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STABILITY AND VIBRATION ANALYSIS OF SLENDER L-FRAMES WITH SEMI-RIGID CONNECTIONS

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Abstract. *The aim of the present work is to study the free and forced non-linear vibration problem of L-frames with semi-rigid connections using a finite element formulation. For this, an efficient non-linear finite element program for the static and dynamic analysis of frames is developed. The equilibrium paths are obtained through continuation techniques together with Newton-Raphson method. The solution of the system of ordinary differential equations of motion is obtained by Newmark implicit numerical integration method together with adaptive strategies for the automatic increment of the time step.*

Keywords*: L- frames, structural stability, semi-rigid connections, vibration analysis, non-linear dynamics.*

1. Introduction

The buckling and post-buckling analysis of frames is an important problem in the design of structures, in particular in the analysis of slender steel frames. It is well known that in many frame structures the analysis of columns or beam-columns as independent members may lead to erroneous results, particularly at large deflections. In some configurations the critical load and in particular the post-buckling behavior is affected by the other members meeting at the ends of the column. In a famous paper, Koiter (1967) showed that L-frames exhibit an asymmetric bifurcation and, as a consequence, its load carrying capacity is affect by imperfections. These results were confirmed experimentally by Roorda (1965). The initial post-buckling behavior of L-frames has been discussed at length by various authors in the past (Koiter, 1967, Roorda and Chilver, 1970, Brush and Almroth, 1975, Bazant and Cedolin, 1991), using asymptotic expansions of either the potential energy or the equilibrium equations governing the non-linear response of the frame-work. These studies are usually concerned with the determination of the initial slope and curvature of the post-buckling response. As shown by Koiter (1967), these two results are enough to characterize the type of bifurcation and can be used to estimate the imperfection sensitivity of the structure. However, these approximate solutions can describe only the initial post-buckling behavior of the frame. The same geometry used by Koiter has also been used in the last two decades by several authors to test the efficiency of several non-linear finite element formulations for planar frames as well as incremental-iterative strategies for the solution of eminently non-linear problems. This interest is due to the highly non-linear response of L-frames under eccentric loads. Nonetheless little is known on the influence of the frame parameters and load and geometric imperfections on the equilibrium and stability behavior of steel frames with flexible connection under static and dynamic loading.

Galvão *et al*. (2005a) conducted a detailed parametric analysis to study the influence of the stiffness of the lateral bracing, boundary conditions as well as load and geometric imperfections on the non-linear response and imperfection sensitivity of this frame. For this an efficient non-linear finite element formulation for the analysis of planar elastic frames was used together with a non-linear solution methodology, which solves the resulting non-linear equations and obtains non-linear equilibrium paths through the Newton-Raphson method together with path-following techniques, such as arc-length schemes (Crisfield, 1991).

The aim of the present work is to conduct a dynamic analysis of L-frames and to study the influence of the stiffness of the connections as well as load and geometric imperfections on the nonlinear response of L-frames. These results provides some insight as to the source and mechanism of asymmetric bifurcation and imperfection sensitivity in some frames and may help engineers to evaluate the importance of the geometrical second order effect and connection flexibility in the analysis of slender frames.

2. The Non-Linear Static Equilibrium and Free Vibration Problems

The non-linear finite element formulation implemented, considering rigid and flexible joints, is based on the works by Yang and Kuo (1994) and Chan and Chui (2000) as implemented by Galvão (2004). The incremental-iterative solution strategy for non-linear static problems adopted in this work is summarized in Fig. 1, where, in the finite element context, the equilibrium of a structural system can be expressed as:

$$\mathbf{F_i}(\mathbf{U}) = \lambda \mathbf{F_r} \tag{1}$$

where $\mathbf{F_i}$ defines a set of generalized internal forces in terms of the corresponding generalized displacement vector **U;** λ is a scalar load multiplier, and $\mathbf{F_r}$ is a fixed load vector (reference vector). To obtain the equilibrium paths of a structure, an incremental solution technique should be used to solve Eq. (1). This is achieved by calculating a sequence of displacement increments $\Delta\mathbf{U}_1$, $\Delta\mathbf{U}_2$, $\Delta\mathbf{U}_3$,... corresponding to a sequence of load parameter increments $\Delta\lambda_1$, $\Delta\lambda_2$, $\Delta\lambda_3$,..... In each increment, however, due to the non-linearities in $\mathbf{F_i}$, the problem must be solved iteratively.

The vibration frequencies and corresponding vibration modes of a pre-loaded structure can be obtained through the solution of the following eigenvalue problem:

$$\left[\left[\mathbf{K}_L + \mathbf{K}_\tau\right] - \omega^2\,\mathbf{M}\right]\boldsymbol{X} = \mathbf{0} \tag{2}$$

where $\mathbf{K}_L$ and $\mathbf{K}_\tau$ are the linear and geometric stiffness matrices, respectively; **M** is the mass matrix, ω is the natural vibration frequency and ***X***, the vibration mode. The numerical procedures for computation of the natural frequencies and mode shapes are also presented in Fig. 1a and 1b.

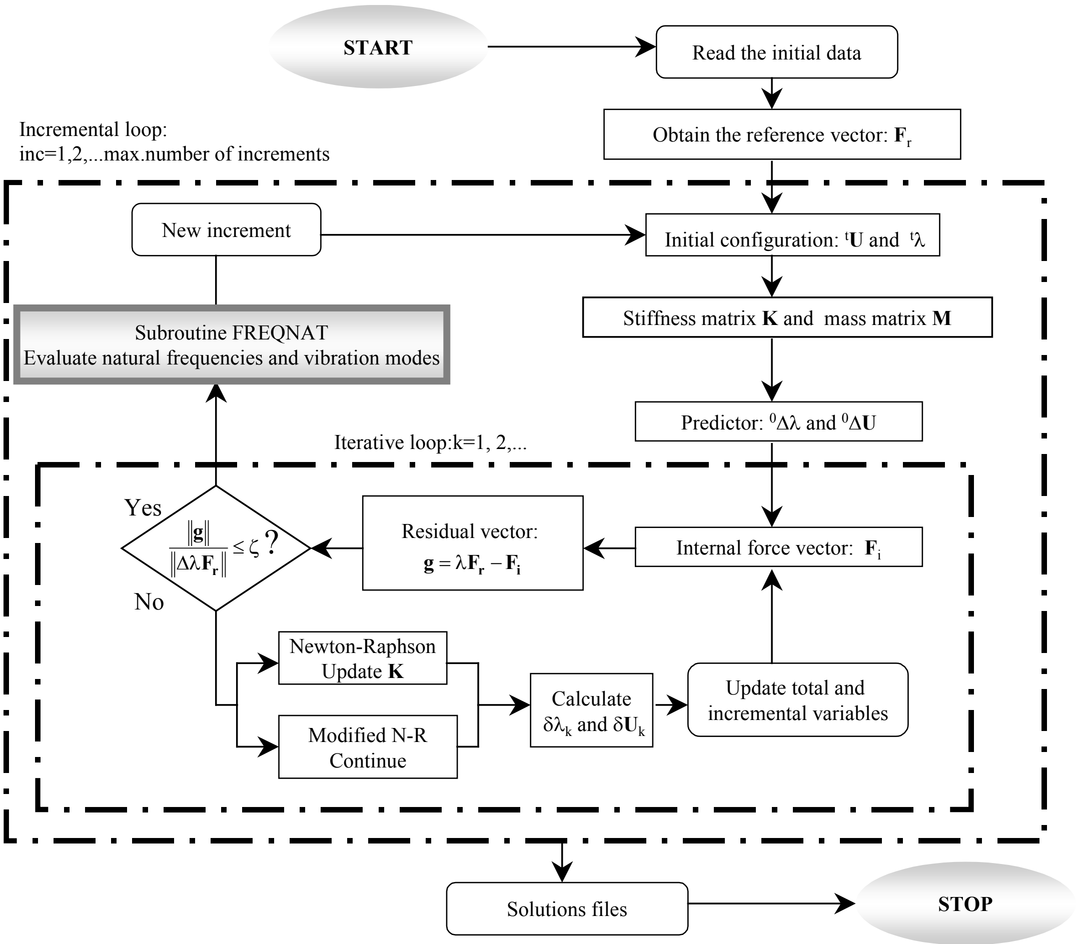
2.1 Numerical example

The numerical strategy used to solve the Eqs. (1) and (2) is now tested in the analysis of the L-frame with flexible joint shown in Fig. 2. In these analyses, the semi-rigid connection is represented by a hybrid element of length L_{sr} = L/1000. Figure 3 shows the non-linear equilibrium paths for increasing values of the joints stiffness Sc. The non-linear relation between the load and the natural frequencies for different values of joint stiffness is given in Fig. 4. This relation not only shows the influence of the static pre-loading on the natural frequencies, but can be used, according to the dynamic stability criteria, to identify stable and unstable branches along the equilibrium paths. Critical conditions are obtained for $\omega^2 = 0$. As observed here, the correct evaluation of the joints stiffness, a key point in the design of metal structures, is essential for the definition of critical conditions. This is particularly important in practical application where damage occur usually at the joints, decreasing their stiffness and changing radically the non-linear behavior of the frame.

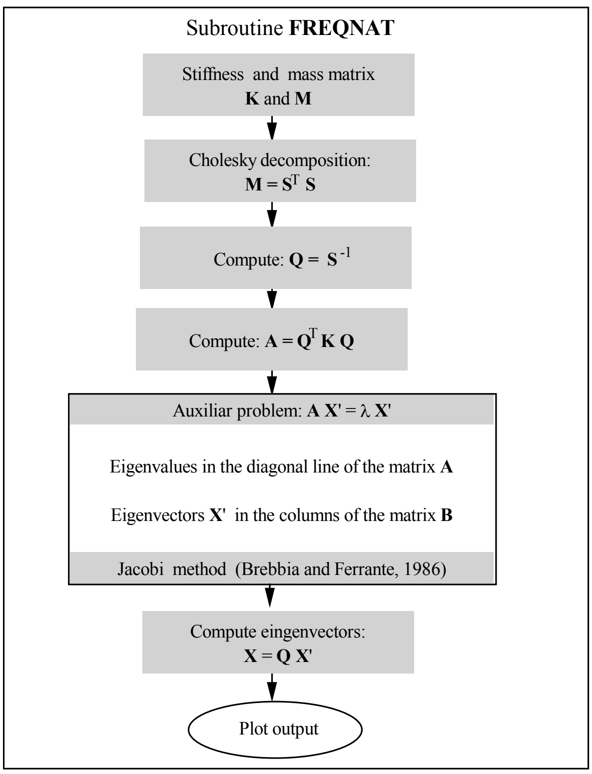
In general, the L-frames usually have a fundamental equilibrium path followed by an asymmetric bifurcation, characterized by an initial slope of the post-critical equilibrium path. Galvão *et al.* (2005a) observed that when the stiffness of the beam-column connection increases the frame critical load increases. Simultaneously, the initial slope of the asymmetric post-critical path increases and the structure becomes more sensitive to initial imperfections. This characteristic can be observed in Fig. 3, where the non-linear equilibrium paths for different connection stiffness parameter Sc are shown. Small perturbations were used in the numerical strategy to obtain the two branches of the post-critical path. Figure 4 shows the variation of the first natural frequency with the load parameter P/P_e for different values of Sc.

Table 1 shows the variation of the first three natural frequencies of the unloaded structure with Sc. When the connection stiffness approaches zero, the joint approaches a perfect hinge, and two first natural frequencies converge to the same value. Among the analyzed frequencies, the second frequency is more influenced by the connection rigidity than the other two. This can be explained by analyzing the three first vibration modes for Sc = 5EI/L and Sc $\cong \propto$ presented in Fig. 5. The variation of the angle between the bars with Sc is larger for the second mode, leading to a larger contribution of the spring to the total stiffness of the system.

The static and dynamic post-critical behavior of the loaded L-frame, with beam-column stiffness connection Sc = 5EI/L, is illustrated in Fig. 6, where the variation of the two first natural frequencies with the load, up to large displacements and rotations is analyzed. The results show that the load has a large influence on both frequencies. Knowing that the regions of the trajectory where the equilibrium is stable all the natural frequencies has real values, and that the corresponding regions with negative values of ω^2 are unstable (natural frequency with imaginary value), the stable and unstable equilibrium configurations can be defined, as illustrated in Figs. 6c and 6d.



(a) Non-linear solution strategy.



(b) Subroutine FREQNAT.

Figure 1. Non-linear static and free vibration solution strategies.

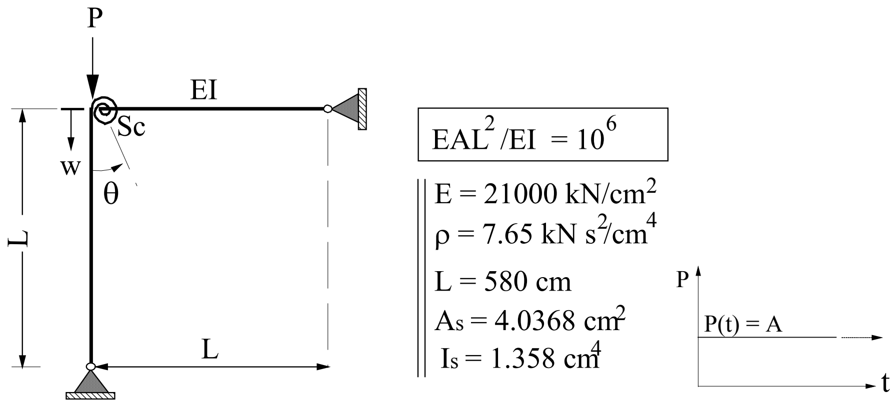


Figure 2. L-frame with semi-rigid connection.

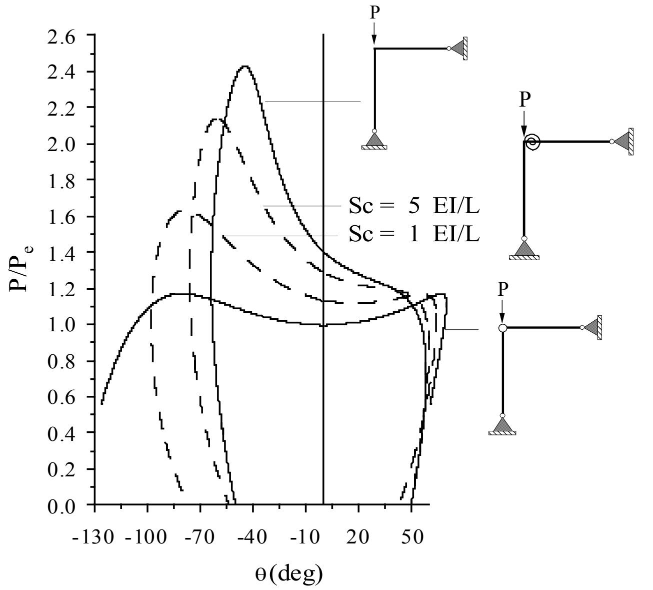


Figure 3. Non-linear equilibrium paths.

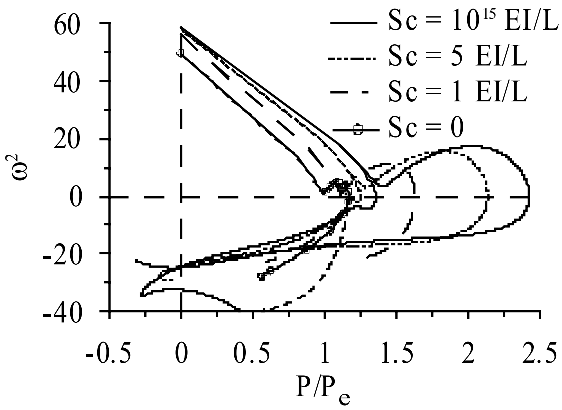
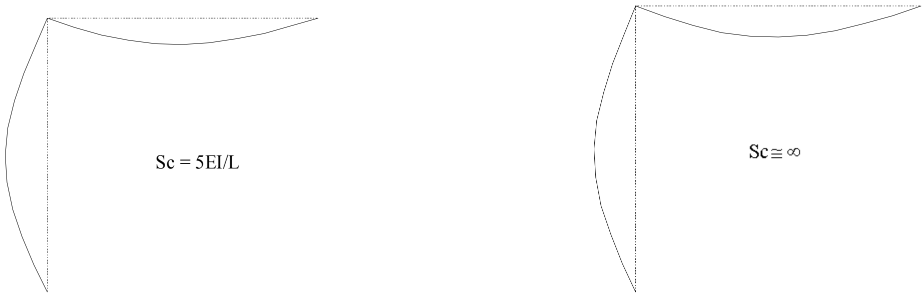


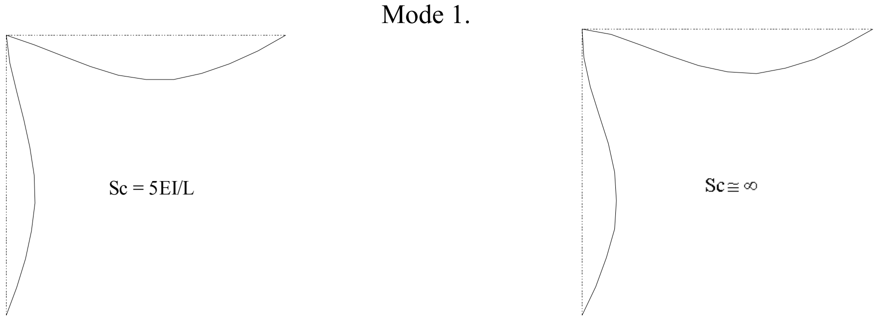
Figure 4. Natural frequency-load relation.

Table 1. Unload L-frame: first three natural frequencies versus stiffness parameter Sc.

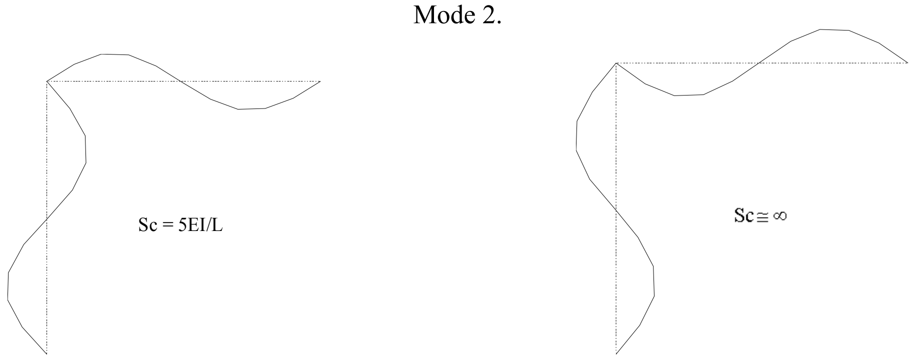
Sc	ω_1	ω_2	ω_3
∞	7.67	13.25	34.56
5 EI/L	7.64	11.53	34.52
1 EI/L	7.49	9.82	34.36
0	7.04	8.92	33.95



Mode 1.



Mode 2.



Mode 3.

Figure 5. L-frame vibration modes for Sc = 5EI/L and $Sc \cong \infty$.

3. The Non-Linear Transient Problem

In the finite element context, the non-linear time response of the structure can be obtained by the solution of the following set of discrete equations of motion:

$$\mathbf{M}\,\ddot{\mathbf{U}} + \mathbf{C}\,\dot{\mathbf{U}} + \mathbf{F_i}(\mathbf{U}) = \lambda(t)\,\mathbf{F_r} \tag{3}$$

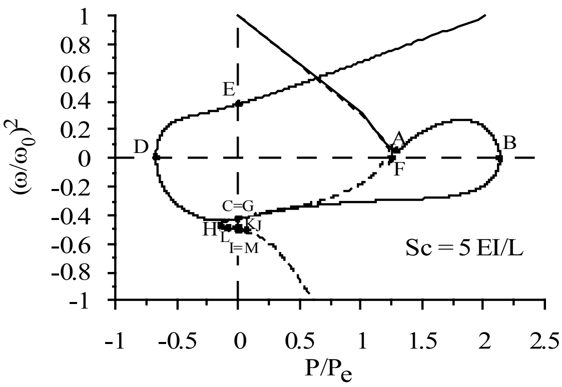
where **M** and **C** are the mass and viscous damping matrices, respectively, and $\mathbf{F_i}$ is the internal force vector which depends on the displacements **U** of the system. Here $\dot{\mathbf{U}}$ and $\ddot{\mathbf{U}}$ are the velocity and acceleration vectors, respectively, and $[\lambda(t)\mathbf{F_r}]$ is the applied load vector. The solution of the non-linear dynamic system (3) can be obtained through a time integration algorithm together with adaptive strategies for the automatic increment of the time step. The numerical methodology used here is presented in Fig. 7. The numerical integration algorithm is based on the work by Dokainish and Subbaraj (1989), while the time increment strategy is based on Jacob (1990). Details of the non-linear dynamic formulation as well as the computational program are presented in Galvão (2004).

3.1. Numerical example

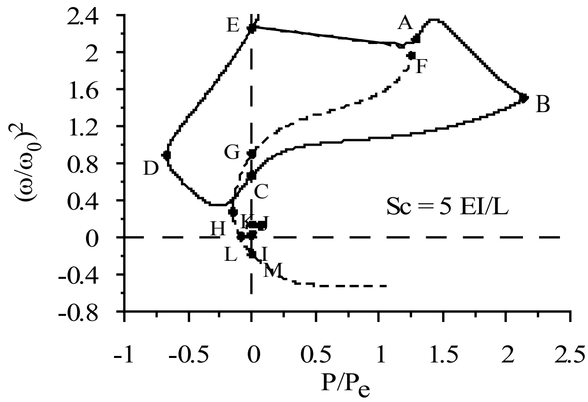
Consider now the same L-frame given in Fig. 2, and a viscous damping matrix proportional to the mass and stiffness matrices, $\mathbf{C} = \text{am}\,\mathbf{M} + \text{ak}\,\mathbf{K}$, defined by Rayleigh coefficients, am and ak, which are calculated here based on the critical damping coefficient ξ = 0,4. The frame behavior under a suddenly applied step load of infinite duration is analyzed. Figure 8 and 9 exhibits the transient L-frame response under different load levels. The frame exhibits large amplitude vibrations during the transient phase and converges to the static configuration corresponding to the applied load level.

4. Conclusions

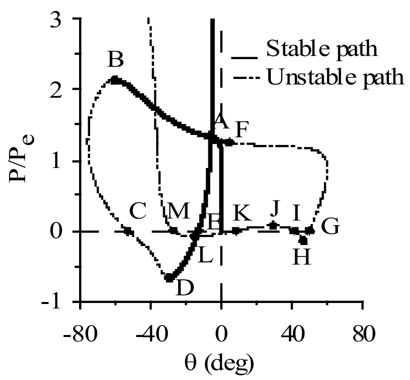
The results presented in this paper for a highly non-linear structural system indicates that the proposed numerical methodologies can be efficiently used to obtain the non-linear static and dynamic responses of slender structural systems with rigid and flexible joints. The results for the L-frame indicate that the precise evaluation of the stiffness of the joints is essential for the calculation of critical conditions. The results also indicate that the loss of stiffness of a joint during the service life of the structure may affect significantly the behavior of the frame under both static and dynamic loads. This is in accordance with the technical literature where structural failures due to support deterioration are reported. The results also show the importance of the consideration of static pre-loading on the vibration characteristics of structural elements liable to buckling.



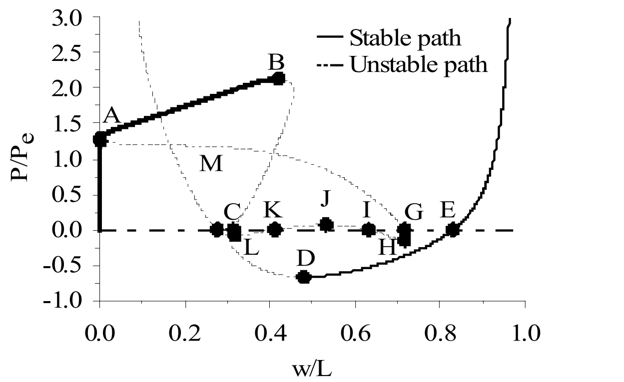
(a) First natural frequency-load relation.



(b) Second natural frequency-load relation.



(c) Non-linear equilibrium path. Displacement θ.



(d) Non-linear equilibrium path. Displacement w.

Figure 6. Frequency-load relations and non-linear equilibrium paths of the L-frame.

a. Initialize ${}^{0}\mathbf{U}$, ${}^{0}\dot{\mathbf{U}}$ and ${}^{0}\ddot{\mathbf{U}}$ and form the matrix $\mathbf{M}$;

b. For each time step Δt:

1. Build matrices $\mathbf{K}$ and $\mathbf{C}$;
2. Calculate the constants:

$$a0 = \frac{1}{\beta \Delta t^2}\,,\ a1 = \frac{\gamma}{\beta \Delta t}\,,\ a2 = \frac{1}{\beta \Delta t}\,,\ a3 = \left(\frac{1}{2\beta \Delta t} - 1\right),\ a4 = \frac{\gamma}{\beta} - 1\,,\ a5 = \frac{\Delta t}{2}\left(\frac{\gamma}{\beta} - 2\right),$$

$$a6 = a0\,,\ a7 = -a2\,,\ a8 = -a3\,,\ a9 = \Delta t(1-\gamma),\ a10 = \Delta t\gamma\,.$$

3. Build the effective stiffness matrix: $\hat{\mathbf{K}} = \mathbf{K} + a0\,\mathbf{M} + a1\,\mathbf{C}$
4. Build the effective load vector: $\hat{\mathbf{F}} = {}^{t+\Delta t}\lambda \mathbf{F_r} + \mathbf{M}\left(a2\ {}^{t}\dot{\mathbf{U}} + a3\ {}^{t}\ddot{\mathbf{U}}\right) + \mathbf{C}\left(a4\ {}^{t}\dot{\mathbf{U}} + a5\ {}^{t}\ddot{\mathbf{U}}\right) - {}^{t}\mathbf{F_i}$
5. Solve for displacement increments: $\hat{\mathbf{K}}\ \Delta\mathbf{U} = \hat{\mathbf{F}}$
6. Iterate for dynamic equilibrium: k = 1, 2, ...

   i. Evaluate the approximation of the acceleration, velocities and displacements:

   $${}^{t+\Delta t}\ddot{\mathbf{U}}_k = a0\,\Delta\mathbf{U}_k - a2\ {}^{t}\dot{\mathbf{U}} - a3\ {}^{t}\ddot{\mathbf{U}}\ ;\ {}^{t+\Delta t}\dot{\mathbf{U}}_k = a1\,\Delta\mathbf{U}_k - a4\ {}^{t}\dot{\mathbf{U}} - a5\ {}^{t}\ddot{\mathbf{U}}\ ;\ {}^{t+\Delta t}\mathbf{U}_k = {}^{t}\mathbf{U} + \Delta\mathbf{U}_k$$

   ii. Evaluate the internal forces vector: ${}^{t+\Delta t}\mathbf{F_i}_{k+1} = {}^{t}\mathbf{F_i} + \mathbf{K}\ \Delta\mathbf{U}_{k+1}$

   iii. Evaluate the residual vector: ${}^{t+\Delta t}\mathbf{R}_{k+1} = {}^{t+\Delta t}\lambda\mathbf{F_r} - \left(\mathbf{M}\ {}^{t+\Delta t}\ddot{\mathbf{U}}_k + \mathbf{C}\ {}^{t}\dot{\mathbf{U}}_k + {}^{t+\Delta t}\mathbf{F_i}_k\right)$

   iv. Solve for the corrected displacement increments: $\hat{\mathbf{K}}\ \boldsymbol{\delta}\mathbf{U}_{k+1} = {}^{t+\Delta t}\mathbf{R}_{k+1}$

   v. Evaluate the corrected displacement increments: $\Delta\mathbf{U}_{k+1} = \Delta\mathbf{U}_k + \boldsymbol{\delta}\mathbf{U}_{k+1}$

   vi. Check the convergence of the iteration process:

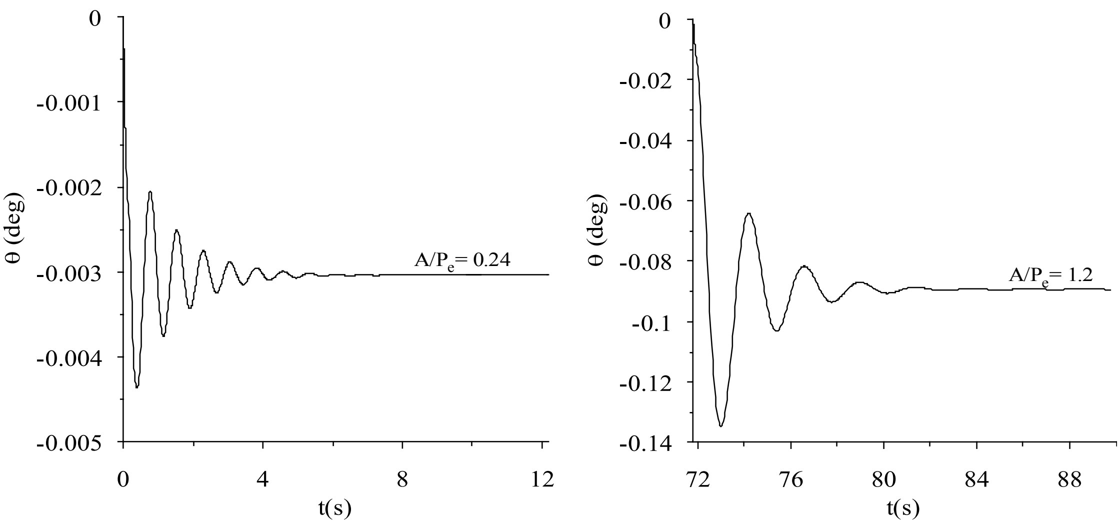
   $$\frac{\left|\Delta\mathbf{U}_{k+1}\right|}{\left|{}^{t}\mathbf{U} + \Delta\mathbf{U}_{k+1}\right|} \le \text{tolerance factor?} \begin{cases} \text{No} \rightarrow \text{Go to } 6 \\ \text{Yes} \rightarrow \text{Continue} \end{cases}$$

   vii. Evaluate the acceleration, velocities and displacements at t+Δt.

c. For the next step:

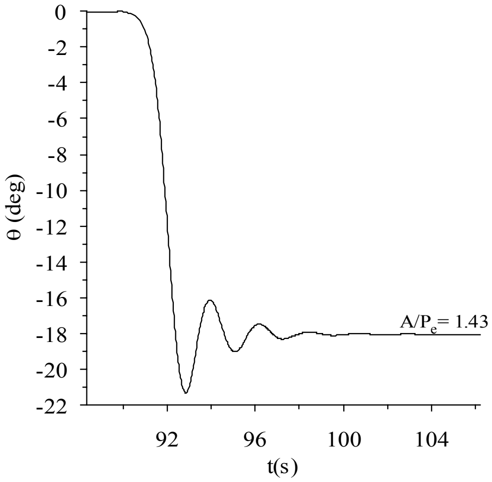
1. Evaluate the internal forces vector ${}^{t+\Delta t}\mathbf{F_i} = {}^{t}\mathbf{F_i} + \mathbf{K}\ \Delta\mathbf{U}$
2. Selects a new time step Δt (adaptative strategy) and return to b.

Figure 7. Non-linear dynamic problem solution algorithm.

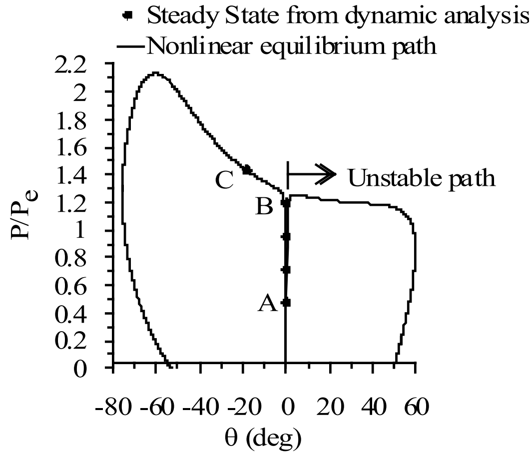


(a) Point A. (b) Point B.

Figure 8. Time response of the L-frame for increasing load levels: initial path.



(a) Point C.



(b) Non-linear static and steady state dynamic response.

Figure 9. Time response of the L-frame for increasing load levels: secondary path.

5. Acknowledgements

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