# GPS BASED WARNING SYSTEMS FOR AIRCRAFT

Polamraju Rajendra Prasad, prasad@dem.inpe.br

### Helio Koiti Kuga, <u>hkk@dem.inpe.br</u>

Instituto Nacional de Pesquisas Espaciais(INPE), DMC, Av.dos Astronautas, 1750 CEP 12227-010, Sao Jose dos Campos-SP, Brasil

Abstract. The concept of exploring the feasibility and applicability of the attitude warning systems from pseudo attitude information for situation awareness was investigated. It could be used to supplement the existing cockpit instrumentation, which are not only error tolerant but also actively aids crew decision-making. The Global Positioning System (GPS) has been demonstrating huge potentiality of satellite based navigational system providing precise position, timing and altitude parameters. GPS attitude estimation is almost an extension of carrier phase position determination. Pseudo-attitude consists of flight path and pseudo roll angles computed from GPS velocity measurements, which are analogs of pitch and roll as measured by conventional attitude estimation. The pseudo attitude parameters, GPS scenario and measurements, velocity computation, traditional attitude determination and Pseudo attitude synthesis. The software is designed for both traditional and pseudo attitude determination and the corresponding modeling are presented. Sensor data was simulated for duration of 5 minutes corresponding to various segments. The results indicate 5° to 7° and 0.5° to 0.7° maximum deviations in roll and pitch angle estimation respectively, between the traditional and that of Pseudo attitude determination. Typical set of results and plots are presented. The accuracy is quite good enough to decision-making in real-time and can serve as an advanced warning system.

Keywords: Attitude Warning Systems, Pseudo attitude, GPS Velocity measurements, Attitude accuracy

### 1. INTRODUCTION

Situation awareness forms the critical input for decision-making. The pilot needs to perceive information about the aircraft and its sub-systems (Johnson et al., 2005) viz. airspeed, position, altitude, route, direction of flight etc. as well as weather, air traffic control clearances, emergency information, and other pertinent elements. One of the most critical parameters for decision-making is aircraft's attitude information. Attitude information for aircraft is generally obtained through gyros for general aviation applications. A vertical gyro is used for pitch and roll while separate directional gyro is used for heading. The display of the information to the pilot is presented by the gyros. Commercial and military aircrafts usually have computer based LCD displays ("glass cockpits") that are driven by an Inertial Measurement Unit (IMU). These attitude systems are precise but expensive. One of the approaches to determine attitude is using GPS measurements. Pseudo-attitude consists of flight path and pseudo-roll angles calculated from GPS velocity measurements, which are highly efficient, fast and accurate enough for display. Two major advantages to implement pseudo attitude system is utilization of commercial off the shelf GPS receiver at the high update rates and secondly near real time processing. The paper presents aspects of attitude estimation, GPS scenario and measurements, velocity computation, traditional attitude determination and Pseudo attitude synthesis. Subsequently modeling and design philosophy employed along with the typical set of results and plots are presented. The analyses indicate that the accuracy is quite good enough in real-time display warning indicators for early decision-making and possible avoidance of accidents emerging from proximity of situation awareness.

# 2. ATTITUDE PARAMETERS

Attitude is the orientation of a vehicle relative to a known set of directions or reference frame. In particular, the attitude of a vehicle is usually expressed as the orientation of the vehicle's body frame to the local level inertial reference frame. Attitude can be expressed in terms of Euler angles (roll ( $\Phi$ ), pitch ( $\Theta$ ), and yaw ( $\Psi$ )) or in terms of quaternion. Fig. 1 represents the reference frame.



Fig-1 Attitude Coordinate System - GCI

## 2.1 Euler's Angles

Euler angles describe a sequence of three rotations about different axes in order to align one coordinate system with a second coordinate system (Fig. 2).



Fig-2 Representation o Euler's Angles

# 3. GPS SYNOPSIS

A number of Navigational improvements are being introduced that improves knowledge of aircraft position and revolutionize traditional procedures, which are made possible by the introduction of satellite navigational system generally known as Global Navigational Satellite System (GNSS). GPS is designed to provide precise position, velocity and time information on a continuous global basis to users. The system currently includes 25 satellites, five of which are always visible from anywhere. GPS operation is based on the concept of ranging and navigation from group of visible satellites. The GPS receiver verifies the integrity of the signal through the process called Receiver Autonomous Integrity Monitoring (RAIM). Fig. 3 depicts a typical GPS constellation configuration.





Each GPS satellite transmits signals in 2 carrier frequencies viz. L1 =1575.42 MHz and L2 =1227.6 MHz. The carrier frequencies are modulated by various spread spectrum signals that contain information necessary to determine position and velocity (De Lorenzo et al., 2003). The basic function of the GPS positioning is the determination of user position from GPS signals. This is accomplished through the usage of propagation delay of GPS signals from an array of satellites to the GPS receivers. For satellite-based systems estimation of navigational parameters accuracy and reliability heavily depends on Geometric Dilution of Precision (GDOP). The other factors that affect the pointing accuracy are error characteristics of L1 carrier phase. The navigational information provides the differences of clocks. The observed

propagation delay  $t_{\rho d}$  from each satellite scaled by velocity of light *C*, corresponds to range measurements referred to as pseudo-ranges  $\rho$  viz.:

$$\rho = C * t_{od} \tag{1}$$

Three dimensional user velocities  $V_U$  are determined from the observed frequency shift of the received GPS carrier signal (Gautier, 2003). The observed carrier frequency differs from the nominal L1 or L2 carrier frequency due to Doppler shift caused by the relative motion of the satellite with respect to the user, as well as a receiver clock frequency bias  $f_U$ . The Doppler shift caused by the relative motion of satellite *i* and user is given by the projection of the relative velocity onto the line of sight scaled by the ratio of the transmitted carrier frequency L1 to the speed of light:

$$\Delta f_{Di} = -\left(\frac{V_i - V_U}{c} * l_i\right) \quad L_1 \tag{2}$$

where  $V_i$  is velocity of satellite "*i*" and " $l_i$ " is the line of sight from user to the i<sup>th</sup> satellite. The satellite velocity vector  $V_i$  can be computed in the receiver using ephemeris information modulated as a navigational message onto carrier signal.

#### 3.1 GPS measurements

A brief description of the GPS measurements and modeling is summarized in the following subsections.

#### 3.1.1 Carrier Phase Measurements

L1 carrier cycles have a wavelength of 19cm. If tracked and measured these carrier signals can provide ranging measurements with relative accuracies of millimeters under special circumstances. All carrier-phase tracking is differential, requiring both a reference and remote receiver tracking carrier phases at the same time (Conway, 1995).

#### 3.1.2 "Δρ" (Delta Range) Measurements

The carrier Doppler phase measurements are extracted by the receiver base band process from the carrier tracking loop using a carrier accumulator. They consist of integer cycle count  $N_{CA}$  and fractional cycle count  $\phi_{CA}$  of the carrier Doppler phase measurements. The delta range  $\delta_i$  is the change in phase in the carrier accumulator during a special time interval  $t_u$  averaged over this interval and sealed with the wavelength  $\lambda_1$  of the  $L_1$  carrier frequency.

#### 3.2 Measurement Modeling

The GPS Observational equations are

$$\rho_{Li} = R_{j}^{i} + b_{j} - B^{i} + I_{j}^{i} + T_{j}^{i} + M_{j}^{i} + V_{j}^{i} 
\varphi_{Li} = R_{j}^{i} + b_{j} - B^{i} - I_{j}^{i} + T_{j}^{i} N_{j}^{i} \lambda + m_{j}^{i} + \varepsilon_{j}^{i}$$
(3)

where

$ ho_{Li}$	Pseudo Range Measurement	$arphi_{Li}$	Carrier Phase measurement at L <sub>1</sub> Frequency
$R^{i}_{j}$	True range from satellite <i>i</i> to user <i>j</i>	$b_{j}$	Receiver offset
$B^{j}$	Satellite clock offset	Ì	Ionosphere delay
Т	Troposphere delay	М, т	Multipath delays
V, <i>E</i>	Receiver Thermal Noise.	$N^i_{\ j} \ \lambda$	Integer Ambiguity

#### 3.2.1 GPS Precise position and velocity

It is possible to use a differential form of GPS for accuracy needs. Pseudo range accuracy is significantly improved by using two antennas and two receivers, situated nearby, both receivers receiving the same signal. The Difference in the Pseudo-ranges is then significantly more accurate, as the effects of atmospheric noise; essentially ionosphere error gets minimized or mostly averaged out. Now the main limitation on differential accuracy is how accurately the two receivers can measure the phase of the C/A code. Position is calculated relative to a receiver on the ground in a differential manner (Yang et al., 1997).

## 3.2.2 GPS Position Computation

The Phase difference comes partially from the distance between the two antennas (Cohen, 1997) in satellite direction ( $S_i$ .  $\Delta X$ ) and X is position, partially from the offset  $\tau$  of local clock,  $\varphi$  carrier phase measurement, thus

$$\begin{bmatrix} S_i & 1 \end{bmatrix} \begin{vmatrix} \Delta X \\ \tau \end{vmatrix} = [\Delta \varphi_i]$$
(4)

By observing "n" satellites, "n" equations are obtained; eq. 5 describes such scenario. As the system is over determined, to solve for  $\Delta X$ , an estimation technique is employed. Estimation process is presented in the subsequent section for the sake of completeness. GPS system is basically designed around position, the velocity estimate is through calculating the difference between current position and the last computed position and divided by time interval. Attitude with carrier phase measurements provides the precise knowledge of the Attitude parameters.

$\begin{bmatrix} S_1 & 1 \\ S_2 & 1 \end{bmatrix}$			$ \begin{bmatrix} \Delta & \varphi_1 \\ \Delta & \varphi_2 \end{bmatrix} $	
	$\left\lceil \Delta X \right\rceil$			(5)
		=		
	$\lfloor \tau \rfloor$			
$\begin{bmatrix} S_n & 1 \end{bmatrix}$			$\Delta \varphi_n$	

#### 4. TRADITIONAL ATTITUDE SYNTHESIS

The aircraft attitude is in body axes and is determined from the attitude in wind axes by performing additional rotation about common y-axes by the angle of attack and the resulting transformation provides flight condition. The final transformation results in following expressions:

$$\sin \theta - \cos \alpha . \sin \theta_{W} + \sin \alpha . \cos \theta_{W} . \cos \phi_{W}$$
(6)
$$\sin \phi - (\cos \theta_{W} . \sin \phi_{W}) / (\cos \theta)$$
(7)

$$\sin(\psi_{\rm W} - \psi) - (\sin\alpha . \sin\phi)/\cos\theta_{\rm W}$$
(8)

from which Attitude in body axes  $(\psi, \theta, \varphi)$  can be calculated if  $\alpha$  is known. Many times angle of attack ( $\alpha$ ) is not available as a measurement and may have to be estimated, instead of using the aircraft lift curves, the flight altitude and aircraft weight.

#### 5. PSEUDO ATTITUDE

The attitude information synthesized from aircraft trajectory has been termed Pseudo-attitude to distinguish from traditional attitude consisting of pitch and roll angles. In contrast to traditional attitude referenced to aircraft body axes, Pseudo-attitude (Kornfield et al., 1999), unlike traditional attitude, provides direct indication of flight path (Johnson et al., 2005). Pseudo attitude parameters representation is described in Fig. 4. Earth's curvature and rotation effect is neglected in the present computation, as most of the flights are sub-sonic or low supersonic speeds wherein Mach number is < 3. The other assumption is that the aircraft is a rigid body.



Fig-4 Pseudo Attitude Parameters Representation

#### 5.1 Reference and coordinate System

The coordinate system is basically an earth fixed system. Its axes are aligned to North, East and local vertical (Down). The NED frame is an Earth fixed local level coordinate system, which has its origin instantaneously, located at the current position of the aircraft center of gravity CG and its axes aligned with the directions of North, East and the local vertical (Down). Due to the flat-earth assumption, the NED frame is, in effect, treated as an inertial reference frame in which accelerations and angular rates are measured. The velocity of the aircraft relative to NED, i.e. with respect to the ground, is denoted  $v_g$  and expressed in NED frame as  $v_g = (v_{gN}, v_{gE}, v_{gD})$ . This is in principle an inertial frame because of the basic assumptions. The velocity of Aircraft is  $V_g$  and is represented as  $(V_{gN}, V_{gE}, V_{gD})$ . The body fixed coordinates have origin at the aircraft centre of gravity and its axes are aligned along the roll ( $X_b$ ), pitch ( $Y_b$ ), and Yaw ( $Z_b$ ) axes of the aircraft. The wind axes are orthogonal axes which have its origin fixed to aircraft CG. The orientation of the body frame with respect to North, East and Down is given by three angles namely Yaw " $\psi$ ", Pitch " $\theta$ " and roll " $\phi$ " angle. These are the Euler angles of body axes. The rotation matrix  $R(\psi, \theta, \phi)$  transforms the vector from North, East, Down to body fixed axes (Conway,1995). Three consecutive rotations are given:

$$R(\psi,\theta,\phi) = R(\psi) \cdot R(\theta) \cdot R(\phi) =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{bmatrix} \bullet \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \bullet \begin{bmatrix} \cos(\psi) & \sin(\phi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(9)

Similarly the other transformations for wind axes frame are also are carried out.

#### 5.1.1 Pseudo Attitude Synthesis

Pseudo-attitude is surrogate for traditional aircraft attitude and is synthesized from the inertial velocity and acceleration vectors  $V_g$  and  $a_g$  respectively. Pseudo attitude is thus completely observable from GPS velocity (Hansman et al., 1999) and acceleration measurements. Its synthesis does not rely on additional knowledge of angle of attack or wind information. High quality GPS information is available from Doppler frequency shift measurements of the GPS carrier tracking loop. Pseudo attitude is referred to the aircraft velocity vector and consists of flight path angle in the longitudinal direction and pseudo roll angle around velocity vector axis in the lateral direction. Flight path angle  $\gamma$  is the angle between velocity vector and local level plane and is used as surrogate for pitch angle. Pseudo roll angle  $\varphi$  is defined as the effective bank angle which is the observed rate of change of velocity vector and is a substitute for regular roll angle. Pseudo attitude unlike traditional attitude provides direct indication of the flight path. Flight path angle is defined as the angle between  $V_g$  and local ground plane and is given as

$$\gamma = \tan^{-I} \left( \frac{-V_{gD}}{\sqrt{V_{gN}^2 + V_{gE}^2}} \right)$$
(10)

Here  $\gamma$  indicates climb angle. Flight path angle is not a measure of pitch but a direct indication of the path of travel of the aircraft.

#### 5.2 Pseudo Attitude System and Major Components

The pseudo attitude system consists of a GPS receiver providing three-dimensional velocity information, a computer executing the pseudo-attitude synthesis algorithm (Rath et al., 1989), and a display showing pseudo attitude information. Fig. 5 describes the major components of the Pseudo attitude system.



Fig - 5 Pseudo Attitude Display System and its components

Aircraft velocity and acceleration are necessary to synthesize the pseudo attitude. Velocity output is fed to Kalman filter which estimates the acceleration. The velocity and acceleration are input to the pseudo-attitude (Conway, 1995) synthesizing algorithm, which, together with Kalman filter is implemented on the computer. The calculated pseudo attitude is displayed.

#### 6. ESTIMATION KALMAN FILTER

The most general problem in state and parameter estimation technique consists of modeling the system, measurement process (Ryu, 2004), and measurement noise characteristics and to determine all the unknown parameters in the above by combining the information in the model output with the measurement in an optimal sense. A discrete measurement model is described by

$$X = F(X, U, \Theta, t) + w(t); X(0) = X_0$$
(11)

$$Z = G(X, U, \Theta, t_k) + v(t_k); \quad k = 1, 2, \dots, N$$
(12)

where X, Z,  $\Theta$ , U, w, and v denote respectively the states, measurements, unknown parameters, control inputs, state and measurement noise matrices of appropriate order. The values at a given time step are denoted by a subscript. The process and measurement noise have zero mean with covariance Q and R respectively. The time evolution of the estimate and covariance of state variable X in terms of the transition matrices  $\phi$  and  $\psi$  obtained by suitable local linearization can be written as (Prasad et al, 2001):

$$X_{k}^{-} = \phi_{k-1} X_{k-1} + \psi_{k-1} U_{k-1}; X(0) = X_{0}$$
(13)

$$\mathbf{P}_{k}^{-} = \phi_{k-1} \mathbf{P}_{k-1}^{+} \phi_{k-1}^{-T} + \mathbf{Q}_{k}; \quad \mathbf{P}(0) = \mathbf{P}_{0}$$
(14)

$$Z_k = H_k x_k + v_k \tag{15}$$

The superscripts (-) and (+) indicate respectively the estimate before and after using the measurement information. Thus we have at time  $t_k$  an estimate  $X_k$  with covariance  $P_k$  and another from the measurement viz.  $Z_k$  with covariance  $R_k$ . When combined statistically (Brown and Hwang 1990) it provides

$$X_{k}^{+} = X_{k}^{-} + K_{k}[Z_{k} - H_{k}X_{k}^{-}] = X_{k}^{-} + K_{k}v_{k}$$
(16)

$$P_{k}^{+} = [I - K_{k} H_{k}] P_{k}^{-} [I - K_{k} H_{k}]^{T} + K_{k} R K_{k}^{T}$$
(17)

where  $v_k$  is the innovation and the residue is  $(X_k^+ - X_k^-)$ . If the optimal Kalman Gain

$$K_{k} = P_{k}^{-} H_{k}^{T} [H_{k} P_{k}^{-} H_{k}^{T} + R_{k}]^{-1}$$
(18)

is used, then

$$P_{k}^{+} = [I - K_{k} H_{k}] P_{k}^{-}$$
(19)

When a linear system is driven by white noise the Gaussian distributed state  $X^-$  evolves with modified mean and covariance. The linear measurement equation also provides a Gaussian distributed Z. Then the conditional probability distribution of X given Z namely p(X|Z) is also a Gaussian. Thus linear systems, measurements, and white noise match

to keep X and Z as Gaussian. But if either the system or the measurement is nonlinear the Gaussian structure is destroyed and one resorts to quasi-linearization over the full data or re-linearise after update at every time point to deal with the problem. These are called respectively linearised and Extended Kalman filter formulation (Yang et al., 1997). The former has a reference state close to and valid over the full time span and the latter has to change the reference state trajectory at each time point. The innovation follows a Gaussian distribution whose probability when maximized leads to the cost function, which is basically a batch processing nonlinear optimization problem and the numerical effort cannot be avoided. Fig. 6 describes the schematic flow of the computational process for adaptive tuning process.



Fig-6 Schematic flow of Adaptive Estimation tuning process

# 7. COMPUTATION FLOW

Following diagram (Fig. 7) provides the computational flow. Initially the GPS receiver data is obtained along with the navigational parameters. Pseudo range computations are executed and measurement modeling is performed. The next activity is to refine the attitude parameters and. display the navigational and pseudo attitude parameters



Fig-7 Software Process-Schematic Flow

# 7.1 Constant Kalman Gain Approach

Constant Kalman gain approach methodology is as follows. Generally after the initial transients the Kalman gain matrix tends to constant values as has been noticed in some of the similar attitude estimation problems. It is noticed in this type of applications as well. These constant gains can be obtained by making the above-normalized innovation cost functions equal to number of measurements. The advantage of the constant gain Kalman approach based on the utilizing the cost function, is that it does not need to propagate the covariance equation at all (Prasad et al, 2001). These can enormously reduce the computational load. In principle this approach can enhance performance.

# 8. COCKPIT DISPLAY

Synopsis of Cockpit display of Traffic information (Whitston, 2000) can be grouped into

- Aircraft "broadcasts" known GPS position along with other navigational and flight parameters
- Radar like display for traffic information in aircraft cockpit
- Other relevant information for situation awareness essentially terrain, weather etc.

A typical display is represented in the figure 8.



Fig-8 Typical representation of Display

#### 8.1 Situation Awareness

One of the definitions for situation awareness is the perception of the elements in the environment within a volume of time and space, the comprehension of their meaning, and the projection of their status in the near future. While a description of these elements has been developed for several classes of military aircraft and air traffic control systems (Theunissen et al., 1997) this has not previously been done for commercial aircraft in a rigorous manner. A clear elucidation of the elements in this definition as they apply to commercial airline pilots is a crucial step towards understanding situation awareness in this unique environment. The objective of this effort was to determine critical parameters in near real time which in turn aids the pilots for early warnings.

#### 8.2 Application of GPS attitude information

The capability to use GPS information for Attitude determination has opened new realm of applications and opportunities. The key element that makes it to happen is very precise carrier phase observables, with the addition of GPS based attitude determination (Brown et al., 1990) to complement of sensors available to pilot for situation awareness. A single GPS sensor has potential to perform much more functions (Whitson et al., 2000) than current instruments in use. A typical example of the scenario is heading and Attitude sensing parameters. The precision landing problem implicitly requires attitude sensing for applying correction to lever arm between antennas and landing gear.

#### 9. PSEUDO ATTITUDE DISPLAY

Conventional attitude determination and display (Kerr et al., 2003) was described in preceding section. Pseudo attitude is displayed in a manner quite similar to traditional attitude. The distinguishing feature of the pseudo attitude display is that its aircraft symbol is referred to as inertial velocity vector. Following is the snap shot of the display and gives a conceptual view (Fig. 9). The right display shows the pseudo attitude of the aircraft in the same flight condition. The roll representation indicates the pseudo roll angle, which in the case shown is very close to the traditional roll angle. The pitch information is replaced with the flight path angle and the horizon is thus coupled with flight path angle. The display is generated through "OpenGL" and the snap shot of the result describes the comparison of the conventional attitude and that of the computed pseudo attitude display. The analysis indicates the relative comparison is fairly close and the pseudo attitude accuracy provided in the display is quite good enough for decision making.



Fig-9 Snap shot of the Display: Comparison of Conventional and Pseudo Roll Attitude Estimation

## **10. RESULTS AND DISCUSSION**

The software is designed based on the mathematical modeling described. The environment is based on OOD (Object Oriented Directives). The simulation was carried out using 12 channel GPS receiver data. Both traditional and pseudo attitude determination models are incorporated for the simulation study. Sensor data was simulated for a duration of 5min. corresponding to various segments. The results indicate about 5° maximum deviations in roll estimation. The deviation in pitch angle estimation was about 0.5° between traditional and pseudo attitude determination. The cockpit navigational parameters along with Pseudo attitude update parameters are displayed. The accuracy is good enough to decision-making, as the pseudo attitude estimation updates in near real time. The results are encouraging and extension of the work is in progress to put all on a single board computer and also an integrated critical parameter display unit in near real time. Typical case study of pitch and roll angles along with the comparisons of both methods are presented. Fig. 10 describes roll angle estimation and Fig. 11 gives pitch angle estimation analysis.



Fig-10A Traditional Attitude determination "Roll"



Fig-10 C Comparison of Traditional and Pseudo "Roll" Estimation



Fig-10 B Pseudo Attitude Determination "Roll"



Fig-11A Pseudo "Pitch" angle estimation







Fig-11C Attitude estimation "Pitch" angle

# **11. CONCLUSIONS**

- Navigational and flight parameters are important components in the decision making process. Attitude of Aircraft is one of the critical parameters in this process.
- Feasibility and applicability of determining the attitude through aircraft velocity information obtained through GPS is investigated.
- Flight path angle is used as a surrogate of pitch angle. Pseudo-roll angle represents the roll angle around the velocity vector axis and is a substitute for traditional roll angle. Pseudo-roll corresponds to the observed lateral rate of change of the velocity vector and is determined from the acceleration vector. The combined use of these attitude or flight control variables is novel and is referred to as pseudo-attitude.

A system based on the concept of Pseudo attitude is designed and developed. Simulation studies are carried out using the GPS data. The results indicate that the attitude computation using GPS velocity measurements are comparable to that of conventional Attitude computation. The results indicate 5° to 7° and 0.5° to 0.7° maximum deviations in roll and pitch angle estimation respectively, between the traditional and that of Pseudo attitude determination.

# **12. ACKNOWLEDGEMENTS**

The authors acknowledge CNPq for its support, keen interest and encouragement provided for the study. The study, part of Grant by CNPq process number 310432/2007-3, is acknowledged gratefully.

# **13. REFERENCES**

1. Walter W. Johnson et al., 2005 "A Cockpit Display Designed to Enable Limited Flight Deck Separation Responsibility", SAE Transactions - Journal of Aerospace, 108 (1).

- 2. Clark Emerson Cohen, 1997 "Attitude determination Using GPS"; PhD. Dissertation.
- 3. J. Rath et al., 1989 "Attitude estimation Using GPS"; Nat. Tech. Mag.
- 4. Andrew Richards Conway, 1995 "Autonomous control of Helicopter using Carrier Phase measurements", PhD.
- 5. R. Brown et al., 1990 "A GPS receiver with built in precision pointing"; IEEE –Plans.
- 6. Kornfield et al., 1999 "Single Antenna GPS based attitude determination"; Journal of Navigation, Vol-45.
- 7. E. Theunissen et al, 1997 "Synthetic vision for general aviation", Delft University report.
- 8. David S. De Lorenzo et al., 2003 "GPS Attitude determination "; Stanford university.
- 9. Jihan Ryu, 2004 "State and parameter estimation for Vehicle dynamics"; Stanford University.
- 10. Jennifer Denise Gautier, 2003 "GPS/INS generalized Evaluation Tool (GIGET)," Stanford university.
- 11. J.Richard Kerr et al., 2003 "Advanced Integrated Vision Systems"; OGI School of science and engineering.
- 12. Hansman.R.J et al., 1999 "The impact of GPS velocity based flight control"; Report no. ICAT-99-05.

13. Lee.C.Yang et al, 1997 "Prototype conflict alerting system for free flight"; Journal of Guidance and control and Dynamics ;Vol. 20, No. 4.

14. Rajendra Prasad et al., 2001 "State and Parameter estimation Perspective applied to CBERS orbit determination problem and analysis; 6-th international symposium on space flight dynamics (ISSFD) in Pasadena, California. 15. Whitston.S., 2000 "Future Flight"; Flight Deck international.