A QUALITATIVE DESCRIPTION OF THE GLOBAL WARMING DYNAMICS

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Abstract. *Global warming is a specific case of the more general term climate change that is induced either by natural processes or by human activities. Global warming is the observed increase in the average temperature of the Earths' atmosphere and oceans. The primary cause of this phenomenon is the greenhouse gases released by burning of fossil fuels, land cleaning, agriculture, among others, leading to the increase of the so-called greenhouse effect. The consequences of this warming are unpredictable, however, one could mention climate sensitivity and other changes related to the frequency and intensity of extreme weather events. This article deals with the modeling of the global warming in a dynamical point of view. A mathematical model based on the daisyworld is proposed representing an archetypal of the Earth behavior. Essentially, this model is able to describe the global regulation that can emerge from the interaction between life and environment. This idea became famous as the Gaia theory of the Earth that establishes self-regulation of the planetary system. In brief, daisyworld represents life by daisy populations while the environment is represented by temperature. Here, two daisy populations are of concern, black and white daisies, and an extra variable related to greenhouse gases is incorporated in the model allowing the analysis of the global warming. Although there are many difficulties related to the system description, their modeling may define at least a system caricature, which may be useful for different goals. Numerical simulations are performed investigating the influence of the greehouse gases in Earth's temperature.*

Keywords: *Ecology, global warming, daisyworld, nonlinear dynamics.*

1. INTRODUCTION

The mechanism of the Earth's heating is related to the energy balance where the main aspects are the radiation energy from the sun and the thermal radiation from the Earth that is radiated out to space. The atmosphere plays an essential role in this process and the presence of greenhouse gases tends to break this balance since they are transparent to the sun short wave radiation, however, they absorb some of the longer infrared radiation emitted from the Earth. Therefore, the increase amounts of these gases makes the Earth cool more difficult increasing the Earth's surface temperature.

Global warming is the observed increase in the average temperature of the Earth's atmosphere and oceans. The primary cause of this phenomenon is the release of greenhouse gases by burning of fossil fuels and large-scale deforestation, leading to the increase of the so-called greenhouse effect that arises as a consequence of the presence of greenhouse gases in the atmosphere. Among others, the greenhouse gases are the carbon dioxide, the methane and the nitrous oxide (Houghton, 2005).

Based on Intergovernmental Panel on Climate Change (IPCC, 2007) data, the amount of greenhouse gases in the atmosphere has significantly increased since the industrial revolution. Moreover, during the 20^{th} century, the Earth's surface mean temperature has increased approximately 0.4 to 0.8°C. The consequences of global warming are unpredictable, however, one could mention climate sensitivity and other changes related to the frequency and intensity of extreme weather events.

The mathematical modeling of biological and ecological phenomena has an increasing importance in recent years (Savi, 2005, 2006). These models may describe time evolution and spatial distribution and may explain some important characteristics of these systems. The mathematical analysis is exploiting the possibility that many of these phenomena may have their roots in some underlying dynamical effect: the so-called dynamical diseases. Although there are many difficulties related to the system description, their modeling may define at least a system caricature, which may be useful for different goals.

This contribution deals with the modeling of the global warming in a dynamical point of view. A mathematical model based on the daisyworld (Lenton & Lovelock, 2000, 2001; Lovelock, 1992) is proposed. The daisyworld is an archetypal of Earth and is able to describe the global regulation that can emerge from the interaction between life and environment. This idea became famous as the Gaia theory of the Earth that establishes self-regulation of the planetary system. In brief, daisyworld represents life by daisy populations while the environment is represented by temperature. Here, two daisy populations are of concern, black and white daisies. Besides, an extra variable related to greenhouse gases is incorporated in the model allowing the analysis of the global warming. Numerical simulations are carried out in order to present a qualitative description of the global warming phenomenon.

2. DAISYWORLD MODEL

Climate system has an inherent complexity due to different kinds of phenomena involved. The equilibrium of this system is a consequence of different aspects related to the atmosphere, oceans, biosphere and many others, and the sun activity provides the driving force for this system. The Earth's heating mechanism may be understood as the balance between the radiation energy from the sun and the thermal radiation from the Earth and the atmosphere that is radiated out to space. The presence of greenhouse gases tends to break this balance since they are transparent to the sun short wave radiation, however, they absorb some of the longer infrared radiation emitted from the Earth. Therefore, the increase amounts of these gases makes the Earth cool more difficult increasing the Earth's surface temperature.

Lovelock (1983a, b) proposes a model to demonstrate that global regulation can emerge from the interaction between life and environment. This behavior was represented by an archetypal model called daisyworld that represents an imaginary planet populated by organisms in stable coexistence. The daisyworld is basically composed by the environment, represented by the temperature, and by populations of daisies representing life. In brief, it is assumed that daisyworld is like the Earth but with less oceans and with the whole surface being fertile. The original daisyworld includes only two populations of daisies but further investigations include herbivores and carnivores as well as daisies.

The first step of the daisyworld modeling is the definition of life, represented by daisies, which evolution is described by the following general equation where $\alpha_i (i=1,2,...N)$ represents the area coverage by daisy populations:

$$\dot{\alpha}_i = \alpha_i \left[\alpha_g \beta(T_i) - \gamma\right] \tag{1}$$

where dot represents time derivative, β is variable growth rate that is temperature dependent, γ is the death rate and α_g is the fractional area coverage of the planet:

$$\alpha_g = p - \sum_{i=1}^{N} \alpha_i \tag{2}$$

Here, p represents the proportion of land suitable for the growth of daisies and N represents the biodiversity related to the number of populations involved in the system.

The mean planetary albedo of the daisyworld, A, can be estimated from the individual albedo of each population (a_i and a_g for the bare ground) by:

$$A = \alpha_g a_g + \sum_{i=1}^{N} \alpha_i a_i \tag{3}$$

Afterwards, the local temperature of each population is defined as follows:

$$T_i^4 = q\left(A - a_i\right) + T^4 \tag{4}$$

$$T_g^4 = q\left(A - a_g\right) + T^4 \tag{5}$$

where T is the globally-averaged temperature of daisyworld, and q is a constant used to calculate local temperature as a function of albedo. Finally, it is important to establish that the daisyworld stays in thermal balance (Foong, 2006), and therefore, the absorbed energy is equal to the energy reradiated given by the Stefan-Boltzmann law:

$$SL\left(1 - A\right) = \sigma T^4 \tag{6}$$

where L is the solar luminosity, and S is the solar constant that establish the the average solar energy, SL; σ is the Stefan-Boltzmann constant.

The functional form for β_i is usually assumed to be a symmetric single-peaked function as follows:

$$\beta_i(T) = \begin{cases} C\left[1-\left(\dfrac{T_{opt}-T_i}{k}\right)^2\right] & \left|T_{opt}-T_i\right| < k \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where T_{opt} is the optimal temperature usually assumed to be $T_{opt} = 295.5\text{K} = 22.5°\text{C}$. The parabolic width k is chosen in order to establish proper life conditions as for example, between 5°C and 40°C (De Gregorio *et al.*, 1992), which is related to $k = 17.5$. In the same way, C alters these values in order to represent different environmental characteristics.

The daisyworld model can be simulated using classical procedures for numerical integration. Here, the fourth order Runge-Kutta method is employed with this aim. In general, the following parameters are assumed: $q = 2.06\times10^9\,\text{K}^4$, $\sigma = 5.67\times10^{-8}\,\text{Wm}^{-2}\text{K}^{-4}$, $S = 917\,\text{Wm}^{-2}$. Other parameters are varied in order to analyze different situations.

2.1. Daisyworld Model with Greenhouse Gases

In order to incorporate the greenhouse gases in the daisyworld, a new population is included into the model. The idea is to represent the albedo increase due to the effect of the greenhouse gases, *G*. Since daisy colors define the amount of energy absorption, the balance between daisy populations can control the planet temperature. A first approach to this archetypal model is to consider only two daisy populations: black daisies that absorb more energy and white daisies that absorb less energy. In this regard, the inclusion of the greenhouse variable *G* has an effect similar to black daisies. Therefore, the model is written as in the classical way, but now there is a function that establishes the greenhouse gases time history:

$$\dot{\alpha}_w = \alpha_w\left[\alpha_g\beta(T_w) - \gamma\right] \tag{8}$$
$$\dot{\alpha}_b = \alpha_b\left[\alpha_g\beta(T_b) - \gamma\right] \tag{9}$$
$$\alpha_g = p - \alpha_b - \alpha_w - G \tag{10}$$
$$G = G(t) \tag{11}$$

The albedo is now influenced by the greenhouse gases as follows:

$$A = \alpha_w a_w + \alpha_b a_b + \alpha_g a_g + G a_G \tag{12}$$

And, the local temperature may be defined as follows:

$$T_w^4 = q(A - a_w) + T^4 \tag{13}$$
$$T_b^4 = q(A - a_b) + T^4 \tag{14}$$
$$T_G^4 = q(A - a_G) + T^4 \tag{15}$$
$$T_g^4 = q(A - a_g) + T^4 \tag{16}$$

where T is the globally-averaged temperature of daisyworld, and q is a constant used to calculate local temperature as a function of albedo. Once again, the daisyworld stays in thermal balance and the absorbed energy is equal to the energy reradiated given by the Stefan-Boltzmann law:

$$SL(1 - A) = \sigma T^4 \tag{17}$$

where the SL average solar energy.

The functional form for β is assumed to be a symmetric single-peaked function as follows:

$$\beta_i(T) = \begin{cases} C\left[1-\left(\dfrac{T_{opt}-T_i}{k}\right)^2\right] & \left|T_{opt}-T_i\right| < k \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

where T_{opt} = 295.5K = 22.5°C and the parabolic width k is chosen in order to establish proper life conditions as for example, between 5°C and 40°C, which is related to $k = 17.5$. In the same way, C alters these values in order to represent different environmental characteristics.

3. CLASSICAL DAISYWORLD: NUMERICAL SIMULATIONS

This section investigates the dynamics of the daisyworld in order to establish a proper comprehension of the self-regulation of the Earth. Greenhouse gases are not considered in this section that treats the classical daisyworld. Initially, constant luminosity is of concern. Parameters used in this simulation are $a_w = 0.75$, $a_b = 0.25$, $a_g = 0.5$, $\gamma = 0.3$ and the initial conditions $\alpha_w = \alpha_b = 0.01$. All simulation employs a time step 0.01. Figure 1 shows the behavior of daisy populations and temperature when for this situation. Note that there is a proper balance between both populations and the temperature tends to be constant.

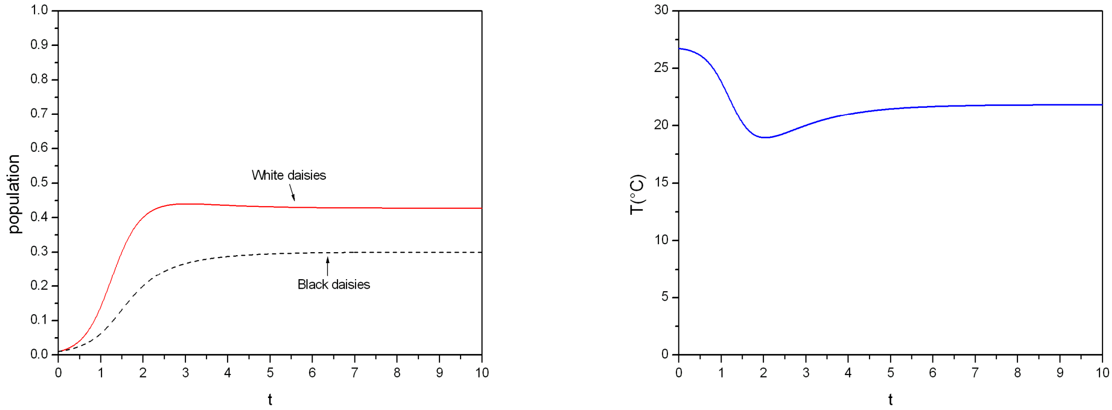

Figure 1. Daisyworld response with constant solar luminosity (L = 1).

A linear increase of the solar luminosity is now in focus, representing the real evolution of the sun. Under this condition, the planet temperature would tend to increase linearly, following the luminosity increase. Nevertheless, daisyworld has self-regulation due to the interaction between life and environment. Therefore, the planetary system tends to maintain a constant temperature due to the interaction between black and white daisy populations. Note that the increase of the black daisies tends to increase the planet temperature since they absorb more energy. This occurs when the solar luminosity has small values. The increase in solar luminosity causes the decrease of the black daisies population and the increase of the white daises. This balance is in such a way that planet temperature remains constant. Figure 2 shows the evolution of the daisy populations and the temperature. It is clear that, when the luminosity is small, black daisies are preponderant. The more luminosity increases, the more white daisies increase. This effect occurs in order to establish a constant temperature. The dead planet, when all daisy populations have died, presents a temperature increase proportional to luminosity increase since life does not have influence in the daisyworld, as presented in Figure 3.

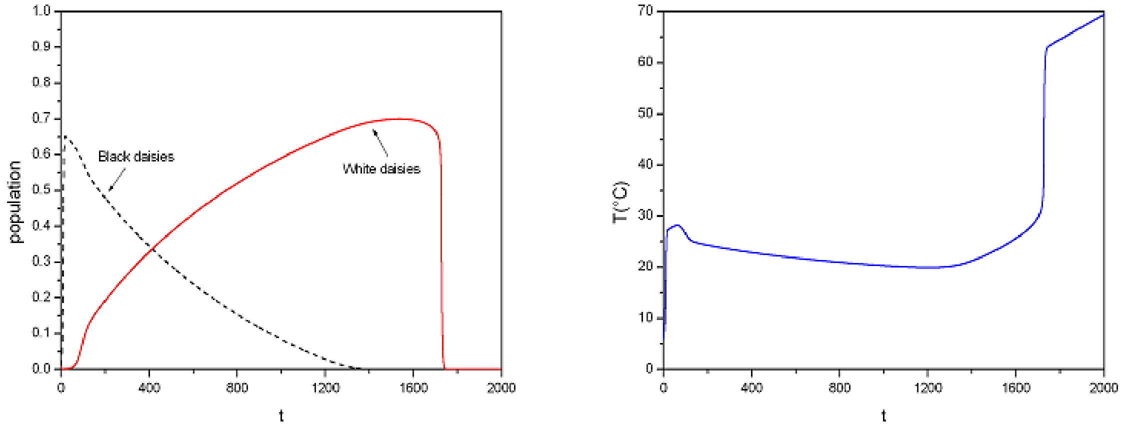

Figure 2. Daisyworld response with linear increase solar luminosity ($0.7 < L < 1.7$).

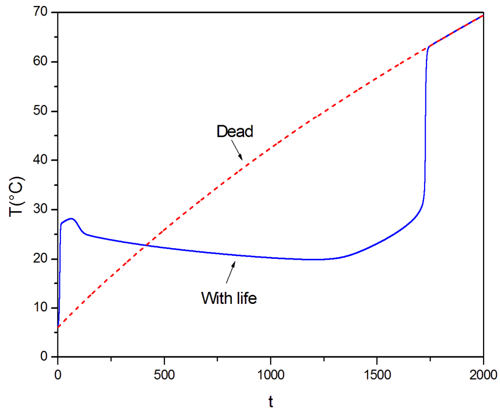

Figure 3. Comparison between dead planet and daisyworld temperatures.

The daisyworld balance is related to all planet conditions. Changes either in environmental characteristics or in life aspects can affect the equilibrium. In daisyworld, all these aspects are related to system parameters. Therefore, our analysis shows some of the dynamical responses related to daisyworld. Lenton & Lovelock (2001) discussed some dynamical aspects as hysteretic behavior and dissipation characteristics in daisyworld. The hysteretic characteristic of the model can be evaluated from a linear loading-unloading process of the solar luminosity. Figure 4 presents results for a linear increase from 0.75 to 1.55 (solid lines), then a linear decrease returning to 0.75 (dashed lines). Luminosity step of 0.001 is assumed. It should be noticed that the system response is characterized by different paths for loading or unloading.

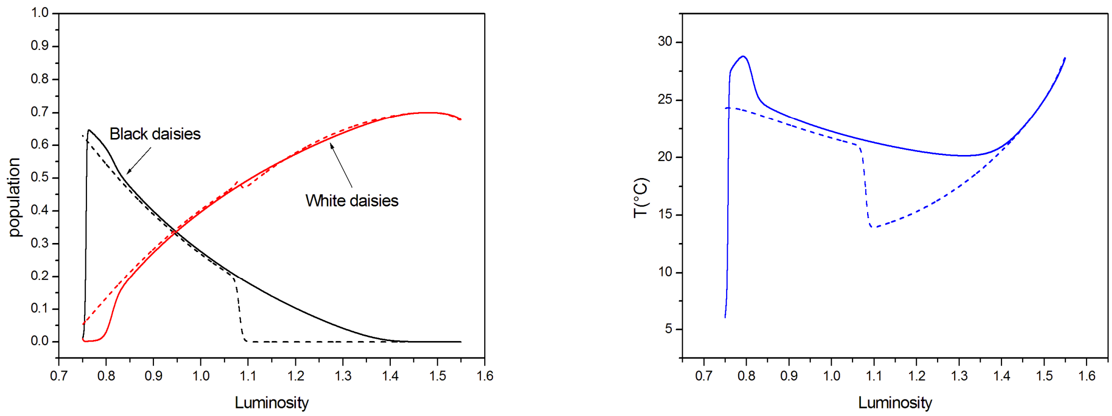

Figure 4. Hysteretic behavior of the daisyworld due to loading (solid line) and unloading (dashed lines).
Daisy populations (left) and planetary temperature (right).

4. DAISYWORLD WITH GREENHOUSE GASES: NUMERICAL SIMULATIONS

This section considers greenhouse gases in the daisyworld. Basically, it is assumed that these gases are known being related to a function or a time series. Experimental values are used as a reference to characterize the general tendency of these gases. In this regard, CO_2 emissions from 1958 to 2009 are used (NOAA, 2009) and Figure 5 presents the average of the annual emission. This Figure is used to define *G* values that consider the general tendency. It should be observed that there is a linear increase in CO_2 emissions, and this information is used in numerical simulations that assume time steps of 0.01. The basic idea of this section is to establish a comparison with results of the preceding section that do not consider greenhouse gases. Initially, a constant luminosity is treated. Figure 6 presents results of this simulation showing that greenhouse gases tend to increase the planet temperature. This increase causes the death of daisy populations earlier when compared to the planet without gases.

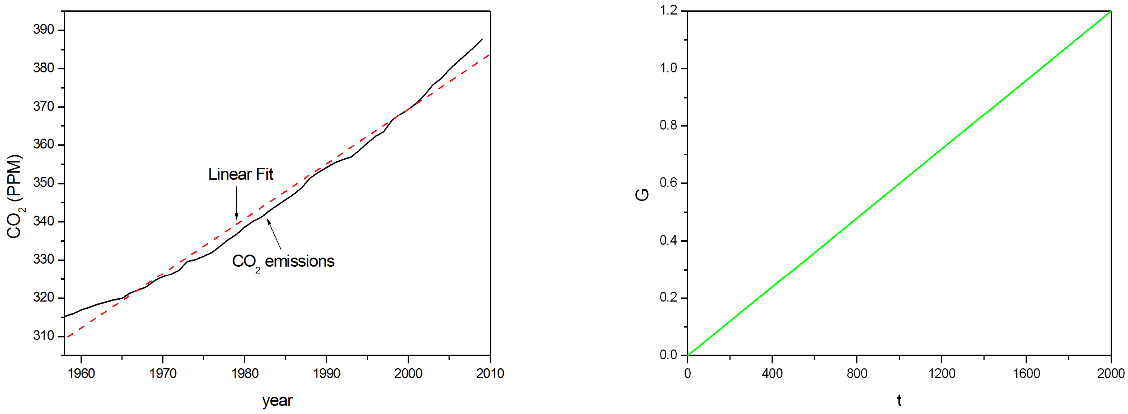

Figure 5. Annual average of the CO_2 emissions.

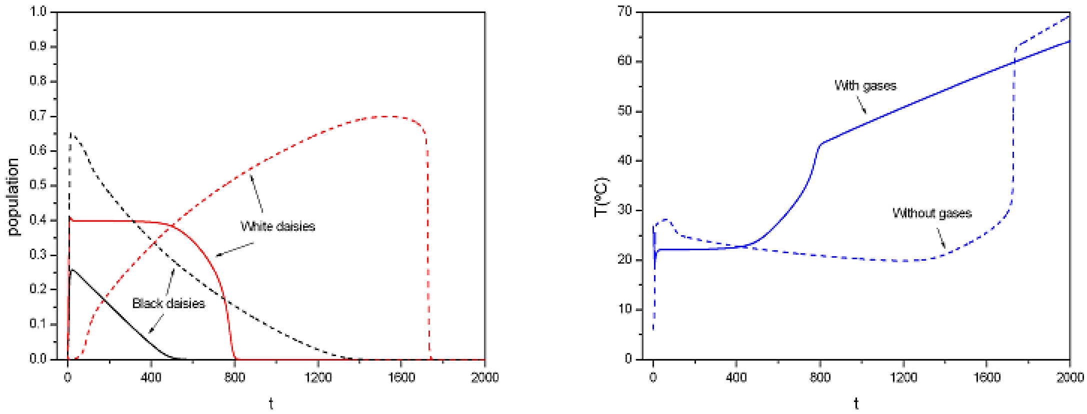

Figure 6. Daisyworld with greenhouse gases (solid lines) and without gases (dashed lines) and constant solar luminosity (L = 1). Daisy populations (left) and the temperature (right).

At this point, a linear increase of the luminosity is in focus. Once again, the populations tend to die earlier, changing the planet balance. Figure 7 shows this behavior that is manifested with the increase of the planet's temperature. Figure 8 establishes a comparison between both models considering the dead planet situation in order to highlight the importance of life in the planet balance.

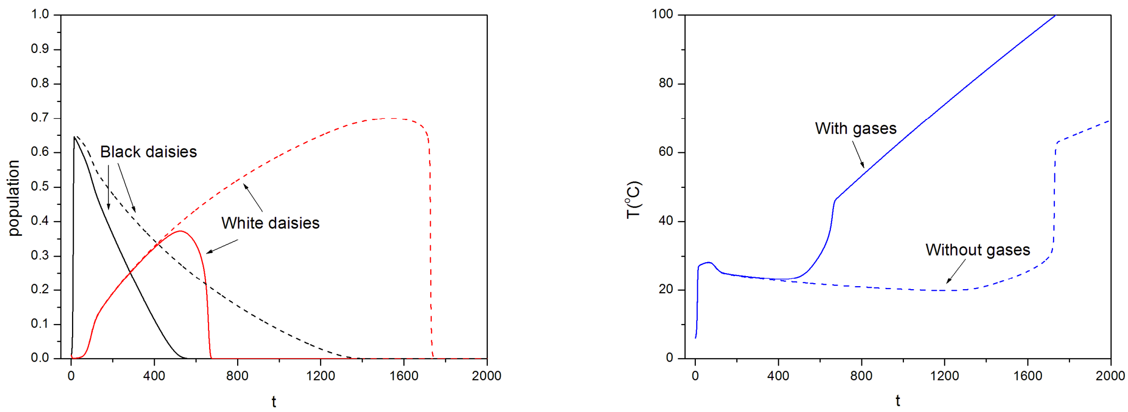

Figure 7. Daisyworld with greenhouse gases (solid lines) and without gases (dashed lines) and linear increase solar luminosity. Daisy populations (left) and the temperature (right).

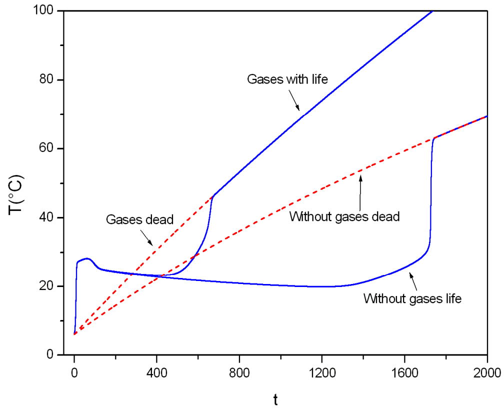

Figure 8. Comparison between dead planet and daisyworld temperatures with and without greenhouse gases.

Hysteretic behavior of the daisyworld can be verified by considering a proper loading-unloading process. Figure 9 shows this simulation where results are compared with those without greenhouse gases.

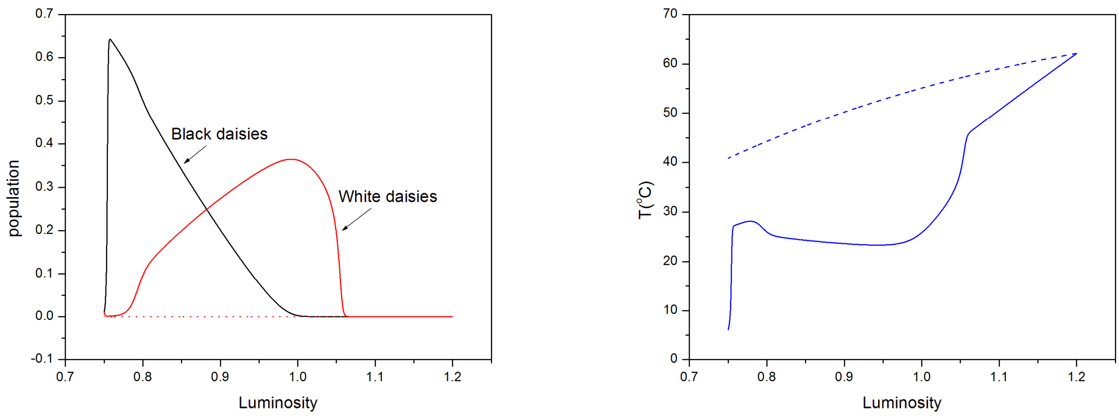

Figure 9. Hysteretic behavior of the daisyworld. Daisy populations (left) and planetary temperature (right).

Global warming is the observed increase in the average temperature of the Earth and the primary cause of this phenomenon is the greenhouse effect related to greenhouse gases. An important question related to the global warming concerns with its reversibility. In other words, it is important to know if the decrease of the greenhouse gases emissions is enough to make the planet to reach temperatures of the past. In order to observe this kind of behavior, we make a simulation establishing a loading-unloading of the greenhouse gases depicted in Figure 10. This simulation considers a constant luminosity ($L = 0.75$) and shows that planet temperature could be stabilized again, although in different values. Figure 11 considers the linear increase of luminosity. Under this condition, a different kind of response occurs that is representative of a more realistic situation.

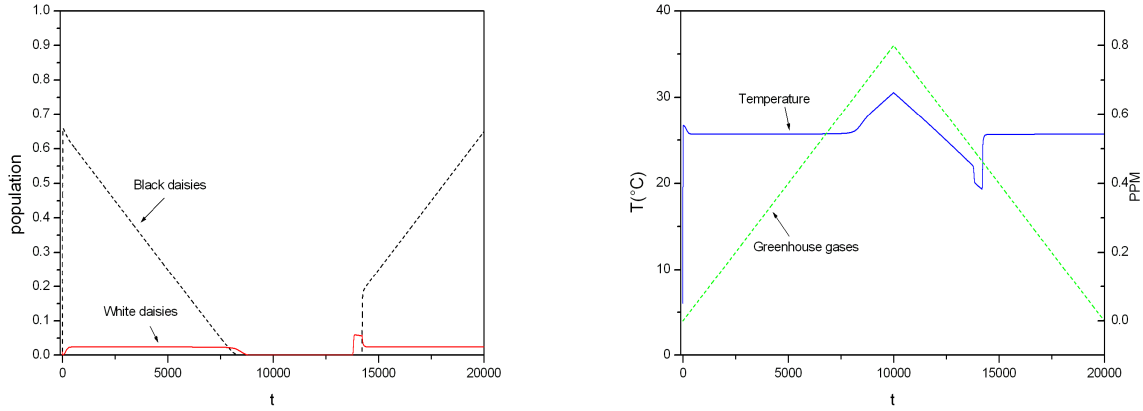

Figure 10. Evolution of daisy populations (left) and temperature (right) with greenhouse gases emissions, considering a constant luminosity.

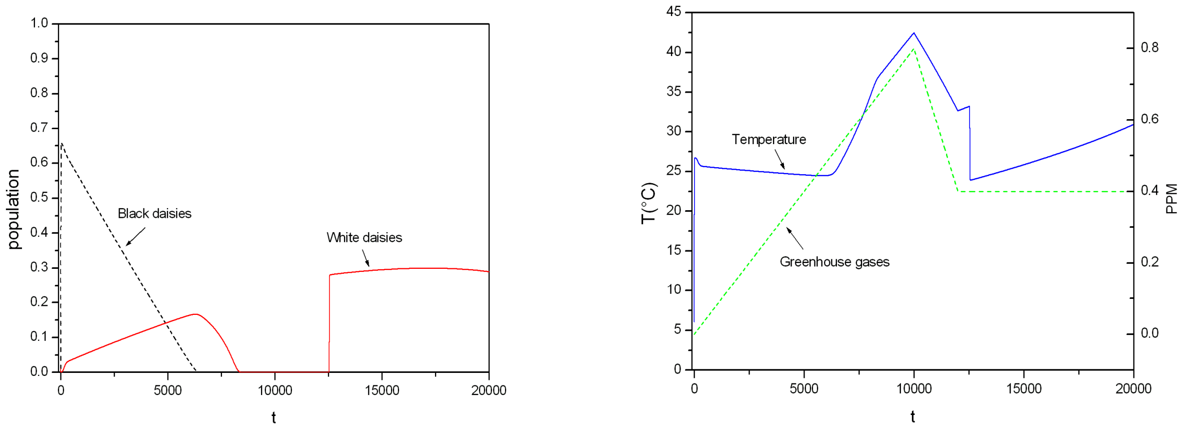

Figure 11. Evolution of daisy populations (left) and temperature (right) with variations in greenhouse gases emissions, considering luminosity with linear increase.

5. CONCLUSIONS

A mathematical model based on the daisyworld is proposed. The daisyworld is an archetypal of the Earth and is able to describe the global regulation that can emerge from the interaction between life and environment. In brief, daisyworld represents life by black and white daisy populations while the environment is represented by temperature. An extra variable related to greenhouse gases is incorporated in the model allowing the analysis of the global warming. A general analysis of the daisyworld is carried out analyzing constant and linear increase of the solar luminosity. Hysteretic behavior related to different paths for loading and unloading responses is identified. Afterwards, the influence of greenhouse gases in daisyworld dynamics is treated establishing a comparison with the classical model. In general, these gases tend to increase the planet temperature, accelerating the death of populations and decreasing the capacity of global regulation. Therefore, the proposed model can be used for the description of the greenhouse effect that is an essential problem of this century.

6. ACKNOWLEDGEMENTS

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