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A Hierarchical Beam Theory for Non-Symmetric and Piezo-Electric Laminates

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Abstract. *A hierarchical theory for layered beams including composite laminates, sandwich beams and piezoelectric active layers is developed. This approach provides a sequence of beam models based on far-field stress and strain solutions called Fundamental State Solutions. Also, through-thickness stress and strain moments of the Fundamental Solutions are used to obtain homogenized axial, flexural and shear stiffness as well as a shear-strain moment correction. The hierarchy of beam models developed are similar to the Timoshenko model. The extension in the present work provides results for non-symmetric laminated beams and piezo-electrically actuated beams. Excellent agreement is shown for all stress and strain components when compared to accurate two-dimensional finite element results.*

Keywords: *beam theory, non-symmetric, piezo-electric*

1. Introduction

Beam, plate and shell theories for layered structures have been treated extensively in the literature. In particular, many models have been developed all of which attempt to address two essential requirements. First, it is desirable to obtain accurate displacement solutions; second, accurate and complete through-thickness stress/strain distributions are crucial in the prediction of failure/delamination. The challenge has been to do both with minimal computational effort. The fundamental difficulty is that the through-thickness stress, strain and displacement fields are in general non-differentiable and may be discontinuous; thus accurately modelling the through-thickness response is complex and using through-thickness polynomials leads in general to poor approximations.

The literature on composite and sandwich analysis is vast; reviews of laminated composite theory may be found in [8, 12, 13, 14]. The appropriate beam theory is due to Timoshenko [16, 17] while the analogous plate results are due to Reissner [15] and Mindlin [11]. Current techniques for modelling layered smart/intelligent structures can be found in [1, 3, 4]. The paper by Chopra [1] is an excellent review of the broader range of smart/intelligent structures.

The de facto standard method of analyzing layered beams, plates or shells is to use a Timoshenko or Reissner-Mindlin formulation, the so called first-order shear deformation theory, with appropriate transverse shear stiffness and shear correction factors and then complete a post-processing calculation in which the in-plane stresses are used to determine approximations for the transverse shear stresses. Smart structure modelling is based on either Euler-Bernoulli or Timoshenko beam theory. The present work adopts a very different point of view and, as will be shown, leads to an extremely accurate modelling capability. Furthermore, the current work provides a unified approach for layered composite, sandwich and active structures.

2. Current Approach

The current approach eliminates the internal modelling inconsistencies of most beam, plate and shell theories; that is, all approximations made in the current approach are internally consistent. For example, solutions obtained are equally accurate for both stress and strain; stress or strain obtained in a post-processing calculation are consistent with the stress and strain approximations used in developing the theory.

An essential feature of the present work is that solutions are expressed as through-thickness moments (averages) of stress, strain and displacement rather than being modelled as through-thickness polynomials; thus difficulties associated with lack of through-thickness differentiability and/or discontinuities are avoided.

The only assumption made is that approximate solutions can be modelled as a superposition of a sequence of calculated far field solutions referred to as Fundamental State Solutions. This assumption has important consequences:

1. No through-thickness polynomial approximations are made.
2. The problem is completely and accurately represented in terms of stress, strain and displacement moments.

3. Accurate ‘average’ structural stiffnesses are calculated based on stress and strain moments obtained from the Fundamental State Solutions.
4. Accurate and internally consistent through-thickness stress and strain distributions are obtained from the Fundamental States and are used in a straight-forward post-processing step to determine an accurate approximation of the stress and strain fields.

3. Stress, Strain and Displacement Moments

Moments of a system variable are evaluated in terms of the through-thickness coordinate z and with respect to the beam middle-surface $z = 0$. The axial coordinate is represented as x while y is normal to the $x - z$ plane.

The n^{th} through-thickness stress or strain moment is defined as

$$s^n(x) \triangleq \int_A \sigma(x,y,z) z^n dA \quad ; \quad e^n(x) \triangleq \int_A \epsilon(x,y,z) z^n dA \tag{1}$$

where $dA = dz\, dy$ and A is the beam cross-sectional area. The n^{th} normalized displacement moments are defined as

$$u_n(x) \triangleq \frac{1}{A_{2n}} \int_A u(x,y,z) z^n \, dA \quad ; \quad w_n(x) \triangleq \frac{1}{A_{2n}} \int_A w(x,y,z) z^n \, dA \tag{2}$$

where A_{2n} is the $(2n)^{th}$ moment of the beam cross-sectional area with respect to z. The displacement moments have been normalized in the manner indicated in order that the final displacement-moment equilibrium equations adopt a form similar to the classical Timoshenko equilibrium equations.

4. Fundamental States

The beam approximation is based on a sum of reference solutions referred to as Fundamental State Solutions. The essence of the Fundamental States is that (y,z) dependent but x-invariant stress/strain distributions exist for all load states corresponding to zero, constant, linear, $\cdots$ flexural states. Such invariant properties always occur for axially invariant beam cross-sections subjected to surface tractions expressed as a polynomial with respect to x. These invariant forms are defined as the Fundamental State Components. The sequence of problems used to generate the set of invariant states are defined in an hierarchical manner corresponding to a complete sequence of bending moments.

4.1 Surface Tractions

The following is illustrative for the case of pressure loads being applied to the top and/or bottom of a beam.

State 0- Constant axial stress resultant: Zero bending
State 1- Pure bending: Constant bending moment
State 2- Pure shear: Linear bending moment
State 3- Constant pressure at top or bottom of beam: Quadratic bending.
State 4- Linearly varying pressure at top or bottom: Cubic bending.
$\vdots$ $\vdots$
State M- (M-3)rd degree pressure : (M-1)st degree bending.

4.2 Piezoelectric Actuation

The following is illustrative for the case of M-2 piezoelectric layers distributed throughout the structure..

State 0- Constant axial stress resultant: Zero bending
State 1- Pure bending: Constant bending moment
State 2- Pure shear: Linear bending moment
State 3- Unit voltage in 1st Piezoelectric layer - Zero voltage in all other Piezoelectric layers
State 4- Unit voltage in 2nd Piezoelectric layer - Zero voltage in all other Piezoelectric layers
$\vdots$ $\vdots$
State M- Unit voltage in (M-2)nd Piezoelectric layer - Zero voltage in all other Piezoelectric layers

4.3 Solution Approximation

The solution approximation includes two sets of variables. The first are the Fundamental State Components which are written as

$$ {}_m\sigma(y,z)\ ,\ \ {}_m\epsilon(y,z) \quad : \quad m = 0, M \tag{3} $$

Note that these components are functions of y and z but are independent of x; the preceding subscript m refers to the Fundamental State. The second set are called the Fundamental State Variables and are written as

$$ \alpha_m(x) \quad : \quad m = 0, M \tag{4} $$

These are the 'amplitudes' associated with each Fundamental State. Also, the Fundamental State Components have been normalized such that $\alpha_m(x)$ corresponds to the magnitude of the stress resultant, bending moment, shear resultant, constant pressure amplitude $\cdots$ or the magnitude of the applied voltage in each of the piezoelectric layers. The total stress and strain fields are approximated as a sum of the Fundamental State Components multiplied by the appropriate Fundamental State Variable. (If mechanical and electrical loads are applied simultaneously the the State numbers used above must be changed appropriately. This should not cause confusion.) Thus

$$ \begin{aligned} \sigma^T(x,y,z) &\approx \alpha_0(x)\,{}_0\sigma(y,z) + \alpha_1(x)\,{}_1\sigma(y,z) + \cdots + \alpha_M(x)\,{}_M\sigma(y,z) \\ \epsilon^T(x,y,z) &\approx \alpha_0(x)\,{}_0\epsilon(y,z) + \alpha_1(x)\,{}_1\epsilon(y,z) + \cdots + \alpha_M(x)\,{}_M\epsilon(y,z) \end{aligned} \tag{5} $$

where the superscript ^{T} means total. It is important to note that of all $\alpha_m(x)$ it is only those of States 0, 1 and 2 (Stress Resultant, Pure Bending and Constant Shear) which are unknown. This is true for both the case of applied surface tractions and piezoelectric actuation and when combined, surface tractions and piezoelectric actuation are treated as a superposition of the above fundamental states.

As noted, the remaining State Variables are uniquely specified by the surface tractions or applied voltages acting on the beam. Thus the beam problem will have at most three undetermined variables; $\alpha_0(x)$, $\alpha_1(x)$ and $\alpha_2(x)$. This means that a Timoshenko like model is all that is required in any situation. The procedure used to determine the fundamental state solutions is completely developed by Hansen and Almeida [6], Kennedy [9], McLean [10]. Finally, once the α_m have been determined then the complete stress and strain fields may be obtained from Eqns. (5) in a straight-forward post-processing calculation.

5. Homogenized Stiffness

Each Fundamental State Solution defines unique inter-relationships between the stress and strain for that state; in particular for States 0, 1 and 2 these inter-relationships are interpreted to define stiffness relations between the stress resultant, bending moment and shear resultant and the corresponding strain moments. It is these relationships which are used to obtain homogenized stiffness properties and which are now developed.

5.1 Homogenized Membrane-Bending Stiffness

The beam cross-section is assumed to be non-symmetric; this implies there will be membrane-bending coupling. Therefore, the coupled axial, membrane-bending and bending homogenized stiffness are defined as a relation between the zeroth and first moments of $\sigma_x(x,y,z)$ and $\epsilon_x(x,y,z)$ corresponding to States 0 and 1. That is

$$ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \triangleq \begin{bmatrix} {}_0s_x^0 & {}_1s_x^0 \\ {}_0s_x^1 & {}_1s_x^1 \end{bmatrix} \begin{bmatrix} {}_0e_x^0 & {}_1e_x^0 \\ {}_0e_x^1 & {}_1e_x^1 \end{bmatrix}^{-1} \tag{6} $$

where a_{ij} are the homogenized stiffness properties. This definition is effectively a 'ratio' of stress moments divided by strain moments and is therefore an extension of the standard definition of material stiffness. The normalization of the Fundamental States (Hansen and Almeida [6] and Kennedy [9]) imposes ${}_0s_x^0 = 1$, ${}_0s_x^1 = 0$, ${}_1s_x^0 = 0$, ${}_1s_x^1 = 1$, therefore Eqn. (6) becomes

$$ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \triangleq \frac{1}{{}_0e_x^0\,{}_1e_x^1 - {}_0e_x^1\,{}_1e_x^0} \begin{bmatrix} {}_1e_x^1 & -\,{}_1e_x^0 \\ -\,{}_0e_x^1 & {}_0e_x^0 \end{bmatrix} \tag{7} $$

Thus an homogenized stress moment - strain moment constitutive relation

$$ \begin{bmatrix} s_x^0 \\ s_x^1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} e_x^0 \\ e_x^1 \end{bmatrix} \tag{8} $$

is obtained. Furthermore, this relation is exact for loading states corresponding to any combination of Fundamental States 0 and 1.

5.2 Homogenized Shear Stiffness

The homogenized transverse shear stiffness is defined as the ratio of the zeroth moments of σ_{xz} and γ_{xz} evaluated for State 2

$$G_{xz} \triangleq \frac{{}_2 s^0_{xz}}{{}_2 e^0_{xz}} \tag{9}$$

In a manner similar to Eqn. (8), this result guarantees that

$$s^0_{xz} = G_{xz} e^0_{xz} \tag{10}$$

is exact for load states corresponding to a state of pure shear (State 2). No shear correction factor is required! Furthermore, for a single layer material, G_{xz} is identical to the exact transverse shear stiffness, independent of the stress and/or strain distribution or the beam cross-sectional geometry. Such a result is not true for conventional laminated or sandwich beam theory.

6. Displacement Representation

Only the x and z displacements enter the analysis. These displacements are decomposed into two components

$$u^T(x,y,z) = u(x,y,z) + \bar{u}(x,y,z) \quad ; \quad w^T(x,y,z) = w(x,y,z) + \bar{w}(x,y,z) \tag{11}$$

where $u^T(x,y,z), w^T(x,y,z)$ are called the total displacements; $u(x,y,z)$, $w(x,y,z)$ are components corresponding to Fundamental States 0, 1, 2 and are ‘unknown’; $\bar{u}(x,y,z)$, $\bar{w}(x,y,z)$ are components corresponding to fundamental states other than 0, 1, 2. and are effectively ‘known’ being defined by the surface tractions. The displacements never enter the analysis explicitly and are not determined as it is the moments

$$u_0(x) = \frac{1}{A_0}\int_A u^T(x,y,z)\,dA \;\; ; \;\; u_1(x) = \frac{1}{A_2}\int_A u^T(x,y,z)\,z\,dA \;\; ; \;\; w_0(x) = \frac{1}{A_0}\int_A w^T(x,y,z)\,dA \tag{12}$$

which are the problem unknowns. The variables $A_0 = A$ and $A_2 = I$ are the beam cross-sectional area and the second moment of the area respectively.

7. Strain Moment – Displacement Moment Relations

Using the classical linear strain-displacement relations, the strain moments of Eqn. (1) and the displacement moments of Eqns. (12) yields

$${}^T e^0_x(x) = A\,\frac{du_0(x)}{dx} \;\; ; \;\; {}^T e^1_x(x) = I\,\frac{du_1(x)}{dx} \;\; ; \;\; {}^T e^0_{xz}(x) = \left[\int_A \frac{\partial u^T(x,y,z)}{\partial z}\,dA\right] + A\,\frac{dw_0(x)}{dx} \tag{13}$$

The first two of the above relations are exact. However [.] in the third relation cannot be determined exactly in terms of $u_0(x), u_1(x)$; therefore, an approximation is required. Noting that [.] depends only on the odd part of $u^T(x,y,z)$ (with respect to z), implies [.] can depend only on $u_1(x)$ and cannot depend on $u_0(x)$. Thus the shear-strain moment correction is defined as

$$C_{xz} \triangleq \left\{ \frac{{}^T e^0_{xz}(x)}{A\left[u_1(x) + \frac{dw_0(x)}{dx}\right]} \right\}_{State\ 2} \tag{14}$$

This correction is the ratio of the exact shear-strain moment to the approximation of the shear-strain moment expressed in terms of $u_1(x), w_0(x)$ all evaluated for State 2 (constant shear). For a rectangular, isotropic beam this correction corresponds exactly to that obtained by Cowper [2] although the interpretation is different. For plates this definition yields a plane-strain correction Guiamatsia and Hansen [5]. This definition is at odds with the usual interpretation of a shear correction factor and also yields slightly different values; therefore, it is given a more accurate name - *shear-strain moment correction*.

8. Total Strain Moment Representation

Based on the strain expansion of Eqn. (5) it follows that the n^{th} total stain moment is

$${}^T e^n = \alpha_0(x)\,{}_0 e^n + \alpha_1(x)\,{}_1 e^n + \cdots + \alpha_M(x)\,{}_M e^n \tag{15}$$

The strain moments are decomposed in the same manner as the displacements of Eqn. (11); that is

$$ {}^{T}e^{n} = e^{n} + \overline{e}^{n} \tag{16}$$

where

$$ e^{n} = \alpha_0(x)\,{}_0e^{n} + \alpha_1(x)\,{}_1e^{n} + \alpha_2(x)\,{}_2e^{n} \quad ; \quad \overline{e}^{n} = \sum_{m=3}^{M} \alpha_m(x)\,{}_me^{n} $$

This decomposition is important for two reasons:

1. $\alpha_0(x), \alpha_1(x), \alpha_2(x)$ are unknown while $\alpha_m(x), m = 3, M$ are known.
2. The stiffness relations defined in Eqns. (7) and (9) relating the stress and strain moments given in Eqns. (8) and (10) are exact only for States 0,1,2. The same holds for the shear strain moment correction of Eqn. (14)

Thus, when the equations of equilibrium are considering it is necessary to express the strain moments in terms of e^n. Thus for the present, the necessary strain moments are

$$ e_x^0 = {}^{T}e_x^0 - \overline{e}_x^0 = A\,\frac{du_0(x)}{dx} - \overline{e}_x^0 \quad ; \quad e_x^1 = {}^{T}e_x^1 - \overline{e}_x^1 = I\,\frac{du_1(x)}{dx} - \overline{e}_x^1 $$

$$ e_{xz}^0 = {}^{T}e_{xz}^0 - \overline{e}_{xz}^0 = C_{xz}\left\{ A\left[u_1(x) + \frac{dw_0(x)}{dx}\right] - A\left[\overline{u}_1(x) + \frac{d\overline{w}_0(x)}{dx}\right]\right\} \tag{17}$$

These relations guarantee exact relations between the stress and strain moments. In these expressions, the terms $\overline{e}_x^0, \overline{e}_x^1$, $\overline{u}_1(x)$, $\overline{w}_0(x)$ represent higher order corrections resulting from surface tractions as well as strain states resulting from piezoelectric actuation.

9. Moment Equilibrium Equations

The moment equilibrium equations are consistent with the Timoshenko approximation. For a beam of rectangular cross-section with thickness $2c$ and unit width they become

$$ \frac{d\,{}^{T}s_x^0}{dx} = -\left[T_{xz}^{+}(x) + T_{xz}^{-}(x)\right] \quad ; \quad \frac{d\,{}^{T}s_x^1}{dx} - {}^{T}s_{xz}^0 = -c\left[T_{xz}^{+}(x) - T_{xz}^{-}(x)\right] \quad ; \quad \frac{d\,{}^{T}s_{xz}^0}{dx} = -\left[T_z^{+}(x) + T_z^{-}(x)\right] \tag{18}$$

where $T_{xz}^{+}(x), T_{xz}^{-}(x), T_z^{+}(x), T_z^{-}(x)$ are the tractions acting on the upper and lower surfaces of the beam. Substitution for stress moments in terms of displacement moments yields

$$ a_{11}\frac{d}{dx}\left[A\,\frac{d\,u_0(x)}{dx} - \overline{e}_x^0\right] + a_{12}\frac{d}{dx}\left[I\,\frac{du_1(x)}{dx} - \overline{e}_x^1(x)\right] = \left[T_{xz}^{+}(x) + T_{xz}^{-}(x)\right] $$

$$ a_{21}\frac{d}{dx}\left[A\,\frac{d\,u_0(x)}{dx} - \overline{e}_x^0\right] + a_{22}\frac{d}{dx}\left[I\,\frac{du_1(x)}{dx} - \overline{e}_x^1(x)\right] - C_{xz}\left\{A\left[u_1(x) + \frac{d\,w_0(x)}{dx}\right] - A\left[\overline{u}_1(x) + \frac{d\,\overline{w}_0(x)}{dx}\right]\right\} $$

$$ = -c\left[T_{xz}^{+}(x) - T_{xz}^{-}(x)\right] $$

$$ C_{xz}G_{xz}\frac{d}{dx}\left\{A\left[u_1(x) + \frac{d\,w_0(x)}{dx}\right] - A\left[\overline{u}_1(x) + \frac{d\,\overline{w}_0(x)}{dx}\right]\right\} = -\left[T_z^{+}(x) + T_z^{-}(x)\right] $$

These equations bear a remarkable similarity to those of Timoshenko beam theory; however, there are two important differences. First, $u_0(x), u_1(x), w_0(x)$ are displacement moments rather than middle-surface displacements. Second, there are higher-order-state corrections $\overline{e}_x^0(x), \overline{e}_x^1(x)$ and $\overline{e}_{xz}^0(x)$ which provide a hierarchy of approximations in the case of surface tractions and yield the complete electro-mechanical input for piezoelectric actuation. Hansen and Almeida [6, 7], Kennedy [9] and McLean [10] provide details.

10. Numerical Examples

The capabilities of this approach will be illustrated for two layered cantilever beams. The fist is a non-symmetric sandwich beam while the second is a symmetric laminated beam with symmetrically located piezoelectric actuators which are anti-symmetrically actuated.

10.1 Sandwich Beam

A cantilever sandwich beam in which the face sheets may have different stiffness is considered. The beam is 100mm long, 10mm thick and 10mm wide. The face sheets are 2.5mm thick while the core is 5mm thick. The top layer has a Young's modulus $E = 70\,GPa$ and a Poisson's ratio $\nu = 0.3$; the core material has $E = 0.01 \times 70\,GPa$ and $\nu = 0.3$; the lower face sheet has $E = \alpha \times 70\,GPa$ and $\nu = 0.3$ with $\alpha = 1.0, 0.1, 0.01$. Thus the structure may have either a symmetric or a non-symmetric cross-section. Comparisons between the present approach and a two-dimensional ANSYS finite element calculation are presented. The finite element model used 200x40 bi-quadratic elements along the length and through the thickness respectively. The sandwich-beam is subjected to a linearly varying surface pressure applied on top of the beam.

Typical results at beam mid-length (x/L = 0.5) are presented in Figs. 2 to 5 when the normal surface traction varies from $100\,kPa$ at the root to $0\,kPa$ at the free end. There is excellent agreement between the beam theory and the finite element calculations. The presented results, axial strain ϵ_x^T, transverse shear strain γ_{xz}^T, transverse shear stress σ_{xz}^T and transverse normal stress σ_z^T have been chosen for presentation as they illustrate characteristics that cannot be captured using existing beam theories. Complete results show that at the beam mid-length the finite element and beam theory predictions are virtually numerically identical for all stress and strain components. At a distance of 1/2 a beam thickness from the cantilever root the maximum worst case error in the shear stress is approximately 11%; this error is attributed to three-dimensional boundary effects. The through-thickness displacement moments have a typical maximum worst case error of approximately 1% over the length of the beam. A complete development for symmetric beams presented in Hansen and Almeida [6, 7] shows equally outstanding accuracy.

11. Five-Layered Bimorph Laminate

A five-layer, bimorph laminated cantilever beam is considered. The beam is constructed of orthotropic elastic and piezoelectric actuator layers. This is illustrated in Fig. 1.

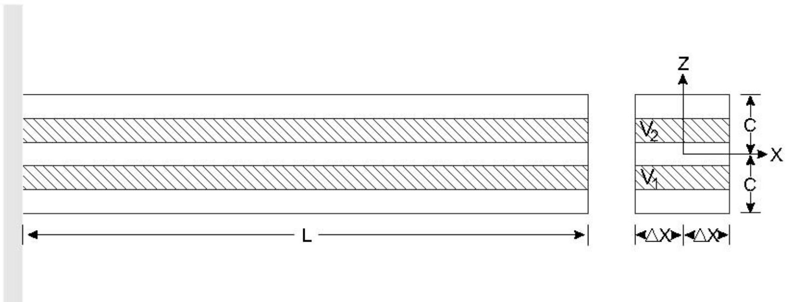

Figure 1. Five-Layered Bimorph Laminate Geometry

The only force applied on this system are the electric fields V_1 and V_2 which activate the two piezoelectric layers. Therefore fundamental states related to the activation of each of the piezoelectric layers are included. In the analysis, each layer is 1 mm thick and 10mm wide while the beam is 100mm long. In addition the piezoelectric layers have the following properties ($\times 10^9$)

$$C_{11}^P = 91.0 \;;\; C_{12}^P = C_{21}^P = 29.8 \;;\; C_{22}^P = 91.0 \;;\; C_{33}^P = 25.6$$

$$D_{31}^P = -0.12 \times 10^{-3} \;;\; D_{33}^P = 0.29 \times 10^{-3}$$

while the elastic layers have the following properties ($\times 10^9$)

$$C_{11}^E = 75.84 \;;\; C_{12}^E = C_{21}^E = 31.04 \;;\; C_{22}^E = 69.87 \;;\; C_{33}^E = 23.0$$

Comparisons between the present beam theory and a two-dimensional ANSYS finite element calculation were completed. The finite element model used 200x40 bi-quadratic elements along the length and through the thickness respectively. Typical results at beam mid-length (x/L = 0.5) were obtained for an applied voltage of +/-10 volts in the upper and lower layers respectively and are presented in Figs. 6 to 8

As in the previous example, the results of these calculations yield extremely accurate comparisons between the ANSYS and the beam theory calculations. In particular, the ability of the present work to calculate the through-thickness strain ϵ_z is to be noted. It should also be mentioned that for this calculation $\sigma_z = \sigma_{xz} = \gamma_{xz} = 0$; these results are also in correspondence between ANSYS and the beam theory

12. Summarizing Comments

This work develops a straight-forward beam theory which yields exceptionally accurate results for average displacements as well as complete two-dimensional stress and strain fields. The present results illustrate this approach for non-symmetric laminated beams and for piezoelectrically actuated beams. The resulting equilibrium equation have a striking similarity to the Timoshenko beam equations although the variables used have a different meaning. The application to layered plates Guiamatsia and Hansen [5] is currently underway, but in general, the plate analysis does not yield a typical Reissner/Mindlin plate theory. For the case of isotropic homogeneous layers a Reissner/Mindlin type model results; however, for general laminated plates a new model is required.

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14. References

Chopra, I., 2002, "Review of State of Art of Smart Structures and Integrated Systems", *AIAA J.*, 40(11), pp. 2145-2187.

Cowper, G.R., "The Shear Coefficient in Timoshenko's Beam Theory", 1966, *Journal of Applied Mechanics*, Vol. 33, pp. 335-340.

Crawley, E., and de Luis, J., 1987, "Use of Piezoelectric Actuators as Elements of Intelligent Structures", AIAA J., 25(10), pp. 1373-1385.

Franco Correia, V.M., Gomes, M.A.A., Suleman, A., Mota Soares, C.M., and Mota Soares, C.A., 2000, "Modelling and Design of Adaptive Composite Structures", *Comput. Methods Appl. Mech. Engrg.*, 185(3-4), pp. 325-346.

Guiamatsia, I. and Hansen, J.S., 2004, "A Homogenization Based Laminated Plate Theory", 2004 ASME International Mechanical Engineering Congress and RD & D EXPO, Anaheim, California , Nov. 13-19.

Hansen, J.S. and Almeida, S.F.M., "A Theory for Laminated Composite Beams", Final Report submitted April 2001, FAPES Grant No. 00/06183-0, São Paulo, Brasil, 157 pages.

Hansen, J.S. and Almeida, S.F.M., 2004, " A Homogenization Based Laminated Beam Theory", 21st International Congress of Theoretical and Applied Mechanics (ICTAM 2004), Warsaw, Poland 16-20 August .

Kapania, R.K., 1989, "A Review on the Analysis of Laminated Shells", *ASME Journal of Pressure Vessel Technology*, Vol. 111, May , pp. 88-96.

Kennedy, G., 2004, "A Hierarchical Beam theory for Beams with Asymmetric Cross-Sections", B.A.Sc. Thesis, Division of Engineering Science, Faculty of Engineering, University of Toronto, Dec. .

McLean, C., 2001. "A Hierarchical Theory for Layered Beams with Piezoelectric Actuation", University of Toronto Institute for Aerospace Studies, M.A.Sc. thesis.

Mindlin,R.D., 1951, "Influence of Rotary Inertia and Shear on The Effect of Transverse Shear Deformation on Flexural Motions of Isotropic, Elastic Plates", *Journal of Applied Mechanics*, Vol. 18, No. 1, pp. 31-38.

Noor, A.K. and Burton, W.S., 1989, "Assessment of Shear Deformation Theories for Multilayer Composite Plates", *Applied Mechanics Reviews*, Vol.42, N0.1, pp. 1-13.

Noor, A.K. and Burton, W.S., 1990, "Assessment of Computational Models for Multilayer Composite Shells", *Applied Mechanics Reviews*, Vol.43, N0. 4, pp. 67-97.

Reddy, J. N., 1990, " A Review of Refined Theories of Laminated Composite Plates", *The Shock and Vibration Digest*, Vol. 22, No 7, pp. 3-17.

Reissner, E., 1945, "The Effect of Transverse Shear Deformation on the Bending of Elastic Plates", *Journal of Applied Mechanics*, Vol. 12, No. 2, pp. 69-77.

Timoshenko, S.P., 1921, "On the Correction for Shear of the Differential Equation for Transverse Vibrations of Prismatic Bars", *Philosophical Magazine*, Vol. 21, pp. 744-746.

Timoshenko, S.P., 1922, "On the Transverse Vibrations of Bars of Uniform Cross-Section", *Philosophical Magazine*, Vol. 43, pp. 125-131.

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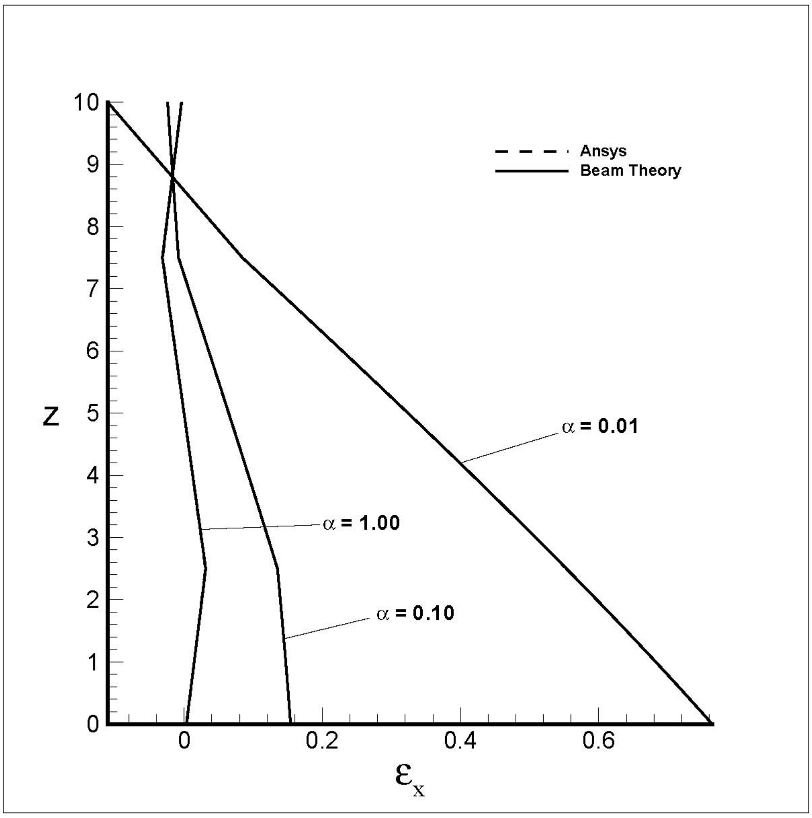

Figure 2. Total Axial Strain ϵ_x^T ($\times 10^{-5}$) at $x/L = 0.5$.

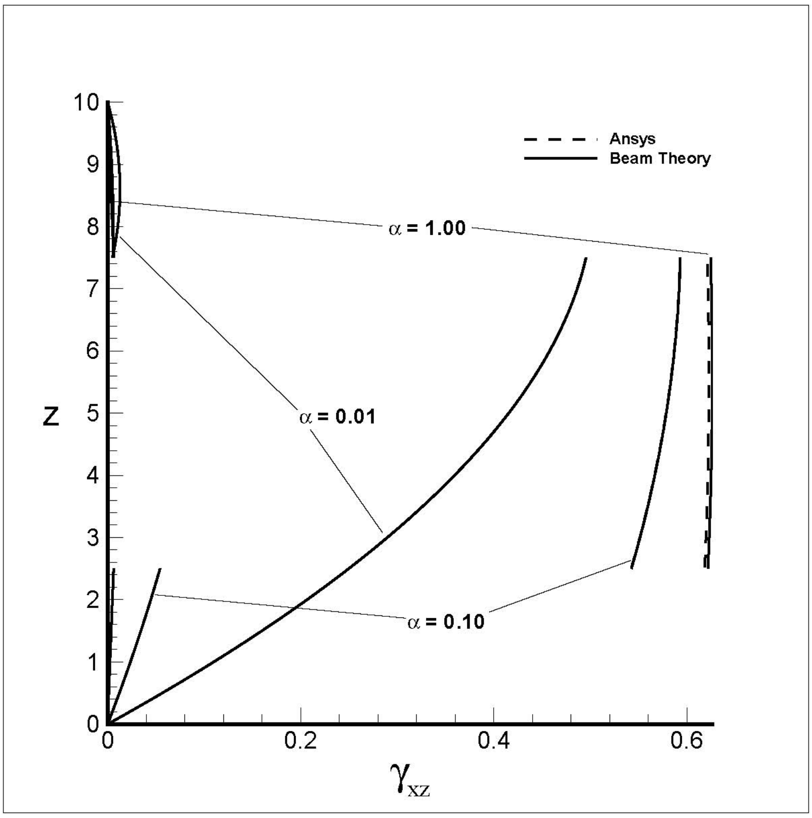

Figure 3. Total Transverse Shear Strain γ_{xz}^T ($\times 10^{-5}$) at $x/L = 0.5$.

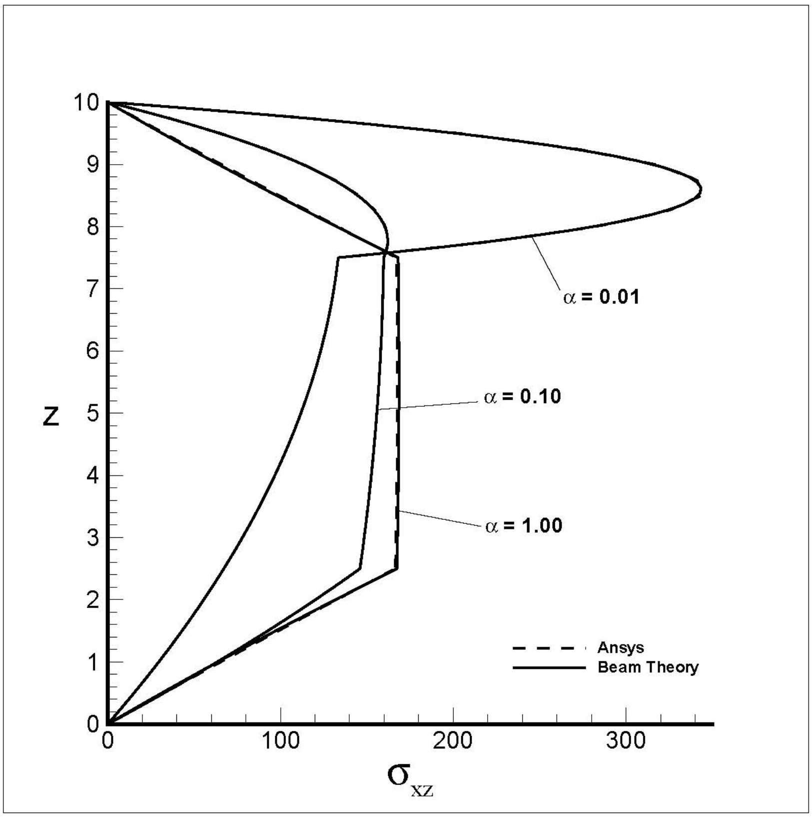

Figure 4. Total Transverse Shear Stress σ_{xz}^T (kPa). at $x/L = 0.5$.

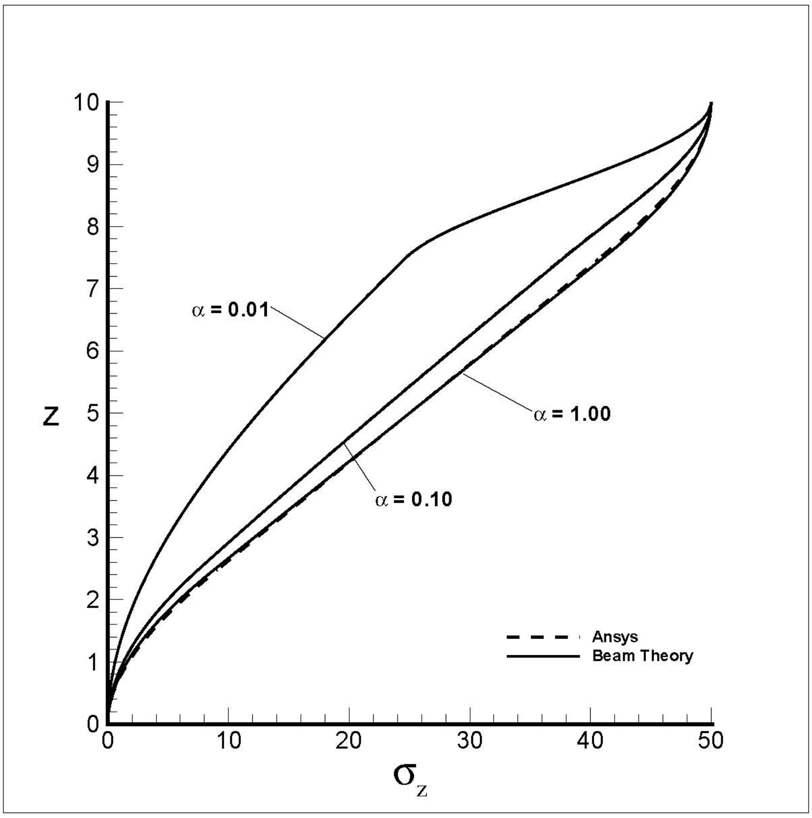

Figure 5. Total Transverse normal stress σ_z^T (kPa). at $x/L = 0.5$.

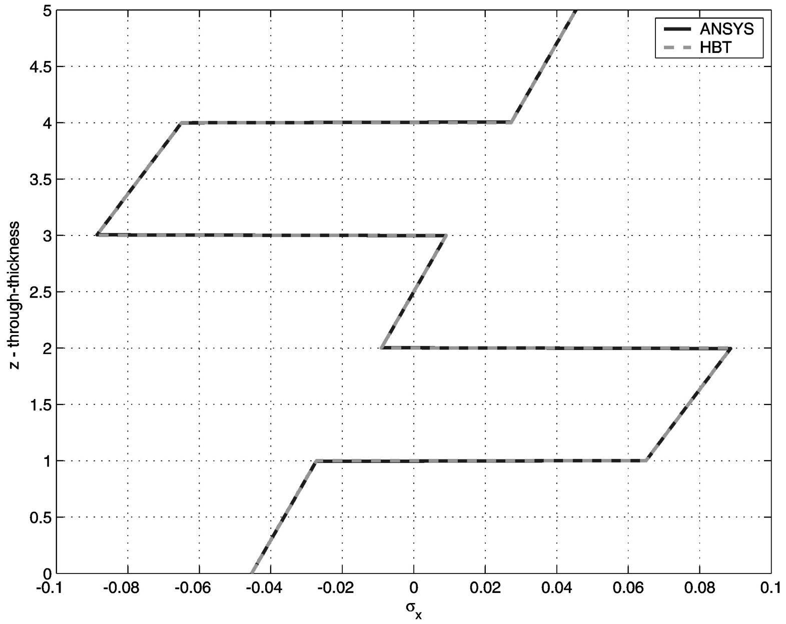

Figure 6. Total Axial Stress σ_x^T at $x/L = 0.5$ for a Five-Layer Bimorph.

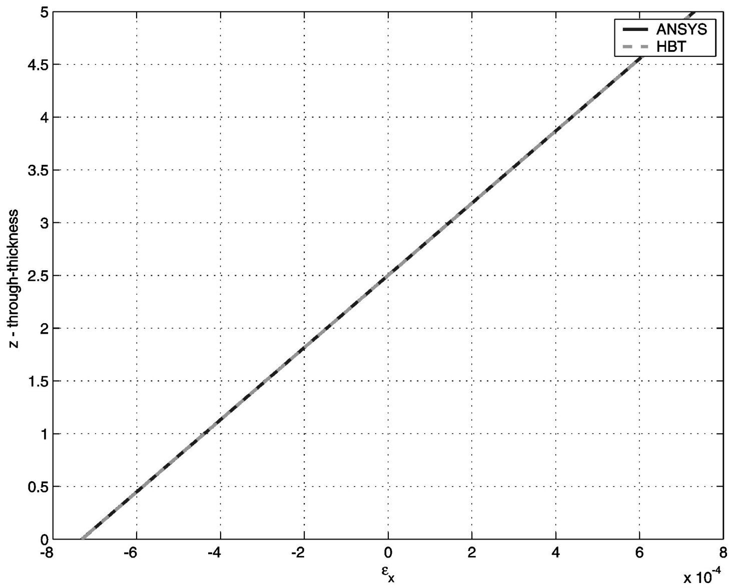

Figure 7. Total Axial Strain ϵ_x^T at $x/L = 0.5$ for a Five-Layer Bimorph.

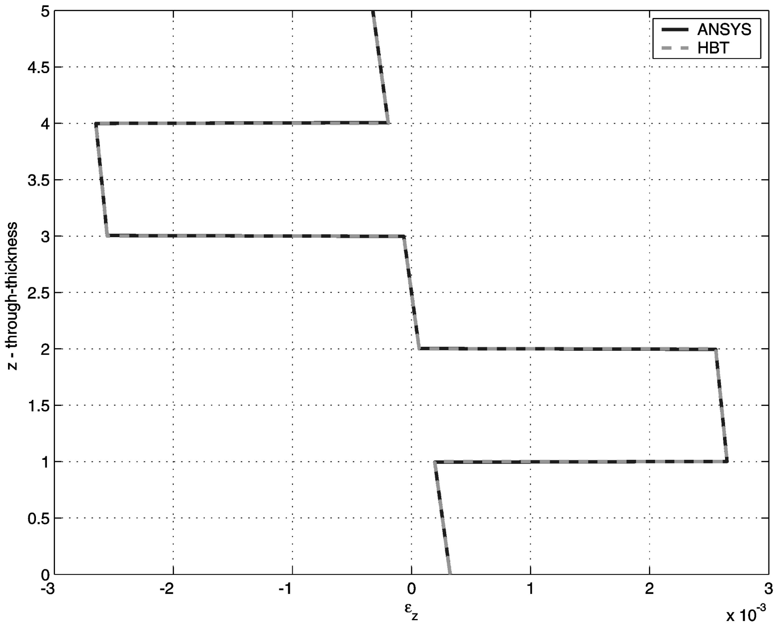

Figure 8. Total Transverse Strain ϵ_z^T at $x/L = 0.5$ for a Five-Layer Bimorph.