NUMERICAL STUDY OF A SELF STANDING RISER SYSTEM

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Abstract. *Nowadays, many of offshore petroleum reservoir discoveries are located in deep water depths. Sometimes, they are placed in ultra deep water depths, over 2500 meters. To succeed to field developments in this scenario, new concepts for exporting and producing risers are demanded. The current paper presents a basic study of a riser tower system (or self standing riser system). Fundamentals of the riser tower dynamics are introduced and numerical simulation developments are presented. Special attention is pointed out for the vortex induced vibration (VIV), and for the combined effect of current and wave which are analyzed for the vertical part of the riser system coupled to a subsurface buoy. Numerical calculations for different environmental conditions are presented and discussions are carried out.*

Keywords*: Production Risers, Offshore Systems, Hydrodynamics, Ocean Waves*

1. Introduction

A Self Standing Hybrid Riser System (SSHR) is basically composed by three components (Hatton, 2002). The first component is a long vertical top tensioned steel pipe riser connected to a wellhead at the sea bottom, and to a subsurface buoy at the riser upper end. The second component is a subsurface buoy and the third one is a flexible jumper as depicted in Figure 1. The floatation of the subsurface buoy gives, with the flexible jumper upward tension component at the gooseneck, the necessary top tension for the riser to stand in vertical configuration. The flexible jumper is connected to a floating production unit at its upper end, and its bottom end is connected to the subsurface buoy throughout the gooseneck.

This riser system composed by a slender cylindrical steel pipe is responsible for transporting produced oil and gas from a petroleum well or a set of wells collected in a seabed placed manifold to a sea surface production facility. Sometimes, this riser system concept is applied for exporting produced oil from a floating production unit to a offloading system by an oil transfer operation.

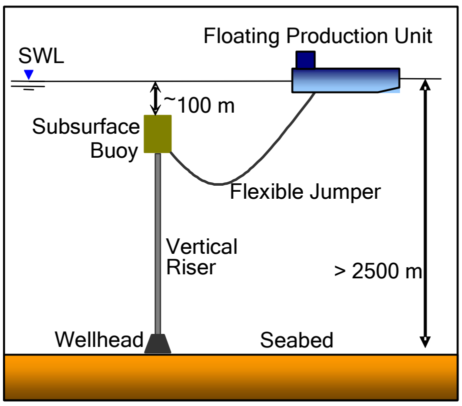


Figure 1. Self Standing Hybrid Riser System.

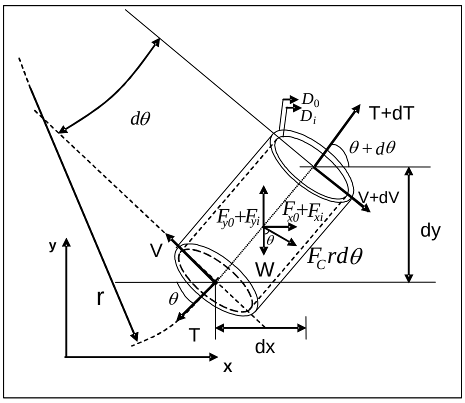


Figure 2. External and internal load in a riser element equilibrium.

The present study considers only the behavior of the second and third part of SSHR, approaching a consideration concerning the vertical rigid riser and the subsurface buoy, as the scheme in Figure 1.

Finally, in the analysis of this study, different current profiles and incident ocean waves with different periods are taken into account and discussions are carried out. There are many previous studies considering this type of load in rigid risers in ultra deep water that can be taken in comparison with this work (Nishimoto, 1989 and Morooka, 2004). The present work is based on a numerical code developed for the dynamics of the top tensioned vertical production riser considering in line and transverse effects due to waves, currents and VIV respectively adapted for this riser tower case. Previous results using this code are presented in previous works (Martins, 2003 and Coelho, 2004).

2. Statics of the Vertical Riser

In general, the vertical steel riser pipe of the SSHR system is usually submitted to external and internal loads of hydrostatic pressure, external environmental loads such as current and waves, and induced loads from other components of the system. Figure 2 shows, schematically, the resulting loads acting in a riser element in equilibrium position. If riser behavior is considered as an Euler Bernoulli beam (Chakrabarti, 1987), the differential equation that describes the static behavior of the vertical riser under action of static current, which loads F_C in horizontal direction (x), can be written as follows:

$$\frac{d^2}{dy^2}\left(EI\frac{d^2x}{dy^2}\right)-(T+p_0A_0-p_iA_i)\frac{d^2x}{dy^2}=F_C+(\gamma_sA_s+\gamma_iA_i-\gamma_0A_0)\frac{dx}{dy} \quad (1)$$

where, p_0 is the external hydrostatic pressure around the riser, p_i is the internal hydrostatic pressure, A_0 is the cross-sectional area of riser, A_i is the internal cross-sectional area of the riser and A_S is the cross-sectional area of riser wall, γ_i is the fluid specific weight in the riser, γ_0 is the specific weight of fluid surrounding the riser (sea water) and γ_S is the specific weight of riser wall (steel).

For instance, if the riser weight is neglected { $(\gamma_0 A_0 - \gamma_i A_i - \gamma_s A_s) = 0$ }, the distribution of axial riser tension along its length is taken as a constant { $(T + p_0 A_0 - p_i A_i) = T$ =Constant}; and the riser diameter, the flexural stiffness (EI) and the external current load (F_C) in the riser are all considered constant, the analytical solution for the Equation (1) can be derived straightforwardly.

The, Equation (1) becomes as the following:

$$\frac{d^4x}{dy^4}-\left(\frac{T}{EI}\right)\frac{d^2x}{dy^2}=\frac{F_C}{EI} \quad (2)$$

In the solution of Eq. (2), boundary conditions for the riser are taken as fixed beam at its bottom end, and free to move at the top end. Then, the analytical solution for Eq. (2) can be obtained as:

$$x=Ce^{ny}+De^{-ny}-\frac{F_C}{2EI}\left(\frac{y^2}{n^2}+\frac{2}{n^4}\right)-\frac{A}{n^2}y-\frac{B}{n^2} \quad (3)$$

where, $n^2 = T/EI$, L is the riser length, $A=-\frac{F_C L}{EI}$, $B=\frac{F_C}{EIn^2}\left[\frac{2+e^{nL}(Ln-1)-e^{-nL}(Ln+1)}{e^{nL}+e^{-nL}}\right]$, $C=\frac{F_C}{EIn^4}\left[\frac{1-nLe^{-nL}}{e^{nL}+e^{-nL}}\right]$, and $D=\frac{F_C}{EIn^4}\left[\frac{1+Lne^{nL}}{e^{nL}+e^{-nL}}\right]$.

2.1. Statics of the Vertical Riser with the Subsurface Buoy

From analogous procedure, considering the riser as a vertical beam and the subsurface buoy as a differentiated element of the riser with a large flexural stiffness, displacements for the riser and for the buoy comes to be, respectively, as follows:

$$x_1=C_1e^{n_1y}+D_1e^{-n_1y}-\frac{F_{C1}}{2E_1I_1}\left(\frac{y^2}{n_1^2}+\frac{2}{n_1^4}\right)-\frac{A_1}{n_1^2}y-\frac{B_1}{n_1^2} \quad (4)$$

$$x_2=C_2e^{n_2y}+D_2e^{-n_2y}-\frac{F_{C2}}{2E_2I_2}\left(\frac{y^2}{n_2^2}+\frac{2}{n_2^4}\right)-\frac{A_2}{n_2^2}y-\frac{B_2}{n_2^2} \quad (5)$$

where x_1 and x_2 are the displacements of riser and buoyancy, respectively, and $n_1^{\,2} = T / E_1 I_2$, $n_2^{\,2} = T / E_2 I_2$. Applying boundary conditions and imposing continuity between riser and buoy, parameters $A_{1(2)}$, $B_{1(2)}$, $C_{1(2)}$ and $D_{1(2)}$ in the Eq. (4) and (5), respectively, can be determined (Appendix A). Figure 3 shows a scheme for riser with buoy.

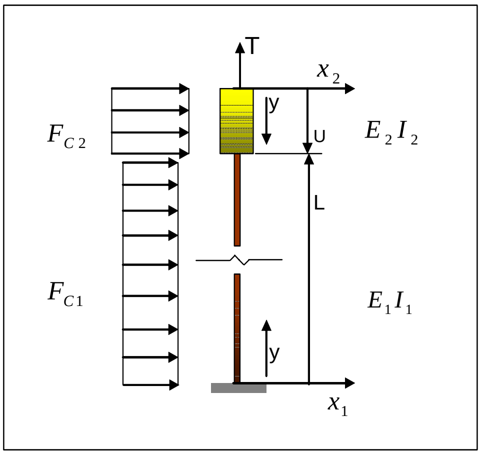


Figure 3. Riser and buoy configurations

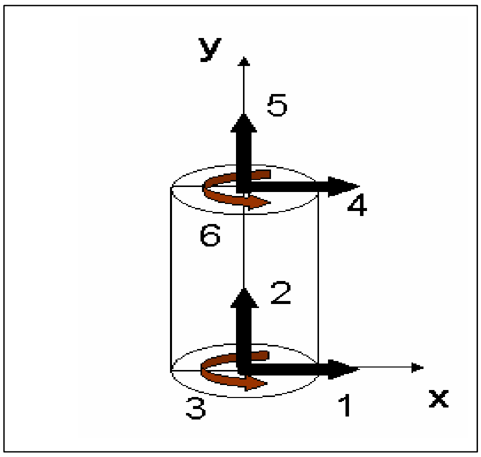


Figure 4. Degrees of freedom

Besides the analytical calculations, numerical calculations have been carried out in the present study. Numerical simulations have been performed by a computer code based in finite element method (Patel, 1989). In these calculations, each element of the riser with six degrees of freedom is taken into account, as illustrated in Fig. 4.

3. Dynamics of the Vertical Riser

3.1. Riser Free Motion Modes

As in the previous description, in the present section, analytical solutions for riser free motion with uniform cross-section and a constant axial tension along the riser length is presented.

In this case, the behavior of free riser motion can be described by the following Eq. (6):

$$EI\frac{\partial^4 x}{\partial y^4} - T\frac{\partial^2 x}{\partial y^2} + \rho A\frac{\partial^2 x}{\partial t^2} = 0 \tag{6}$$

and the analytical solution for Eq. (6) obtained from (Rivin, 1999) is as shown bellow:

$$X = C_1 senh(r_1 y) + C_2 \cosh(r_1 y) + C_3 sen(r_2 y) + C_4 \cos(r_2 y) \tag{7}$$

If the riser is considered fixed at its bottom end and free to move at its top end, it can be inferred that $C_1 = -\frac{r_2}{r_1} C_3$; $C_2 = -C_4$; $C_3 = qC_4$, and $q = \frac{r_2(r_2^2 + n^2) sen(r_2 L) - r_1(r_1^2 - n^2) senh(r_1 L)}{r_2\left[(r_1^2 - n^2)\cosh(r_1 L) + (r_2^2 + n^2)\cos(r_2 L)\right]}$. The analytical solution now has the following form:

$$X = C_4 q\left[\frac{\cos(r_2 y)}{q} + sen(r_2 y) - \frac{1}{q}\cosh(r_1 y) - \frac{r_2}{r_1} senh(r_1 y)\right] \tag{8}$$

where, $r_1 = \sqrt{\frac{n^2}{2} + \sqrt{\frac{n^4}{4} + k^4}}$; $r_2 = \sqrt{-\frac{n^2}{2} + \sqrt{\frac{n^4}{4} + k^4}}$; $k = \sqrt[4]{\frac{\rho A^* \omega^2}{EI}}$; $n^2 = T / EI$.

In the constants above, ρA^* is the addition of the masses that intervene together with the additional mass. Then, the characteristic equation comes to be as follows:

$$-\left[r_2^2+r_1^4+n^2(r_2^2-r_1^2)\right]+r_1r_2(r_1^2-r_2^2-2n^2)senh(r_1L)sen(r_2L)-\left[2r_1^2r_2^2+n^2(r_1^2-r_2^2)\right]\cos(r_2L)\cosh(r_1L)=0 \quad (9)$$

Equation (9) is a function of riser motion frequency only. Then, natural frequencies ω of the riser can be obtained throughout an iterative procedure.

The numerical solution for riser free motions can be obtained according to Eq. (10), in matrix form:

$$[M]\{\ddot{x}(t)\}+[K]\{x(t)\}=0 \quad (10)$$

where, $[M]$ is the mass matrix, $[K]$ is the stiffness matrix, $x(t)$ is harmonic solutions such as $\{x\}=\{x_0\}\cos(\omega.t-\beta)$, and β is the riser motion phase angle. For this case, the characteristic equation to be solved becomes:

$$det\left([K]-\omega^2[M]\right)=\left|[K]-\omega^2[M]\right|=0 \quad (11)$$

3.2. Equation for Riser Dynamic Behavior

Equations for riser in line and transverse dynamic behavior are described in this section. The riser motion behavior for the in line and transverse directions can be written as follows:

$$[M]\{\ddot{x}\}+[B]\{\dot{x}\}+[K]\{x\}=\{F\}_x \quad (12)$$

$$[M]\{\ddot{y}\}+[B]\{\dot{y}\}+[K]\{y\}=\{F\}_y \quad (13)$$

where, *x* is the riser motion for the in line direction, and *y* for the transverse direction which is perpendicular to the *x* direction. The direction of current and incident waves coincides with the in line riser motion direction. In Equations 12 and 13 $[B]$ is the structural damping matrix.

For the sake of simplicity in the present work, only hydrostatic force that contributes to the riser axial tension and the forces due to waves and currents estimated by Morison's Equation are taken into account. For the riser transverse direction, Ferrari and Bearman approach (Ferrari, 1999) is applied. Then, the hydrodynamic forces for the two directions can be described by the following equations:

$$F_x = C_M A_I \frac{\partial u}{\partial t} + C_D A_D |V_r| (u+U_c - \dot{x}) - C_A A_I \ddot{x} \quad (14)$$

$$F_y = F_{VIV} \underbrace{- C_D A_D |V_r| \dot{y} - C_A A_I \ddot{y}}_{Fluid \quad Re\,action} \quad (15)$$

where, F_x is the in line force, F_{VIV} is the transverse force due to Vortex Induced Vibration (VIV), V_r is relative velocity, *u* is wave velocity, U_C is current velocity, $\dot{x}$ ($\dot{y}$) is riser structure velocity in x (y) direction , C_A and C_M additional mass and inertial coefficient. Finally, $A_I = \frac{\pi D^2 \rho_0}{4}$ and $A_D = \frac{1}{2}\rho_0 D$ (D external diameter, ρ_0 the density of sea).

4. Results

In the following results, calculations have been performed to the vertical rigid riser and subsurface buoy. The effect of flexible jumper in the riser system is replaced by a static reaction force at the subsurface buoy.

Table 1 shows main dimensions of the riser and the subsurface buoy in the initial calculations. The subsurface buoy, in this case, is placed at 50 meters bellow the sea level.

Figure 5 shows comparisons for displacements and rotation angles along the riser length, with buoy and without buoy conditions, from static behavior calculations. In one hand the axial tension along all the riser length is taken constant in this calculation. As expected, current viscous effect is larger with the buoy than without it. And, rotation angles along the buoy are constant and this angle is almost vertical. In the other hand, without the buoy, the riser angle at its upper part follows all over the riser rotation behavior.

Table 1. Main Dimensions of the Riser and the Subsurface Buoy in the Initial Calculations

Riser	
Length [m]	100.0
Outer Diameter [m]	0.25
Inner Diameter [m]	0.21
Top Tension [kN]	178.0
Elasticity Modulus [kPa]	2.1E+8
Current Velocity [m/s]	1.0
Density of water [kg/m3]	1025.0
Drag Coefficient C_D	0.7

Subsurface Buoy	
Length [m]	10.0
External Diameter [m]	1.5
Elasticity Modulus [kPa]	2.1E+12
Buoyancy [kN]	178.0

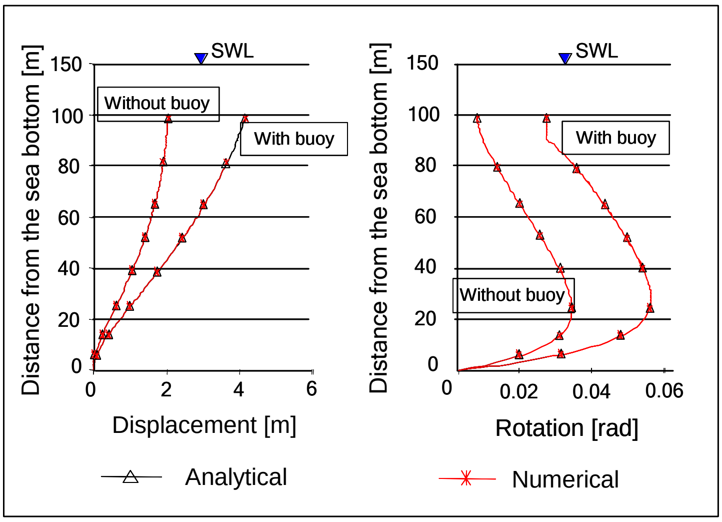
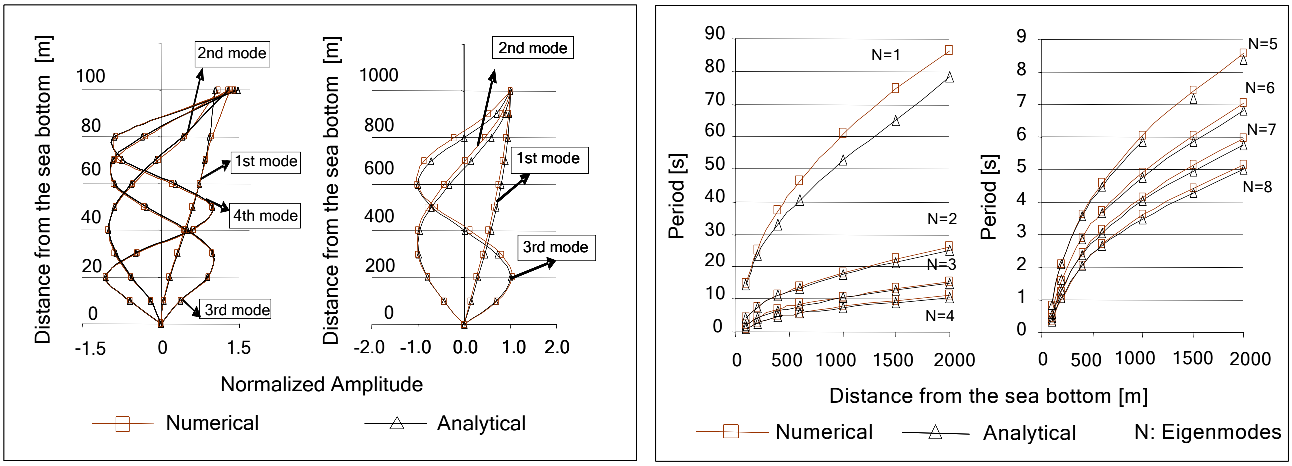


Figure 5. Comparisons of displacements and rotation angles through the riser length, with and without the subsurface buoy, with constant riser axial tension.



(a) (b)

Figure 6. (a) Natural modes of the beam without buoy with constant tension (b) Natural Periods versus length of riser

Figure 6 shows calculation results of riser dynamics for two different lengths, one with 100 meters length and the other with 1000 m. Calculations for the riser without the buoy by analytical solution and by numerical one are compared. The riser axial tension is taken constant for this case, as in the previous. Figure (6a) shows free riser motion modes in terms of normalized amplitude by the term C_4q in the Eq. 9. Figure (6b) depicts riser natural period that increases proportionally to the riser length. And, for a given riser length, the eigen modes are smaller for higher natural periods and vice versa.

With these previous results and analysis, a numerical code based on the dynamic models described above has been used for the dynamic analysis of a vertical rigid top tensioned riser with a subsurface buoy. Then, numerical calculations have been performed for a vertical riser with subsurface buoy in a very deepwater condition. Table 2 shows main dimensions for the riser and buoy, and the hydrodynamic coefficients used in the calculations. The riser is fixed at the bottom end and it is free at the top. The considered water depth is 2800 meters, and the subsurface buoy is submerged 100 meters bellow the water level.

Table 2. Main Dimensions of the Riser and Subsurface buoy for Deepwater Case

Riser	
Water level [m]	2800.0
Length [m]	2700.0
External Diameter [m]	0.45
Internal Diameter [m]	0.40
Elasticity modulus [kPa]	2.10E+08
Density of external fluid [kg/m3]	1025.0
Density of internal fluid [kg/m3]	970.43
Density of Riser Material [kg/m3]	7846.05
Drag Coefficient C_D	1.2

Buoyancy can	
Length [m]	37.0
External Diameter [m]	6.4
Elasticity Modulus [kPa]	2.10E+13
Material Density [kg/ m^3]	170.0

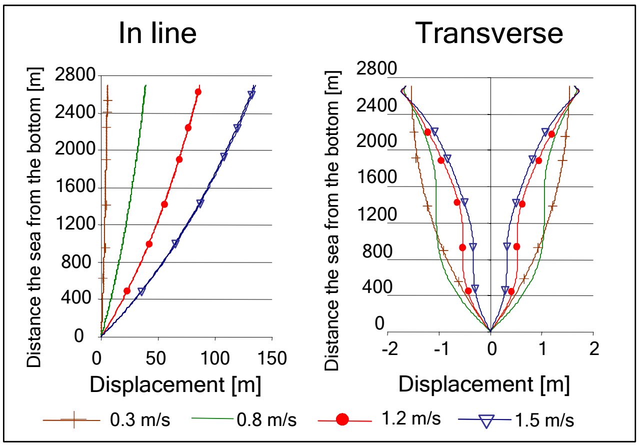


Figure 7. Maximum and minimum motions for differents current profiles

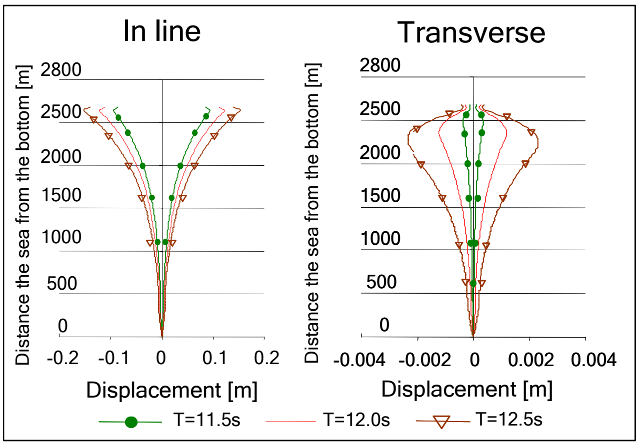


Figure 8. Maximum and minimum motions for incident wave periods with 4.0m wave height

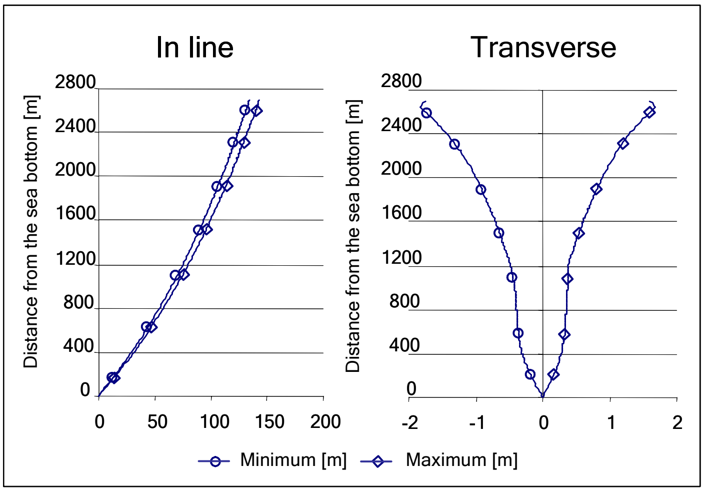


Figure 9. Maximum and minimum motion for simultaneous wave and current loads

Figures 7, 8 and 9 show the riser system maximum and minimum motion envelopes. Figure 7 shows the maximum and minimum motions of the riser with buoy for four different current profiles. In line and transverse direction results are shown and it can be observed that motion in the in line direction increases with the increase of current velocity.

Figure 8 shows the maximum and minimum riser motion with the buoy for four different incident wave periods. Wave height is 4.0 meters for all incident waves.

Figure 9 shows riser motions when wave and current are present simultaneously. Wave period of 12.2 seconds and wave height of 14.6 meters were taken into account. Constant current profile with 1.5 m/s velocity is imposed from the position of bottom of the buoy to the free surface.

5. Conclusions

The current work presented a fundamental study for a self standing riser system concept throughout numerical procedure developments for the vertical riser and subsurface buoy of that system. Ocean wave and current effects were taken into account. Previous analytical investigations have been performed in order to verify the numerical developments. As next steps of the research, complete SSHR system should be numerically modeled and compared with experimental results.

6. Acknowledgements

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8. Responsibility notice

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9. Appendix

Parameters in the solution of the Static Behavior of the Vertical Riser with the Subsurface Buoy

From the boundary conditions at bottom end of the riser, A_1 and B_1 can be determined:

$$(x_1=0;\ y=0) \quad \Rightarrow \quad A_1 = n_1^3 (C_1 - D_1) \tag{A1}$$

$$\left(\frac{dx_1}{dy} = 0;\ y=0\right) \quad \Rightarrow \quad B_1 = n_1^2 (C_1 + D_1) - \frac{F_{C1}}{E_1 I_1 n_1^2} \tag{A2}$$

The free boundary condition of the upper part of the buoy gives C_2 and A_2 as follows:

$$\left(\frac{d^2 x_2}{dy^2} = 0 \ ; \ y=0\right) \qquad \Rightarrow C_2 = \frac{F_{C2}}{E_2 I_2 n_2^4} - D_2 \tag{A3}$$

$$\left(\frac{d^3 x_2}{dy^3} - n_2{}^2 \frac{dx_2}{dy} = 0 \ ; \ y=0\right) \qquad \Rightarrow A_2 = 0 \tag{A4}$$

With A1 ~ A4 into Equations (5) and (6):

$$x_1 = C_1\left(e^{n_1 y} - n_1 y - 1\right) + D_1\left(e^{-n_1 y} + n_1 y - 1\right) - \frac{F_{C1} y^2}{2E_1 I_1 n_1^2} \tag{A5}$$

$$x_2 = D_2\left(e^{-n_2 y} - e^{n_2 y}\right) - \frac{B_2}{n_2^2} + \frac{F_{C2}}{2E_2 I_2 n_2^4}\left(2e^{n_2 y} - n_2^2 y^2 - 2\right). \tag{A6}$$

The continuity between riser and buoy gives, $x_1(L) = x_2(U)$; $\left.\frac{dx_1(y)}{dy}\right|_{y=L} = -\left.\frac{dx_2(y)}{dy}\right|_{y=U}$;

$$\left.\frac{d^2 x_1(y)}{dy^2}\right|_{y=L} = J^4 \left.\frac{d^2 x_2(y)}{dy^2}\right|_{y=U}; \quad \left.\frac{1}{n_1^2}.\frac{d^3 x_1(y)}{dy^3} - \frac{dx_1}{dy}\right|_{x=L} = -\left.\left(\frac{1}{n_2^2}.\frac{d^3 x_2(y)}{dy^3} - \frac{dx_2}{dy}\right)\right|_{x=U}.$$

where $J^4 = (E_2 I_2)/(E_1 I_1)$.

By solving the following equations in matrix form, C_1, D_1, D_2, B_2 can be obtained.

$$\begin{bmatrix} e^{n_1 L} - n_1 L - 1 & e^{-n_1 L} + n_1 L - 1 & e^{n_2 U} - e^{-n_2 U} & \frac{1}{n_2^2} \\ n_1\left(e^{n_1 L} - 1\right) & n_1\left(1 - e^{-n_1 L}\right) & -n_2\left(e^{-n_2 U} + e^{n_2 U}\right) & 0 \\ n_1^2 e^{n_1 L} & n_1^2 e^{-n_1 L} & n_2^2 J^4\left(e^{n_2 U} - e^{-n_2 U}\right) & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} C_1 \\ D_1 \\ D_2 \\ B_2 \end{Bmatrix} = \begin{Bmatrix} \frac{F_{C1} L^2}{2E_1 I_1 n_1^2} + \frac{F_{C2}}{2E_2 I_2 n_2^4}\left(2e^{n_2 U} - n_2^2 U^2 - 2\right) \\ \frac{F_{C1} L}{E_1 I_1 n_1^2} - \frac{F_{C2}}{E_2 I_2 n_2^3}\left(e^{n_2 U} - n_2 U\right) \\ \frac{F_{C1}}{E_1 I_1 n_1^2} + J^4 \frac{F_{C2}}{E_2 I_2 n_2^2}\left(e^{n_2 U} - 1\right) \\ -\frac{F_{C2} U}{E_2 I_2 n_2^2 n_1} - \frac{F_{C1} L}{E_1 I_1 n_1^3} \end{Bmatrix} \tag{A7}$$