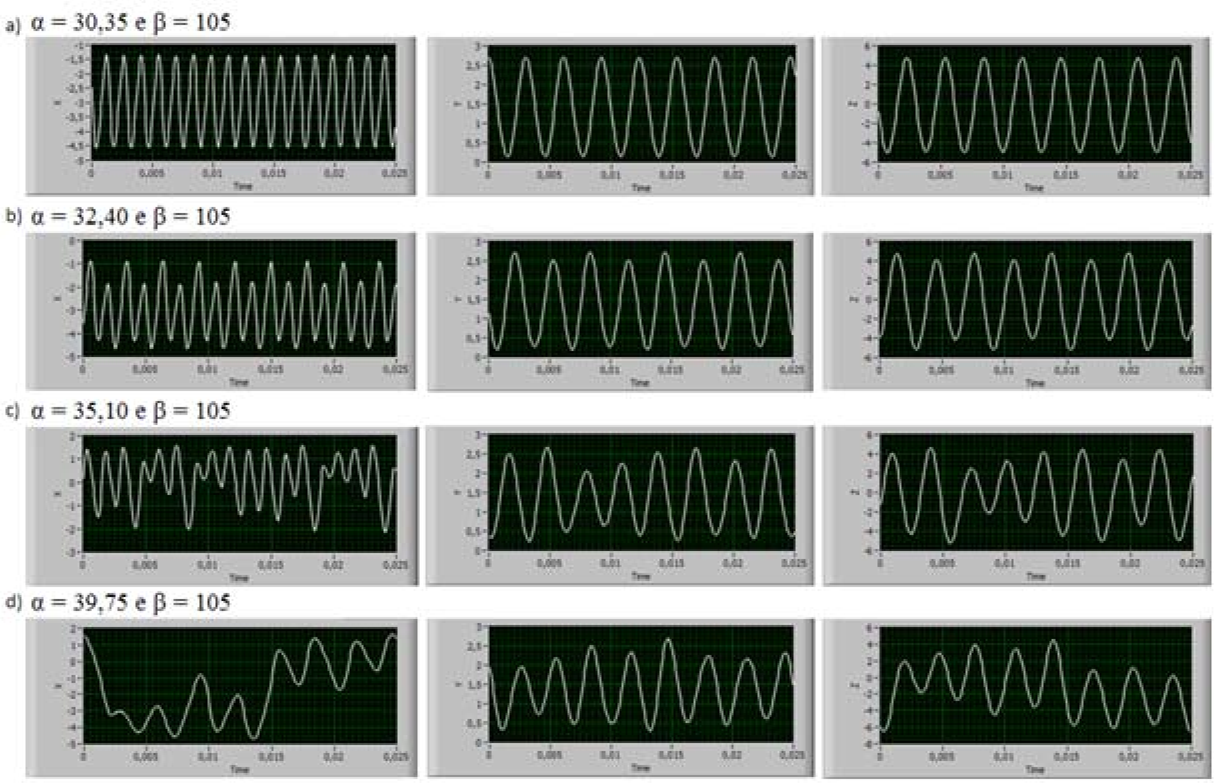

Figure 5. Time waveforms generated in analogous Chua's circuit: a) $\alpha = 30.35$ and $\beta = 105$; b) $\alpha = 32.40$ and $\beta = 105$; c) $\alpha = 35.10$ and $\beta = 105$; d) $\alpha = 39.75$ and $\beta = 105$.

fig. 6. The series of phase portraits presented in the fig. 7 shows as the behavior of the analogous Chua's circuit evolutes from a periodic to a chaotic motion when the parameter $a(\alpha)$ varies from $0.540V(18.90)$ to $1.403V(49.10)$ while the parameter $b(\beta)$ is fixed in $3V(105)$. The circuit presents a periodic behavior in the interval $0.54V < a < 0.926V(18.90 < \alpha < 32.40)$, happening a bifurcation for double-period orbit when $a = 0.893V(\alpha = 31.35)$. The dynamical behavior evolves to a Rossler-type attractor in the interval of $0.949V < a < 1.003V(33.20 < \alpha < 35.10)$. The transition of

Rossler to double-scroll occurs at $a = 1.014V (\alpha = 35.50)$, existing two windows in $a = 1.091V (\alpha = 38.20)$ and $1.201V < a < 1.213V (42.05 < \alpha < 42.45)$ where the behavior is periodic before of the circuit becomes unstable for $a > 1.403V (\alpha > 49.10)$.

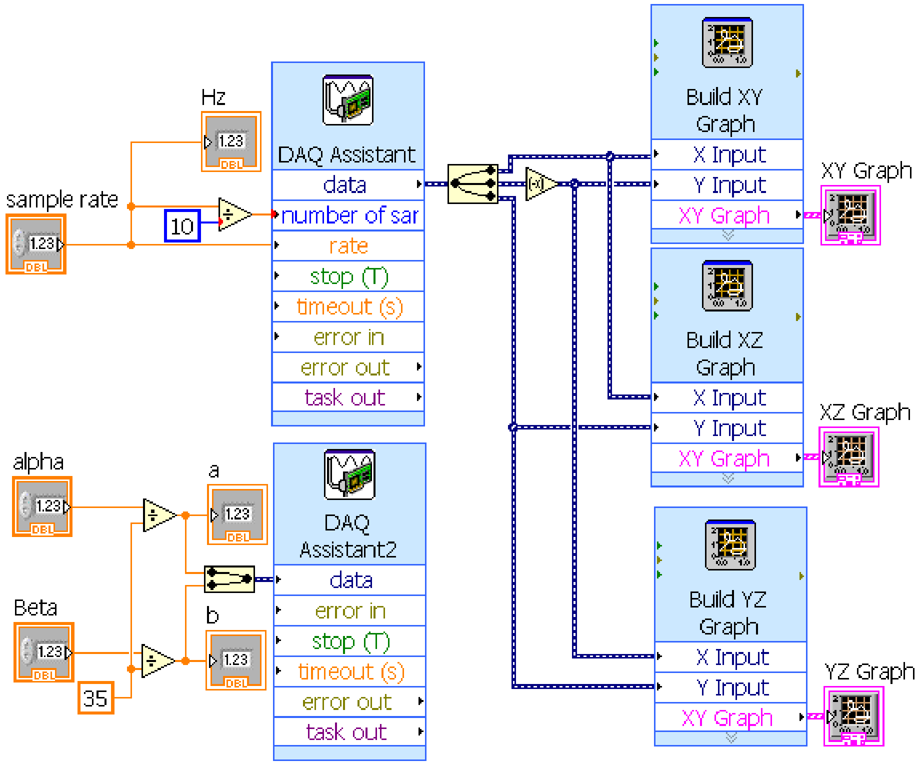

Figure 6. LabVIEW block diagram for visualization of the phase portraits Y vs X.

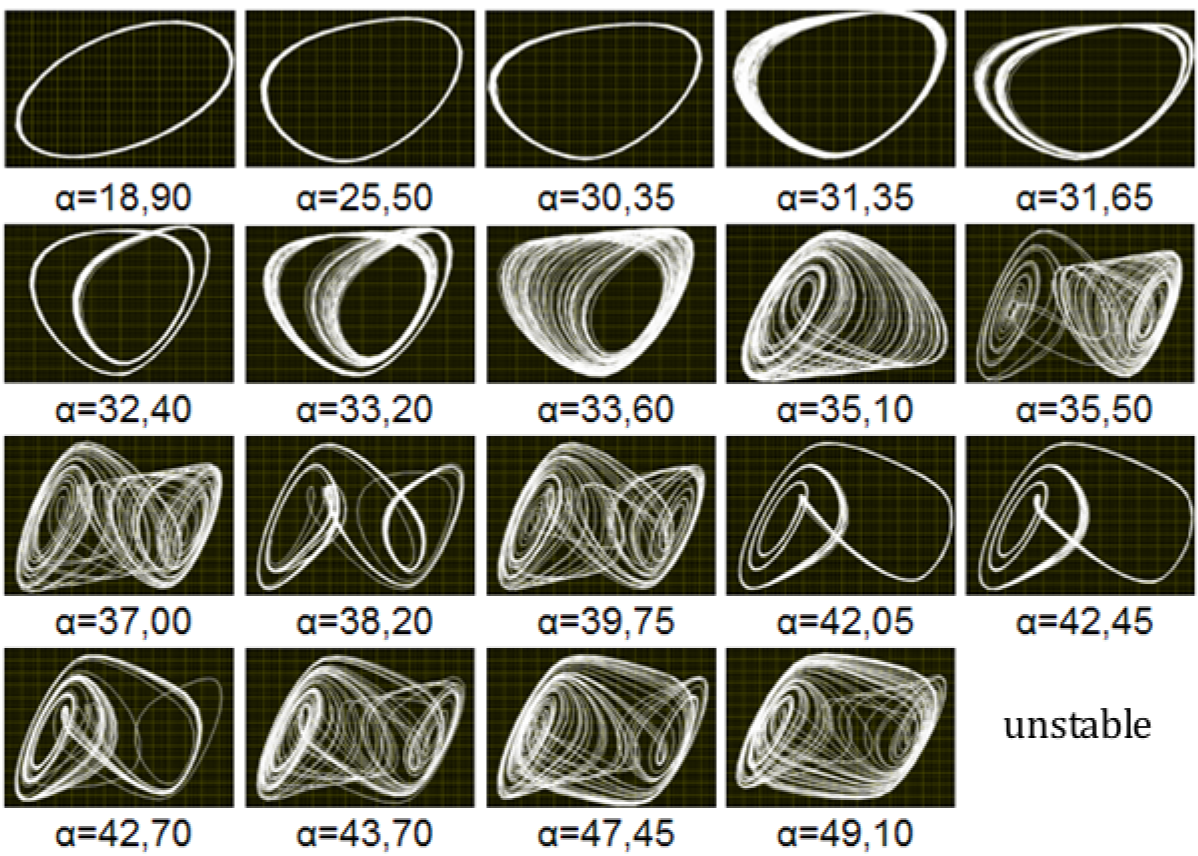

Figure 7. Route to chaos for $\beta = 105$ and $18.90 < \alpha < 49.10$.

4.3 Frequency spectra

The spectral methods are the most used tools for signal analysis. Since any waveform can be represented by the summation of an unique combination of sine waves, the magnitude and phase evaluation of each frequency component of the measured signal provides important informations about the nature of a system. Since only nonlinear systems introduce new frequencies in the frequency spectrum, it can be easily identified from a spectral analysis, which also allows to distinguish periodic, non-periodic, chaotic and stochastic behaviors: while the noises present a random frequency spectrum, the chaotic signals are wide-band signals, and both can be easily distinguished from periodic signals by inspection of their frequency spectra. The magnitude and phase of each frequency component are obtained from the Fourier transform, which can be approximately computed using an algorithm so-called Fast Fourier transform (FFT). The spectral analysis of a real signal can be directly realized using spectrum analyzers and some digital storage oscilloscopes, since the FFT algorithm is available in these instruments. In the LabVIEW environment, the block diagram to obtain the graph of the frequency spectrum is shown in the Fig. 8, where the block ”Spectral Measurements” must be adequately configured. The spectral graphs of two parameter configurations of the analogous Chua’s circuit are presented in the fig. 9. For $a = 0.729V(\alpha = 25.50)$ and $b = 3V(\beta = 105)$, the periodic behavior of the circuit is clearly characterized by well defined frequencies in the spectrum of the measured signal. However, the frequencies are not well-defined and occupy a wide band of the spectrum for $a = 1.057V(\alpha = 37)$ and $b = 3V(\beta = 105)$, which characterizes a chaotic behavior.

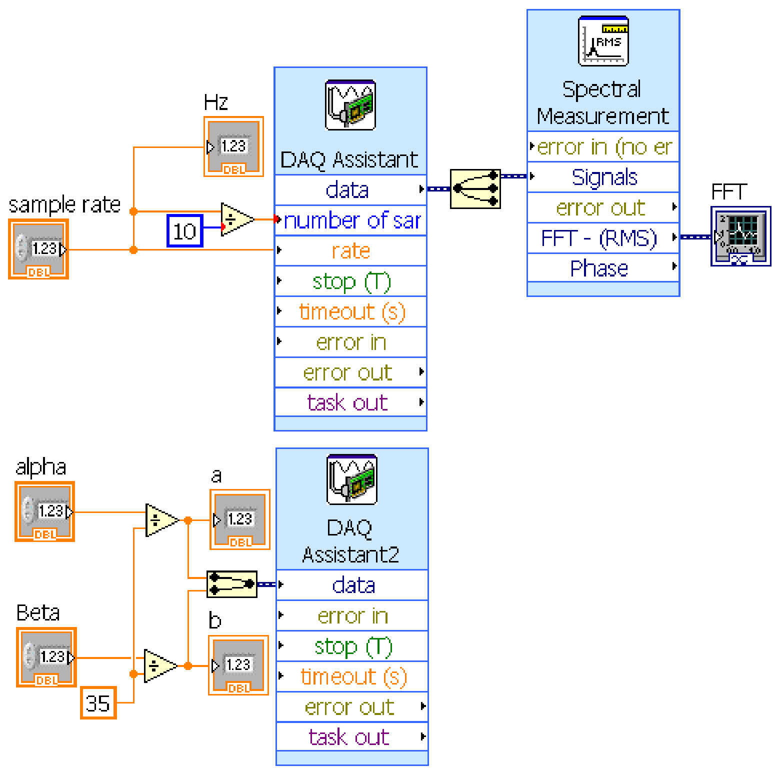

Figure 8. LabVIEW block diagram for spectral analysis

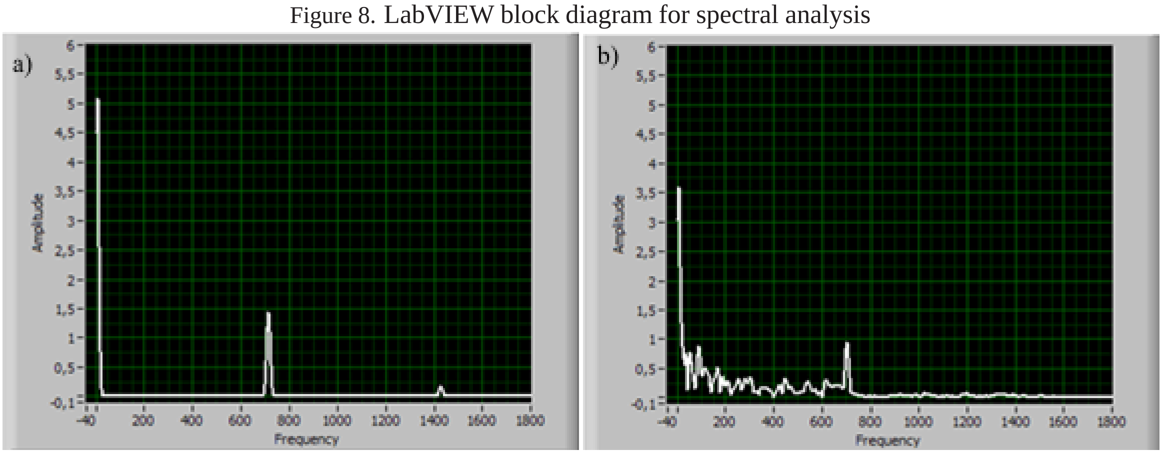

Figure 9. Frequency spectra of signal X: a) $\alpha = 25.5$ and $\beta = 105$; b) $\alpha = 37$ and $\beta = 105$.

4.4 Poincaré section

From a two dimensional projection of an attractor (effectively a phase portrait), a periodic orbit can be confidently identified for the most cases. However, this approach does not allow usually a definitive conclusion about a torus or a chaotic attractor. In this case, it is necessary the examination of the Poincaré section, a two dimensional plane that intersects the steady-state trajectories (sometimes referred as the attractor) as shown in the Fig. 10, converting a continuous flow into a discrete-time mapping. By definition, only one direction of crossing of the trajectory must be considered in the construction of a Poincaré section. Observing the stroboscopic distribution of the discrete points on the Poincaré section, it is possible to identify the behavior of the system. For a periodic motion, it is observed a finite number of points on the Poincaré section that adequately reflect the periodicity of the motion. A closed loop on the Poincaré section is observed for a quasi-periodic motion (torus). If the motion is chaotic, it is verified a large number of irregularly and densely located points on the plane. At this point, the limitations of this approach must be emphasized, since the results depend on the choice of the Poincaré section. Namely, different Poincaré sections will provide different diagrams, resulting in different kinds of information in the analysis. Although its experimental observation may appear non-trivial, techniques for visualization of the Poincaré section using an oscilloscope can be found in the bibliography (Tse, 2002). For the visualization of a Poincaré section using the LabVIEW, the block "XY Chart Buffer" must be used for storage and plotting of the values of X and Y, as shown in the fig. 11. The new points will substitute the old points in the plotting of the graph when the quantity of stored data reaches its maximum value, a parameter that must be carefully selected for a good visualization of the Poincaré section. Considering the plane $Z = 1$, the Poincaré sections for two parameter configurations of the analogous Chua's circuit are shown in the fig. 12. For $a = 0.540V(\alpha = 18.90)$ and $b = 3V(\beta = 105)$, the attractor always hits the plane $Z = 1$ in the two points (little variations can be explained by noises), which characterizes a periodic behavior with period 1. The behavior of the circuit is chaotic for $a = 1.057V(\alpha = 37.00)$ and $b = 3V(\beta = 105)$, since the Poincaré section presents several different points irregularly distributed in the plane $Z = 1$.

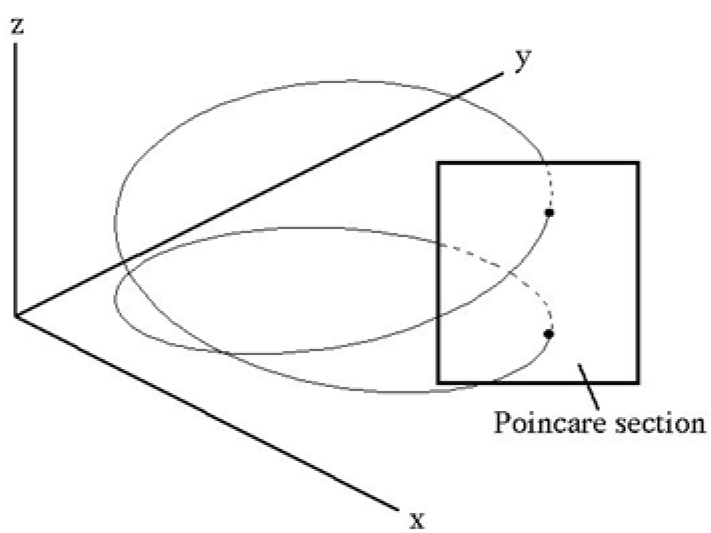

Figure 10. Poincaré section (Tse, 2002).

4.5 Bifurcation diagrams

Since the qualitative dynamical behavior of a system changes with the parameter variations, a global description of the system involves the knowledge of all possible behaviors for several parameter values. A convenient way of displaying the variety of behaviors exhibited by a system is the bifurcation diagram, a graphical representation that shows the possible long-term values (equilibrium/fixed points, periodic orbits, chaotic orbits) of a system as function of a bifurcation parameter. A bifurcation diagram consists basically in a visual summary of the values that a determined system variable visits during the asymptotic behavior for each value of the bifurcation parameter, which allows an easy identification of the way in which the system's qualitative behavior changes when some chosen parameter is varied. A typical bifurcation diagram has its horizontal axis corresponding to the bifurcation parameter and its vertical axis corresponding to the sampled steady-state values of a variable from the system. After discard the initial transients, a large number of samples of a system variable must be captured for each value of the chosen bifurcation parameter, and plotted in the bifurcation diagram. The sampling of the system variable can be realized using a Poincaré section. An extra electronic circuit is required aiming to display an experimental bifurcation diagram using an oscilloscope (Tse, 2002). An experimental bifurcation diagram can be exhibited in the LabVIEW environment using the block diagram shown in the Fig. 13, where a sufficient quantity of

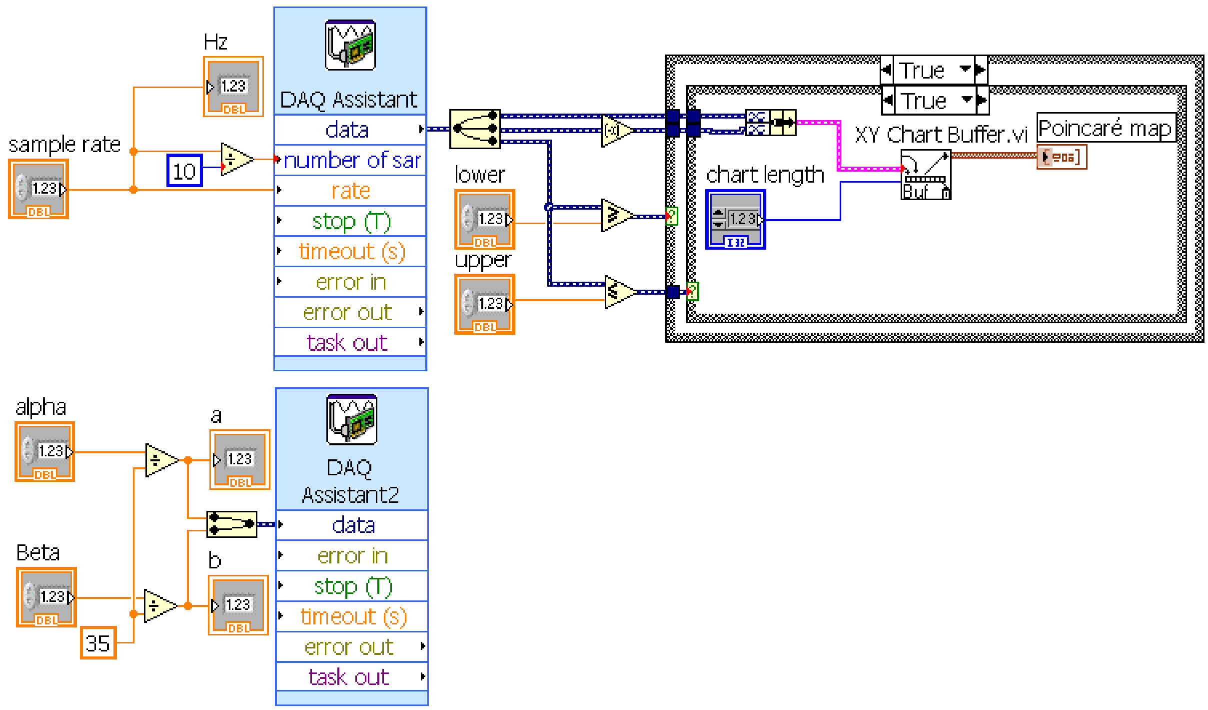

Figure 11. LabVIEW block diagram to plot a Poincaré section.

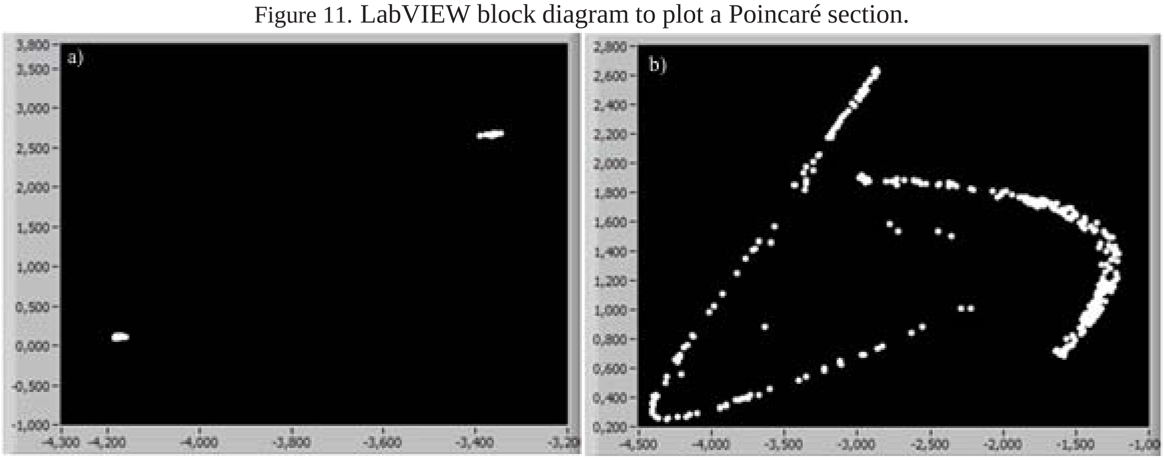

Figure 12. Poincaré sections for plane $Z = 1$: a) $\alpha = 18.90$ and $\beta = 105$; b) $\alpha = 37.00$ and $\beta = 105$.

points of the system variable is sampled in a Poincaré section $Z = 1$ for each bifurcation parameter, which is slowly swept by a stepwise sawtooth. The experimental bifurcation diagram X vs α for the analogous Chua's circuit is presented in the Fig. 14. The bifurcation voltage a (α) varies from $0V$ (0) to $1.403V$ (49.10) with an incremental step of 7.143mV (0.25), while b (β) is fixed in 3V (105). It is observed the conventional route to chaos by period doubling, where the punctual behavior of the circuit becomes periodic at $a \approx 0.514V(\alpha \approx 18)$ with period-1. In the interval $a > 0.857V(\alpha > 30)$, the periodic behavior doubles for period-2, period-4 and period-8, until finally reaches the chaos at $a \approx 0.943V(\alpha \approx 33)$.

5. CONCLUSIONS

The behavior of the analogous version of the Chua's circuit proposed by Rocha and Medrano (Rocha and Medrano-T, 2008) is experimentally characterized using usual laboratories techniques for investigation of nonlinear phenomena. These techniques, normally based in the use of oscilloscopes, are adapted to LabVIEW, which provides an adequate environment for the analysis of the time series produced by experimental implementation and captured by a data acquisition system (DAQ). Time waveforms, phase portraits, frequency spectra, Poincaré sections, and bifurcation diagrams can be simultaneous observed in a low expensive LabVIEW-based platform, allowing a real-time identification of equilibrium points, periodic and chaotic attractors, and bifurcations. Thus, the behavior of the circuit is experimentally mapped considering the variation of a parameter, where is observed the route to chaos by period double, a characteristic of the Chua's circuits. This work contributes for the development of experimental activities for study of other real systems, since the presented platform consists in an attractive and low expensive tool for data acquisition of times series and real-time

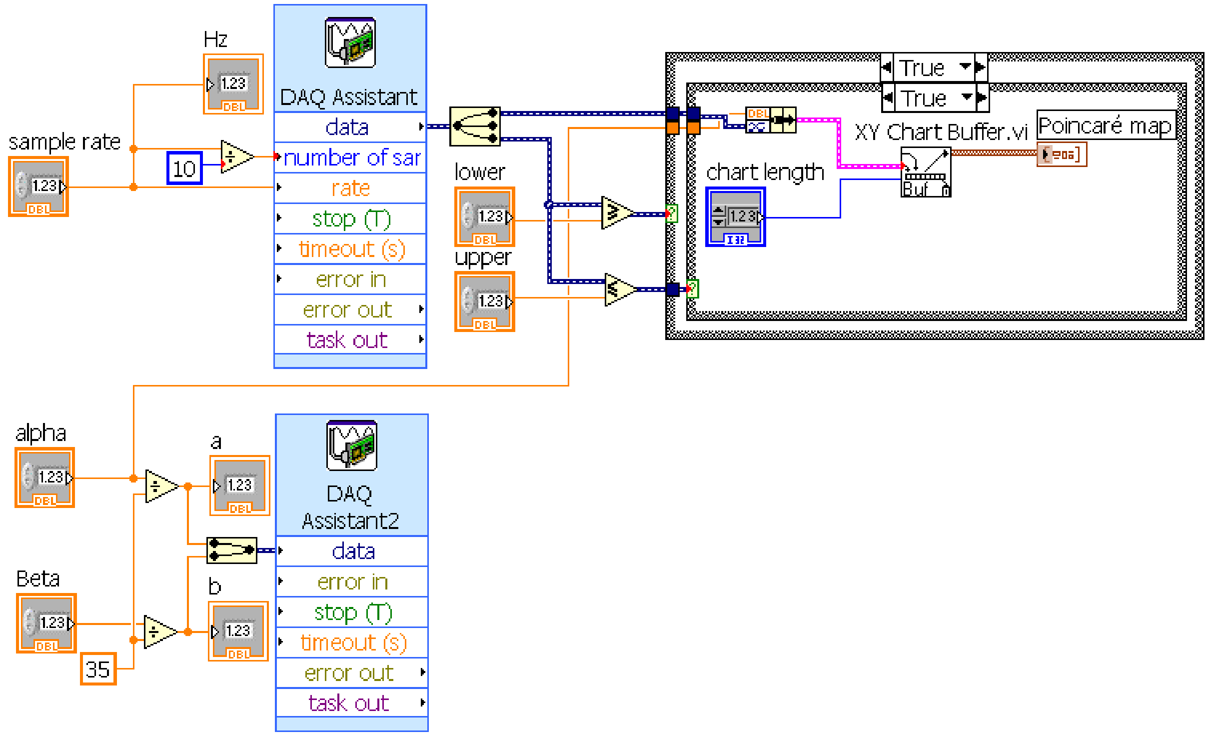

Figure 13. LabVIEW block diagram for plotting of the bifurcation diagram.

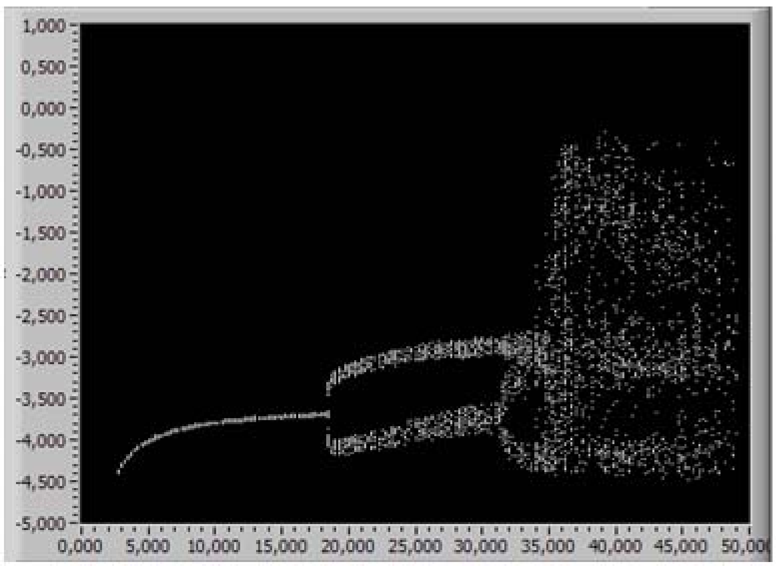

Figure 14. Bifurcation map for $\beta = 105$ and $0 < \alpha < 49.10$.

analysis of nonlinear phenomena if compared with conventional laboratory techniques using oscilloscopes.

6. ACKNOWLEDGEMENTS

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