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A COMPARISON OF TWO MODAL CONTROL STRATEGIES FOR THE ACTIVE VIBRATION CONTROL OF A TRUSS STRUCTURE

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Abstract. *In this paper two strategies are described for the active vibration control of a truss structure using smart materials. The control algorithms have been implemented for a truss structure model developed using the Finite Element Method (FEM). A FEM of the piezoelectric stack actuator is also described. The aim of this works is to compare two methods of modal control, which are Independent Modal Space Control (IMSC) and Efficient Modal Control (EMC). The essence of these methods is that a feedback force is applied according to a weighting factor in some specific modes i.e., in each method a different weighting factor is used. To verify the effectiveness of these methods, a numerical example is presented for a truss structure and the results show that the maximum feedback control voltage required with EMC is smaller than that compared to IMSC for similar control performance.*

Keywords: *Active vibration control, Piezoelectric active bar, Intelligent truss structure, Independent Modal Space Control, Efficient Modal Control*

1. Introduction

A truss structure is one of the most commonly used structures in aerospace and building engineering, (Yan and Yam, 2002). Because it is desirable to use the minimum amount of material for construction, the trusses are becoming very lightweight and flexible. These characteristics mean that the structures are susceptible to structural vibration. The conventional form of external passive damping is not preferred as the addition of damping materials adds to the overall weight of the system. This has led to extensive research into active vibration control.

In active control, the effect of an unwanted, uncontrolled disturbance is cancelled by addition of another controlled disturbance. Piezoelectric active members introduced into the structure act as actuators to control the response of the structure. Application of a voltage to a piezoelectric stack produces a force which can change the stiffness and damping characteristics of the passive structure. These changes, if properly adjusted, can reduce the amplitude of vibrations, (Singh et al. 2003).

In many practical systems the lower modes of vibration have the most energy and are more critical. For this reason, these modes are targeted for active control so that energy is not wasted in controlling the higher modes of vibration. The system is thus transformed into modal space and its individual modes are controlled using modal space control strategies.

In this paper, the performance of the Independent Modal Space Control (IMSC) method, which has received much interest, is examined and compared with the Efficient Modal Control (EMC) method to control a truss structure. A similar work was done by Singh et al. (2003), which was controlled a cantilever beam mounted with a PZT patch. The IMSC method combines modal decomposition with a classical Linear Quadratic Regulator (LQR) control law. The control gains for each mode can be found by solving a second order Riccati equation. The computational load is greatly reduced if the IMSC algorithm is used to design the control system, since the control design is carried out on the lower frequency modes (Fang et al. 2003).

The essence of the Efficient Modal Control (EMC) method is that a feedback force may be applied in different modes according to the vibration amplitude in the respective modes i.e., modes having a small amplitude of vibration may have a smaller control force. This weighting may be done on the basis of either displacement or energy present in the different modes.

The results of numerical simulations indicate that the implementation of the EMC algorithm is simple and convenient, and its design can be carried out following the same procedure as for IMSC.

2. Model of the system

A smart truss structure is composed of active and passive members. A model of the passive members is easily obtained using finite elements and, with some modification it can also be used to model the active members, which are commonly composed of a piezoelectric stack and two metallic bars. A piezoelectric active member is shown in Fig. 1. The stiffness matrix for an active member is formulated by combining the stiffness matrix for the piezoelectric stack with the stiffness matrices of two metallic bars; each piezoelectric active member has an additional electrical degree of freedom. The local mechanical-electric stiffness matrix for the piezoelectric active member, described in (Lammering, Jia and Rogers, 1994), is:

$$\mathbf{K}^{\mathbf{el}} = \begin{bmatrix} \frac{k_1 k_2 k_3}{k_1 k_2 + k_1 k_3 + k_2 k_3} & \frac{-k_1 k_2 k_3}{k_1 k_2 + k_1 k_3 + k_2 k_3} & \frac{-k_1 c_2 k_3}{k_1 k_2 + k_1 k_3 + k_2 k_3} \\ \frac{-k_1 k_2 k_3}{k_1 k_2 + k_1 k_3 + k_2 k_3} & \frac{k_1 k_2 k_3}{k_1 k_2 + k_1 k_3 + k_2 k_3} & \frac{k_1 c_2 k_3}{k_1 k_2 + k_1 k_3 + k_2 k_3} \\ \frac{-k_1 c_2 k_3}{k_1 k_2 + k_1 k_3 + k_2 k_3} & \frac{k_1 c_2 k_3}{k_1 k_2 + k_1 k_3 + k_2 k_3} & \frac{-c_2^2 (k_1 + k_3)}{k_1 k_2 + k_1 k_3 + k_2 k_3} - \bar{c}_2 \end{bmatrix} \tag{1}$$

where $k_2 = {E_2 A_2}/{L_2}$, $c_2 = {e_2 A_2}/{L_2}$, $\bar{c}_2 = {\varepsilon_2 A_2}/{L_2}$ (2a,b,c)

and e is the piezoelectric coefficient, ε is the dielectric coefficient, E is the elastic modulus, A is the cross-section area and L is the length of each part of the elements. The subscripts 1, 2 and 3 correspond to the left metallic bar, the piezoelectric stack and the right metallic bar, respectively; and k_1 and k_3 are defined similarly to k_2 in Eq. (2a).

Figure 1. Piezoelectric active member

Assembling the equations of motion of the truss structure with piezoelectric and passive members in global coordinates, the global equation of motion for a smart truss structure is given by:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f} - \mathbf{Q}\mathbf{v} \tag{3}$$

where $\mathbf{q}$, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ are the displacement, velocity and acceleration vectors, respectively; **M**, **D** and **K** are the mass, the damping, and the stiffness matrices, respectively; **Q** is the control coefficient matrix that relates the actuator control force to the corresponding nodal degree of freedom; **v** is the vector of applied voltages to the actuators, and **f** is the force vector.

For control system analysis and design purposes, it is convenient to represent the flexible structure equations in state-space form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B_1}\mathbf{w} + \mathbf{B_2}\mathbf{u} \tag{4a}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} \tag{4b}$$

where **A** is the dynamic matrix, $\mathbf{B_1}$ is the disturbance input matrix, $\mathbf{B_2}$ is the control input matrix, **C** is the output matrix, **w** is the disturbance input vector, **u** is the control input vector, and **y** is the output vector.

The state vector **x** of modal coordinates consists of n independent components, $\mathbf{x}_i$, that represent the state of each mode and are given by:

$$\mathbf{x_i} = \begin{Bmatrix} q_i \\ \dot{q}_i \end{Bmatrix} \tag{5}$$

where q_i is the ith modal displacement and $\dot{q}_i$ is the ith modal velocity.

The modal state-space realization is characterized by the block-diagonal dynamic matrix and the related input and output matrices:

$$\mathbf{A} = \mathrm{blockdiag}(\mathbf{A_{mi}}),\quad \mathbf{B} = \begin{bmatrix} \vdots \\ \mathbf{B_{mi}} \\ \vdots \end{bmatrix},\quad \mathbf{C} = \begin{bmatrix} \cdots & \mathbf{C_{mi}} & \cdots \end{bmatrix},\ \text{where } \mathbf{A_{mi}} = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\zeta_i\omega_i \end{bmatrix} \tag{6}$$

where ζ_i is the ith modal damping ratio, ω_i is the ith natural frequency, and the subscript $(.)_{mi}$ is relative to the ith mode. $\mathbf{A}_{mi}$ is one possible form of the block-diagonal dynamic matrix, more information about this topic can be found in Gawronski (1998).

The order of the state-space representation is generally very large, causing numerical difficulties. Therefore, the determination of a low order model is fundamental for successful implementation of a controller. A reduced-order model can be obtained by truncating the states. Let $\mathbf{x}$ and the state ($\mathbf{A}$, $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{C}$) be partitioned considering the canonical modal decomposition.

$$\begin{Bmatrix} \dot{\mathbf{x}}_\mathbf{c} \\ \dot{\mathbf{x}}_\mathbf{r} \end{Bmatrix} = \begin{bmatrix} \mathbf{A_c} & \mathbf{0} \\ \mathbf{0} & \mathbf{A_r} \end{bmatrix} \begin{Bmatrix} \mathbf{x_c} \\ \mathbf{x_r} \end{Bmatrix} + \begin{bmatrix} \mathbf{B_{1c}} \\ \mathbf{B_{1r}} \end{bmatrix} \mathbf{w} + \begin{bmatrix} \mathbf{B_{2c}} \\ \mathbf{B_{2r}} \end{bmatrix} \mathbf{u} \tag{7a}$$

$$\mathbf{y} = \begin{bmatrix} \mathbf{C_c} & \mathbf{C_r} \end{bmatrix} \begin{Bmatrix} \mathbf{x_c} \\ \mathbf{x_r} \end{Bmatrix} \tag{7b}$$

where the subscripts $(.)_c$ and $(.)_r$ are for controlled modes and residual modes, respectively.

3. Independent modal space control (IMSC)

As mentioned previously the design of a structural control system is usually carried out based for the first few modes only. The control force vector in physical space is given by, (Jianhu, 1990):

$$\mathbf{V} = \mathbf{L_c u}, \quad \text{where} \quad \mathbf{L_c} = \left(\mathbf{L^T L}\right)^{-1} \mathbf{L^T} \tag{8}$$

where $\mathbf{L} = \mathbf{M_m^{-1}} \boldsymbol{\varphi}^\mathbf{T} \mathbf{Q}$, $\mathbf{M_m}$ is the modal mass matrix, $\boldsymbol{\varphi}$ is the modal matrix and $(.)^\mathbf{T}$ denotes the transpose of the matrix. The feedback control parameters of displacement and velocity gains, are selected as modal gains. The modal feedback force in each mode u_i is dependent on q_i and $\dot{q}_i$ alone:

$$u_i = -G_{di} q_i - G_{vi} \dot{q}_i \tag{9}$$

where G_{di} and G_{vi} are the displacement and velocity modal gains, respectively.

This procedure avoids re-coupling of modal equations through feedback. Thus, an independent controller can be designed for each mode. The modal control forces u_i are determined using optimal control theory, which determines feedback control gains by minimizing a quadratic performance index J_i given by:

$$J_i = \frac{1}{2} \int_0^\infty \left(\mathbf{x_i^T Q_i x_i} + R u_i^2 \right) dt = \frac{1}{2} \int_0^\infty \left(\omega_i^2 q_i^2 + \dot{q}_i^2 + R u_i^2 \right) dt \tag{10}$$

where the diagonal matrix $\mathbf{Q}_\mathrm{i}$ is the weighting matrix for the ith modal state vector; R is the weighting factor of the ith modal control force, i.e., the importance of reducing the system energy with respect to the required control effort. The structural energy is used frequently to define a unique state-weighting matrix $\mathbf{Q}_\mathrm{i}$. When $\mathbf{Q_i} = \mathrm{diag}\left(\omega_i^2 \;\; 1\right)$, the performance index is related with the sum of the potential energy $\omega_i^2 q_i^2$ and the kinetic energy $\dot{q}_i^2$ of the vibrating system as well as the required input control effort u_i^2.

The *ith* modal control force can be determined by:

$$u_i = -R^{-1} \mathbf{B}_{\mathbf{2}mi}^T \mathbf{P}_i \mathbf{x}_i \tag{11}$$

where $\mathbf{P}_i$ is the semi-definite symmetric matrix, and it is governed by the Riccati equation (Xu and Jiang, 2004):

$$\mathbf{P}_i \mathbf{A}_{mi} + \mathbf{A}_{mi}^T \mathbf{P}_i + \mathbf{Q}_i - \mathbf{P}_i \mathbf{B}_{2mi} R^{-1} \mathbf{B}_{2mi}^{-1} \mathbf{P}_i = 0 \tag{12}$$

Using the feedback gains from Eq. (9), the closed-loop state space equation corresponding to the ith mode is described by:

$$\dot{\mathbf{x}}_{\mathbf{i}} = \mathbf{A}'_{\mathbf{mi}}\mathbf{x}_{\mathbf{i}}, \quad \text{where} \quad \mathbf{A}'_{\mathbf{mi}} = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 - G_{di} & -2\zeta_i\omega_i - G_{vi} \end{bmatrix} \tag{13}$$

This equation can be solved to determine the control response of the system in any mode. A recent study (Fang et al. 2003) shows that the modal control forces may increase the amplitudes of higher modes of vibration (uncontrolled modes) if the IMSC algorithm is used to design a structural control of a multi-degree-of-freedom system. Therefore, the responses of the controlled structure may be underestimated if the effects of control forces on higher modes are not considered in the response analysis.

4. Efficient modal control (EMC)

When controlling a vibrating system using modal feedback gains selected according to the IMSC algorithm, where a number of modes are excited, the physically applied voltages can become large (Singh et al. 2003). The information about the uncontrolled system response can be utilized to tailor the control forces. The objective is to reduce the amplitude of vibration to some acceptable level by the application of the smallest control force in a minimum period of time. It can be observed from the results of IMSC that optimal feedback gains are higher for higher modes of vibration, but the amplitude of vibration is generally low in these modes.

If the criterion of an acceptable level of vibration is set as the goal, the time taken to achieve this amplitude depends upon the initial amplitude of vibration for the same value of damping. Hence, when controlling a number of modes simultaneously, a reduced control force can be applied to modes that have smaller vibration amplitudes. Such a technique is proposed as an EMC strategy, in which modal feedback gains are weighted according to relative modal displacement in that mode.

Here, it is important to note that the EMC method, just like any modal control method, requires modal quantities to be measured. In the case of free vibration, each mode vibrates at its own frequency, and the total response is a combination of all the modes excited. Frequency filters can be used to separate modal displacements. In forced vibrations the response of each mode can be estimated using an observer (Singh et al. 2003).

The control force can be weighted according to displacement in each mode, so, the Fourier Transform of the time response of the uncontrolled system is taken to estimate these. The ratio of the amplitudes in different modes is calculated with respect to the mode having maximum amplitude. The feedback gains in modes having smaller amplitudes are reduced by their respective ratios. Thus, if the feedback control gains are to be applied for controlling *i*th, *j*th and *k*th modes, the gain ratios are set by:

$$G_r(i) : G_r(j) : G_r(k) = 1 : \frac{displacement(j)}{displacement(i)} : \frac{displacement(k)}{displacement(i)} \tag{14}$$

where $G_r(i)$ is the gain ratio of the mode i.

5. Numerical Simulation

In the numerical simulation presented here the control design of a 2-bay truss structure with 31 bars in Fig. 2 is considered. Nodes 1 to 4 are clamped. The materials properties and dimensions are given in Tab. 1. The tubes are made of steel with diameter of 6 mm and for each node there is a centralized mass block of 0.3 kg. The piezoelectric active member is made of 150 circular piezoelectric patches. It is considered that the damping is proportional to stiffness and mass matrices ($\mathbf{D}=10^{-6}.\mathbf{K}+10^{-4}.\mathbf{M}$). The aim of these numerical simulations is to demonstrate the applicability and the theory described in the previous sections. The state-space structural model was obtained through a program developed in Matlab® using the finite element method (FEM).

Table 1. Material properties and dimensions of the smart truss structure.

	Structure	Active Member
Elastic modulus (GPa)	210	52.6
Mass density (kg/m^3)	7800	7600
Cross-section area (m^2)	2.83 x 10^{-5}	5.65 x 10^{-5}
L (m)	0.25	0.15
Piezoelectric coefficient (N/V.m)	---	44
Dielectric coefficient (F/m)	---	2.12 x 10^{-8}

Each node has three degrees of freedom, translation in *x*, *y* and *z* direction, so, the truss structure has 24 active dofs, and the model in the form of state-space results in an order of 48. The control design to attenuate the vibration considers only the first six modes (12 states) of the structure. The eighteen remaining modes (36 states) are considered as residual modes in the model. In this application one "PZT wafer stack" and two sensors were used. The placement of actuator and sensors is shown in Fig. 2.

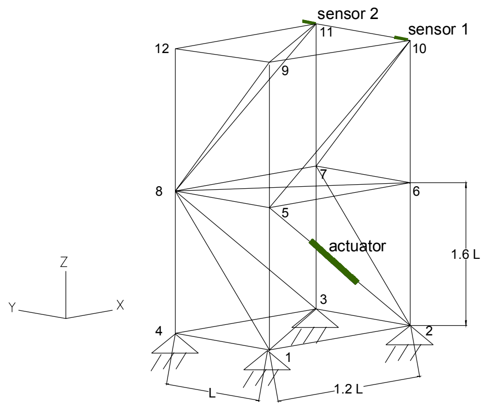

Figure 2. Truss structure showing the actuator and sensor location

The optimal placement of active members with the objective of controlling the first and third modes of this structure, using one actuator and two sensors, was developed using the H_2 norm for each sensor/actuator candidate position. This methodology is described in Carvalhal et al. (2005). These positions correspond to a situation where the actuator and sensors couple into and sense the motion to be controlled effectively.

Using Eq. (12) with $R=10^{-5}$ the feedback gains for the two controlled modes of the structure are given in Tab. 2.

Table 2. Optimal feedback gains for the two controlled modes.

Mode number (i)	G_{di} (10^3)	G_{vi}(10^3)
1	1.24	0.42
3	3.92	0.43

One can apply both modal feedback gains to control two modes of the system, or a single modal feedback gain to control the dominating mode. To achieve simultaneous control of more than one mode, the modal feedback forces are calculated for each individual mode. These feedback forces are converted to physical forces at the same actuator. The physical actuator forces for the different modes are summed to give the resultant force required at the actuator location. This should be done because different modes of the system are linearly independent for a given structure. Each force has its own independent effect. Since the forces are vectorially added and these forces are directly proportional to the voltage to be applied in the actuator, the voltages can be added to give the total voltage to be applied by the actuator.

An impulse load is applied at node 9 in the *y* direction of the structure. This load excites some modes of the system. The uncontrolled response at sensor 1 is given in Fig. 3. The FFT of the uncontrolled response is shown in Fig. 4.

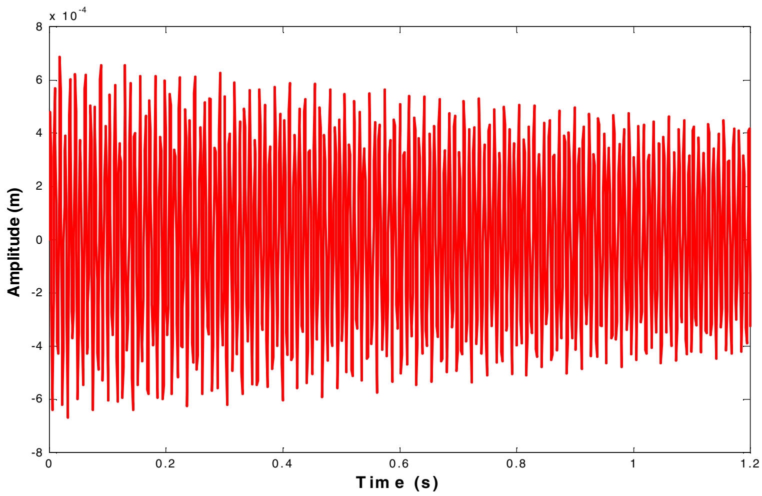

Figure 3. Uncontrolled response for the reduced model with an impulsive load

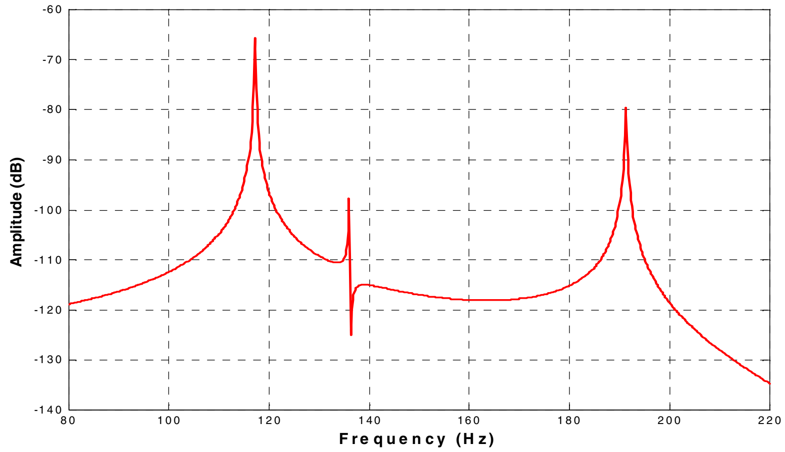

Figure 4. FFT of the uncontrolled response

The control of the first and third modes of the system is achieved using IMSC. Optimal modal feedback gains are applied to the modal displacement and modal velocity in each mode. The physical feedback voltages calculated for each mode are summed and applied to the actuator. Figure 5 shows the controlled response of the truss structure measured with sensor 1. With this configuration and weighting factor, the overall amplitude dies from the initial value of about 6.9 x 10^{-4} (m) to 0.6 x 10^{-4} (m) in about 0.5 s. Figure 6 shows the feedback voltage applied to the actuator. The maximum control voltage requirement is 2200 V.

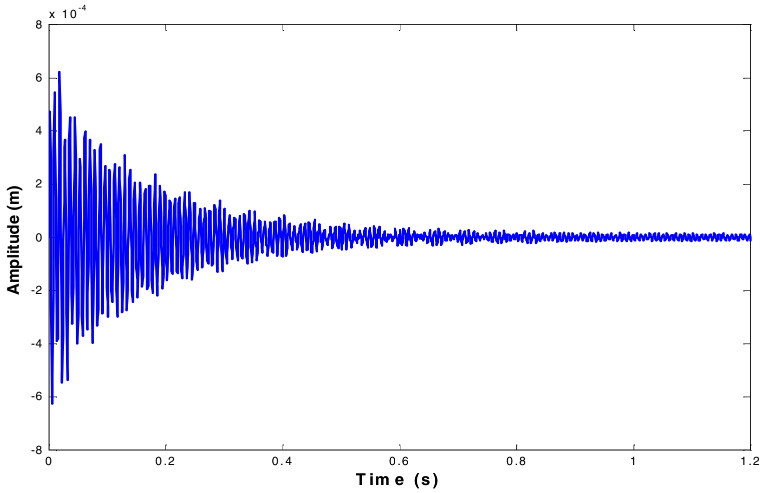

Figure 5. Controlled response for the structure due to feedback force applied according IMSC

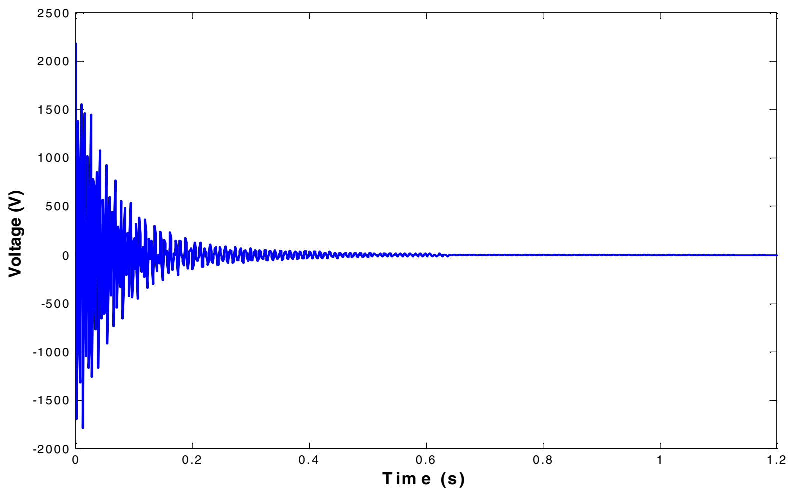

Figure 6. Feedback voltage applied according IMSC

The displacement amplitudes of the two modes (Fig. 4) have a ratio of 1:0.172. Thus, optimal gain in the third mode is reduced by these ratios as proposed in Eq. (15). Figure 7 gives the response of the structure when the same gains are applied with this new consideration. The maximum control voltage of 640 V can be seen in Fig. 8.

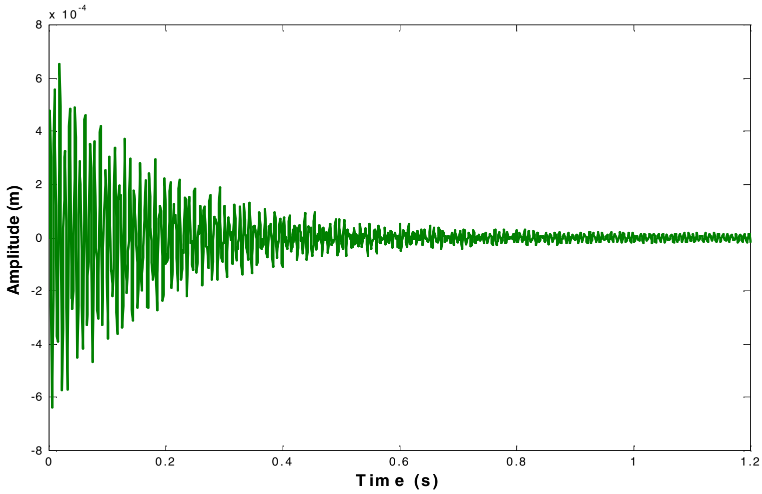

Figure 7. Controlled response for the structure due to feedback force applied according EMC

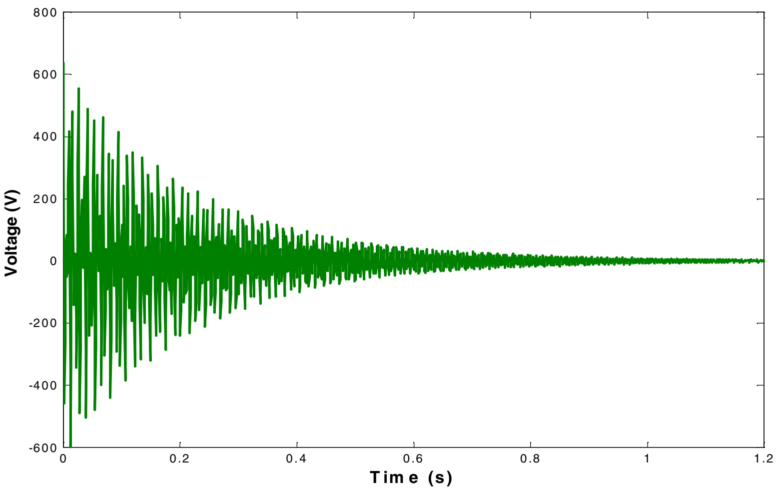

Figure 8. Feedback voltage applied according EMC

The closed-loop response of this structure, for both algorithms IMSC and EMC, in frequency domain is shown in Fig. 9.

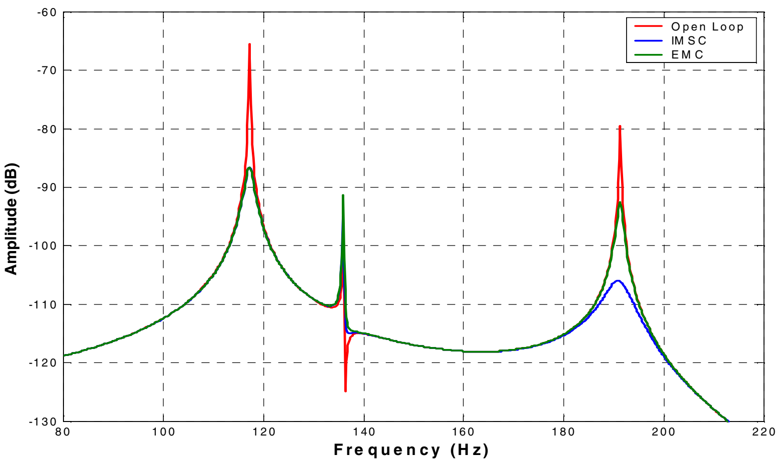

Figure 9. FRFs of the uncontrolled and controlled responses for both algorithms IMSC and EMC

A comparison of Fig. 5 and 7 indicates that EMC gives almost the same settling time as IMSC. The difference lies in the vibration attenuation achieved on third mode, for which there is a much reduced control effort applied. Figures 6 and 8 clearly show that there is a large reduction in the maximum feedback voltage applied in the actuator for EMC. The maximum control voltage required in EMC was 640 V. The same attenuation of 20 dB in the amplitude of the first mode was obtained for both algorithms IMSC and EMC. The third mode was attenuated 26 dB by the IMSC algorithms and 12 dB by the EMC. This shows that for the EMC algorithm the control effort for this mode is much less.

6. Conclusions

In this paper two control algorithms have been compared for modal control of a truss structure; IMSC and EMC. Optimal feedback gains were found to be independent of the kind of applied force for the IMSC algorithm. When feedback was applied to control two modes of vibration, the voltage was found to be very high. This can be much reduced by using the EMC algorithm. In this method the displacement of each mode is used to weight the feedback control force. Comparing these methods for the control of the first and third modes of a space truss structure shows that there is a large reduction in the maximum voltage applied to the actuator in the case of EMC without a significant change in control performance.

7. Acknowledgements

The first two authors would like to acknowledge the financial support from Research Foundation of the State of São Paulo (FAPESP-Brazil). Prof. Brennan acknowledges the support from FAPESP (process number 04/133242-3 – Visiting Professor) and the Leverhulme Trust, UK.

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