

GAS TURBINE IDENTIFICATION WITH LINEAR AND NON-LINEAR TECHNIQUES

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Abstract. *The discussion present in this paper explains how identification can be applied on gas turbine dynamics, considering a single-spool gas turbine. This gas turbine have one shaft, with a centrifugal compressor and an axial turbine. Its dynamic behavior have linear and non-linear characteristics, where the non-linear behavior presents some dificulties in construct a complete gas turbine model. Some linear models provide tolerable transient behavior to the identification, such rotor, pressure and temperature dynamics. These dynamics characteristics can be described by linear identification, such as Box-Jenkins and Output-error models. This models can only present linear dynamics, and it's applicable only to small disturbances around a design point. To realize a non-linear identifcation a Narmax identification is adopted to obtain a wide range model, and a comparison between the linear and non-linear models is important to distinguish the main features in both identifications. The identifications are made in an open-loop engine, with especific manipulations in the fuel range to obtain the different dynamic features. Firstly a linear identification is obtained, acquiring a significant number of important models to design and off-design point operation. To the non-linear identification, a Narmax identification is proposed, with posterior simulations and satisfatorial validation of both linear and non-linear models.*

Keywords*: gas turbine model, identification, linear, non-linear, design point, transient behavior.*

1. Introduction

Gas turbines are rotating machines that convert fuel energy in mechanical work, sometimes to provide shaft power in an electrical generation duty, sometimes to supply thrust during aircraft flight and maneuvering.

The considered aeronautical single-spool gas turbine is compounded by an air intake, a centrifugal compressor, anular combustion chamber, axial turbine and an exhaust nozzle, with a shaft that links the compressor with the turbine. The components features of the gas turbine are illustrated in Fig. 1. The colors differentiate the engine sections.

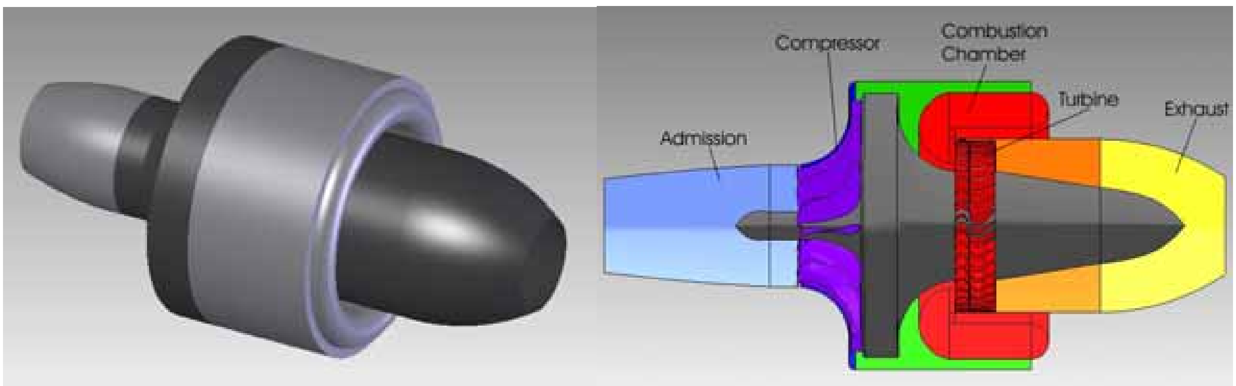

Figure 1 Single-spool gas-turbine.

Gas turbine modeling has been a key factor for its dynamic investigation, and great efforts are realized to aim the construction of more and more complex models that delineate and accomplish the dynamic behavior of these thermodynamic systems. High costs and expensive investments in developing new gas turbines guided to an undesirable commercial risk. By this fact, a high fidelity acquisition of a model more accurate is crucial to analyze and predict the engine performance without over arising costs with prototypes and aiming control laws development to the engine and optimize commercial parameters like fuel consumption or material quantity used to construct the metallic structure. All of these items are to carry out with the limited requirements demanded by the clients, project requirements, certification entities and environmental laws.

The single-spool gas turbine is selected in this report because it's simple to study and have many applications (e.g. APU-Auxiliary Power Unity, drone engine and UAVs-Unmanned Aerial Vehicles), or electrical generation in small or medium powerplants. This adoption makes easier the dynamic observation because of its simplest conception between gas turbines.

The control system and actuators, often present in aeronautical or electrical powerplants, are not considered in this case, so the identification is in open-loop, which means that the engine runs without a control system. The gas turbine plant is identified by induced fuel injection. Thus, the identification can be more precisely and complete, practically ranging all the system frequencies, making the model more accurate. This frequencies detection is very important because gas turbine have many physical factors that influence performance.

In case of aeronautical gas turbines, for example, models without heat transfer occurring on surfaces components and flows with unvalued friction are tolerable and satisfactory. In case of regenerative cycles, this consideration can't be adopted, with possible penalties on performance prediction. This is due to difficulties of theoretical analyses and experimental data acquisition for the study of heat transfer between gas and metal in the gas path during changes on engine operating conditions.

The frequencies that occur in models and are more important in simulations are in the following ranges, as defined by Horobin (1998) and Alves and Barbosa (2000):

- 1 Hz – thermal transient;
- 1-5 Hz - shaft transient operation;
- 5-50 Hz - gas dynamics transient.

The "Transient" term is used to describe models in the 5 Hz range, and "Dynamic" or HIFEM (High Frequency Engine Model), describes models that are capable in reach frequencies in the 30-50 Hz range.

Transient shaft are considered in aeronautical models where the operation is in free-load running mode. In electrical powerplants the load due electrical generator and accessories can't be despised. Thermal transient are rarely considered in models, because there are difficulties in implement this characteristic. The gas dynamics is an important characteristic to consider due to the mass and energy stored in volumes.

Regarding to the identification strategies, there are four main identification approaches to find a gas turbine model, as related by Arkov *et al*. (2000):

- System identification using ambient noise excitation;
- Multi-sine and frequency-domain techniques, for both linear and non-linear models;
- Extended least-squares/optimal-smoothing algorithms for finding time-varying linear models;
- Multi-objective genetic programming for the selection of the nonlinear model structure.

These strategies are being the most common ways to identify dynamics in gas turbines. Identification of aircraft gas turbine dynamics has historically relied on sine-wave testing at a fairly large number of frequencies. This approaches results in Bode plots, which describe the dynamics of the engine and are familiar to control designers. Low-amplitude sine-wave test results can be made intensively to the non-linearities in the engine response and to noise. However, they have severe drawbacks, the primary one being the expense associated with very long test duration to allow the decay of initial transients at each frequency and achieve sufficient insensible noise. More than this, there is the necessity to spanning the range of engine speeds from idle to 100% of power to find all frequency response at each of several operating condition.

In this paper, the identification techniques used for models acquisition are the multi-sine and frequency-domain for both linear and non-linear models. The dynamic relationship is about the shaft speed, compressor pressure discharge, combustion chamber temperature and thrust. The frequency of fuel-demand signal is about 5-50 Hz. The 3 or more level signal, used to identification, involves the dynamics around 75-100% of speed range.

The identification is realized using a gas turbine simulation and performance program, aiming the generation of data for gas turbines studies. Using a program to simulate gas turbine running to obtain models are cheap, considering that there are no costs in an engine running. However, the accuracy can have some distortion and all the non-linearities are not present on the gas turbine simulation software. For instance, however, models from the identification can provide good approximation and supply much information about gas turbine design and performance, as the necessary information to control strategies that can be adopted for the engine.

2. Identification techniques

Some linear techniques are used to identification and models construction. These linear models are based on input and output signals, whose must be considered for black-box identification. This identification generates an equation in frequency domain that is its transfer function, which can represent the dynamic of the system studied.

The transfer function of a system is its impulse response Laplace transformation. The time response is obtained from the inverse transformation of the same transfer function.

Thus, a system time response *y(t)* is given by an impulse input and the its system transfer function product is:

$$y(t) = L^{-1}\{Y(s)\} = L^{-1}\{G(s)U(s)\} \quad (1)$$

where *Y, G* e *U* are the Laplace transform of *y(t), g(t)* e *u(t)*, respectively. The *U(s)* is the perturbation signal or input signal that needs to be manipulated very carefully to obtain a good identification and the *G(s)* is the identified plant transfer function. It is necessary to know the Laplace transform of the input too, to obey the law dictated by Eq. (1).

Depending on the transfer function order, it can be decomposed in a combination of lower order transfer function, which can ease the dynamic analysis. So it's important know the behavior of lower transfer function, or the first order transfer function, that have form:

$$G(s) = \frac{K}{\tau s + 1} \quad (2)$$

The term K is a gain constant and τ is the transfer function time constant. These equations are very important because their values determine the dynamics behavior.

Other relevant characteristic in transfer function models is the delay in exponential form, given by the form in Eq. (3):

$$Delay = e^{-Ts} \quad (3)$$

Combining Eq. (1) and (2), we can have a good approximation for dynamic shaft speed, as proposed in Kulikov and Thompson (2004). This parametric modeling can be applied on static modeling of turbo jet identification, and the equation is:

$$G(s) = \frac{K}{(T_1 s + 1)} e^{-Ts} \quad (4)$$

The time constants T and T_1 are identified from transient gas turbine response. This function is a good approximation for gas turbine models considering small perturbations around an operation point. The exponential term represents the delay responsible for high transient frequencies generated by volume dynamic and is an approximation. These high frequencies are difficulty to identify due to chaotic behavior and non-linear characteristics in gas flow dynamics in each engine volume and changes in blade-tip clearance, for example.

Other kind of identification are the approximations via discrete representations, such as Armax, Arx, Output-error and Box-Jenkins models, whose offers tolerable results depending on the system to be identified.

These models are good mathematical representations suitable to system identification using algorithms known for parameters estimation, as specified in Aguirre (2004) and Ljung (1994).

The general form of discrete representation is:

$$A(q)y(k) = \frac{B(q)}{F(q)}u(k) + \frac{C(q)}{D(q)}\upsilon(k) \quad (5)$$

The q is a delay operator, ν is the white noise and polynomial functions *A(q), B(q), C(q), D(q)* e *F(q)* are presented below:

$$A(q) = 1 - a_1 q^{-1} - \ldots - a_{n_y} q^{-n_y};$$
$$B(q) = b_1 q^{-1} - \ldots - b_{n_u} q^{-n_u};$$
$$C(q) = 1 + c_1 q^{-1} - \ldots - a_{n_v} q^{-n_v};$$
$$D(q) = 1 + d_1 q^{-1} - \ldots - d_{n_d} q^{-n_d};$$
$$F(q) = 1 + f_1 q^{-1} - \ldots - f_{n_f} q^{-n_f}.$$

Equation (5) can be written as:

$$y(k) = \frac{B(q)}{F(q)A(q)}u(k) + \frac{C(q)}{D(q)A(q)}\upsilon(k)$$
$$y(k) = G(q)u(k) + H(q)\upsilon(k) \quad (6)$$

where *G(q)* is the transfer function process and *H(q)* is the noise transfer function. In block diagrams representation, Eq.(2) have the form presented in Fig. 2.

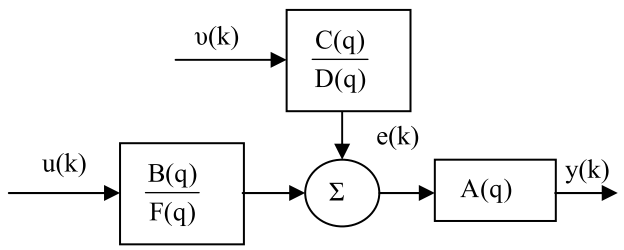

Figure 2 Block Diagram of a generalized linear model.

Between discrete models, there are different manners in representing the linear dynamic *y(k)* , in terms of polynomial functions. This representation will depends on the input signals and how it will be treated considering the kind of noise chosen to identification.

The models used to linear identification in this paper are the Box-Jenkins (BJ) and Output-error (OE) which revealed the best results in gas turbine identification.

2.1. Box-Jenkins identification model

Considering the general linear model and making *A(q)*=1:

$$y(k) = \frac{B(q)}{F(q)}u(k) + \frac{C(q)}{D(q)}\upsilon(k) \quad (7)$$

This method is classified as an output-error model because the response *y(k)* does not depends on polynomial term *A(q)* , and by the fact that the error function is added directly in output.

2.2. Output-Error identification model

The Output-error is the simplest form of output errors representation, and has both polynomial terms *A(q)* and *H(q)* equal to 1.

$$y(k) = \frac{B(q)}{F(q)}u(k) + \upsilon(k) \quad (8)$$

The parametrical models *B, C, D* and *F* present in both models are terms parametrically independents of transfer functions. They have no common parameters, because are independently obtained by *B/F* and *C/D* relations, whose are the process transfer function and noise transfer function respectively.

2.3. Narmax identification model

The general representation of Narmax models is described by the following non-linear equation:

$$y(k) = F^{j}\begin{bmatrix} y(k-1),\ldots,y(k-n_y),u(k-\tau_d),\ldots \\ \ldots,u(k-\tau_d-n_u+1),e(k-1),\ldots,e(k-n_e) \end{bmatrix} + e(k) \quad (9)$$

The term *e(k)* outside the brackets in Eq. (9) is the sum of effects that can't be well represented by the function F^j, which is a polynomial function of *y(k), u(k)* e *e(k)* with a non-linearity degree $j \in N$. Thus, the deterministic equation terms of the Narmax model can be expanded with non-linearity degree ranging in the interval of $1 \leq m \leq j$. Each term with degree m can have degree p of $y(k-1)$ and a factor degree $m-p$ of $u(k-1)$ term, multiplied by an estimated parameter $c_{p,m-p}(n_1,\ldots,n_m)$.

3. Gas turbine identification

The single-spool gas-turbine model considered have approximately 1000 Newtons of thrust and run in a nominal rotational speed of 45000 rpm (at design point). Information about the engine can be observed in Tab. 1.

Tabela 1 Characteristics of design point operation

Maximum operation Altitude [m]	5000
Nominal shaft speed [rpm]	45000
Maximum operational Temperature [K]	1060
Nominal Thrust [Newtons]	≈1000

The simulations realized in the program to this aeronautical application considered two different altitudes, with small variations in fuel demand around each operation point. The fuel demand signal ranged the 75-100 % rotational speed level, and the dynamics characteristics were acquired for each level.

The use of multi-sine signals as input signals are often used because there are many embedded frequencies, whose excite many of the engine modes. The multi-sine signal is an arbitrary sine or cosines combination, harmonically listed by an input signal function $u(t)$:

$$u(t) = \sum_{k=1}^{F} A(k) cos(i(k)\omega_0 t + \phi(k))$$

In gas turbine identification this signal is more useful when combined with step input because the engine have non-linearities in acceleration and deceleration behaviors, and the step function can detect this non-linearities in gas turbines. So, the input function can be:

$$u(t) = u_{STEP}(t)\left(\sum_{k=1}^{F} A(k) cos(i(k)\omega_0 t + \phi(k))\right) \qquad (10)$$

3.1. Identification data

An example of running signals acquired by simulation is presented in Fig. 3, considering the fuel flow input as a signal generated by Eq. (10).

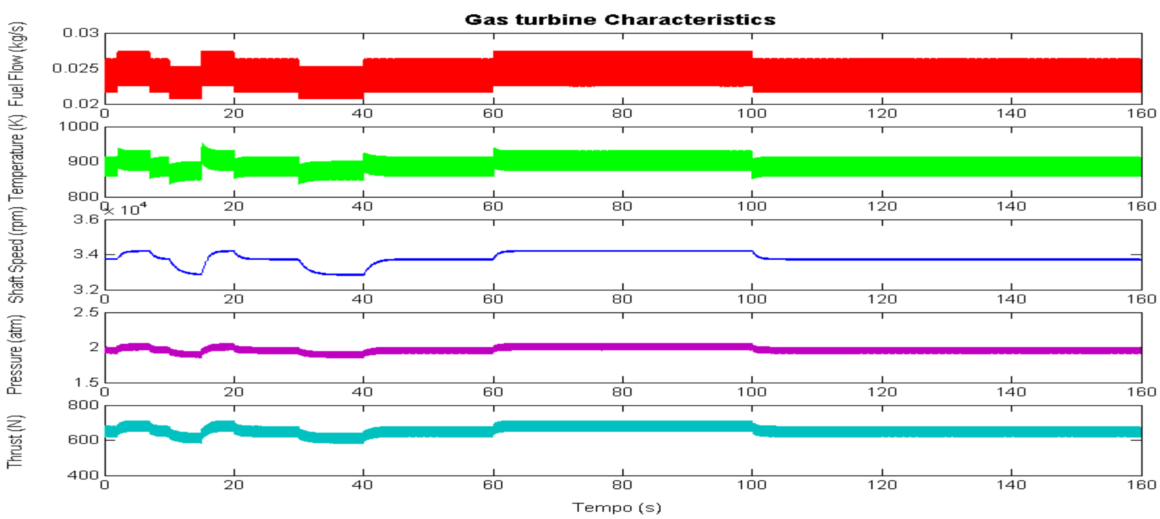

Figure 3 Example of transient behavior of state variables considered to the gas turbine identification.

The linear and non-linear models found considering fuel flow as input and the others variables as output have different coefficients related to the different operation points. Some coefficients are listed in Tab. 2, and represent the BJ, OE models and the Narmax model identified only for Condition 1. The models were obtained from two different conditions:

- Condition 1: before the take-off (Alt = 0; Mach number = 0);
- Condition 2: condition of maximum thrust and Mach number in design point (Alt = 5000; Mach number = 0.7).

Tabela 2 Identified parameters of linear and non-linear models for Condition 1

	Condition 1	
	Linear Models	**Non-linear Models**
Exhaustion turbine temperature	$B(q) = 26480\ q^{-1} - 30260\ q^{-2} – 17030\ q^{-3}$ $22280\ q^{-4} - 4445\ q^{-5} + 2984\ q^{-6}$ $F(q) = 1 - 1.138\ q^{-1} - 0.6443\ q^{-2} + 0.8378\ q^{-3} - 0.167\ q^{-4} + 0.1107\ q^{-5} - 0.003596\ q^{-6} + 0.003635\ q^{-7} + 0.00467\ q^{-8} + 0.000678\ q^{-9} - 0.005987\ q^{-10} + 0.001611\ q^{-11}$	y(k) = +3.7923 y(k-1) - 981.52 + 7247800 u(k-3)u(k-3)u(k-1) + 40.078u(k-1)y(k-3) - 15808000u(k-3)u(k-3)u(k-3) + 766.22u(k-3)u(k-3)y(k-1) + 2.2465e(k-2) + 2.1906e(k-1)
Shaft speed	$B(q) = 50759\ q^{-1} - 1186\ q^{-2} + 1957\ q^{-3}$ $C(q) = 1 + 0.1813\ q^{-1} + 0.3629\ q^{-2} + 0.177\ q^{-3}$ $D(q) = 1 - q^{-1}$ $F(q) = 1 - 0.9878\ q^{-1}$	y(k) = -5.3021y(k-1) - 4.1751y(k-3) + 115480 + 28050u(k-1) - 11.899u(k-1)u(k-1)y(k-1) + 1.1194e(k-5) + 0.098011e(k-3) + 0.073547e(k-1) - 0.0048105e(k-2)
Compressor discharge pressure	$B(q) = 0.3966\ q^{-3} - 0.09958\ q^{-4} - 0.2833\ q^{-5}$ $C(q) = 1 + 0.02006\ q^{-1} + 0.02113\ q^{-2}$ $D(q) = 1 - q^{-1}$ $F(q) = 1 - 1.135\ q^{-1} - 0.6806\ q^{-2} + 0.8159\ q^{-3}$	y(k) = + 1.9638y(k-1) + 0.11505y(k-3)y(k-3) - 7.8632u(k-3) - 0.94277 - 0.0021384y(k-3)y(k-3)y(k-3) + 12.611u(k-1)y(k-3) - 0.35525y(k-3)y(k-1) - 17.415u(k-1) - 143.72su(k-3)u(k-1) - 0.54003e(k-1) - 0.53926e(k-2) + 0.0010339e(k-4) - 0.00097506e(k-5) + 0.00068481e(k-3)
Thrust	$B(q) = 233.4\ q^{-1} - 226.9\ q^{-2}$ $C(q) = 1 - 0.04175\ q^{-1} + 0.006176\ q^{-2} - 0.0003223\ q^{-3}$ $D(q) = 1 - 1.045\ q^{-1} + 0.04543\ q^{-2}$ $F(q) = 1 - 1.964\ q^{-1} + 0.964\ q^{-2}$	y(k) = -4.7794y(k-1) + 35221u(k-1) + 38.162 -209.04u(k-1)y(k-1) + 0.0061535y(k-1)y(k-1) + 2.4316u(k-1)y(k-1)y(k-1) - 2124.8u(k-1)u(k-1)y(k-1) + 1345000u(k-1)u(k-1) - 0.00069475e(k-3) - 0.0036678e(k-2) - 0.00024315e(k-4) - 0.00021365e(k-1)

The models contain some parameters of different degrees, which are responsible for the polynomial adjustment to the real transient response. The linear models have parameters that are simple combinations of *u(k)* and *e(k)* which are linear model characteristic, while the non-linear models have combinations of *u(k)*, *y(k)* and *e(k)* that represents the characteristics of non-linearities in the model. In non-linear models, the lonely terms are the linear dynamics. In Fig. 4, the comparison of each kind of model can be visualized and the discrepancies between the linear and non-linear models can be noted by the difference and the error in reach the set-points leaded by the fuel flow signal.

3.2. Results

The results from static operation models can be visualized in the following figures.

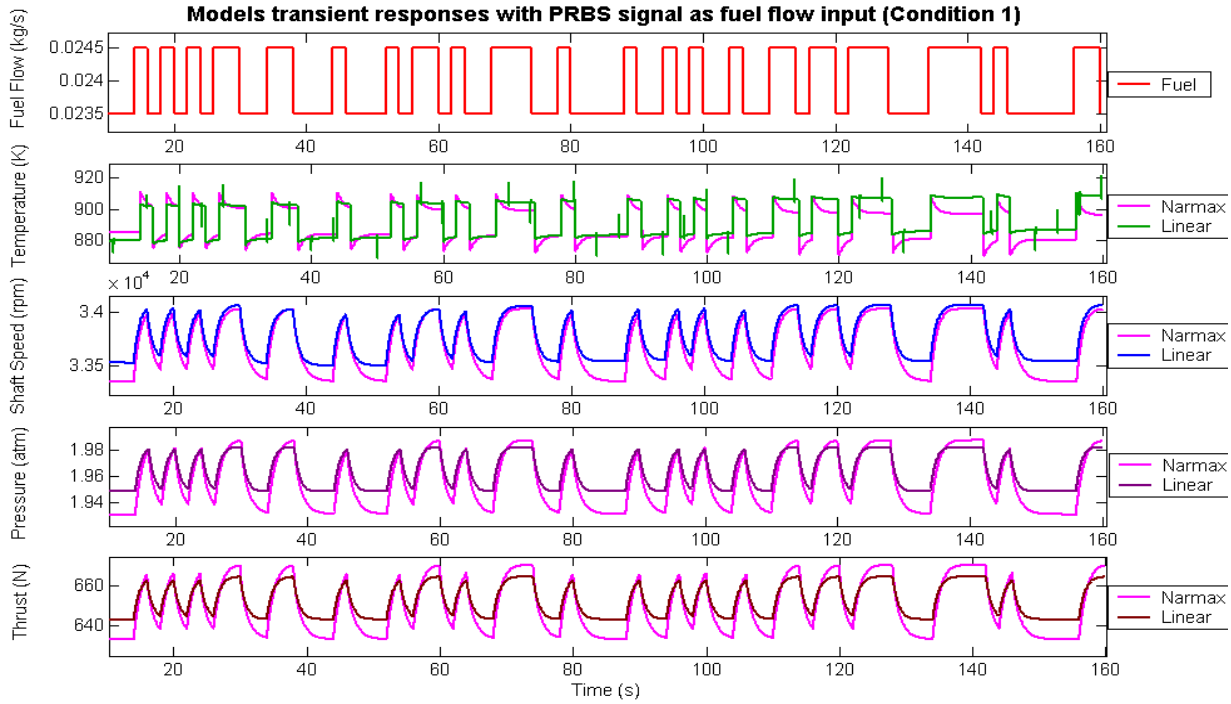

Figure 4 Transient responses of linear and non-linear models identification using PRBS signals on Condition 1.

The models identified related to Condition 2 had similar responses and are presented in Fig. 5. The same PRBS signal was used aiming a comparison between the conditions. This comparison is important because there are non-linear behaviors between operation points due to external variations that can influences the transient behavior, such as varying mass flow of air and altitude. To these conditions the parameters and the models can substantially change, which demand a better analysis.

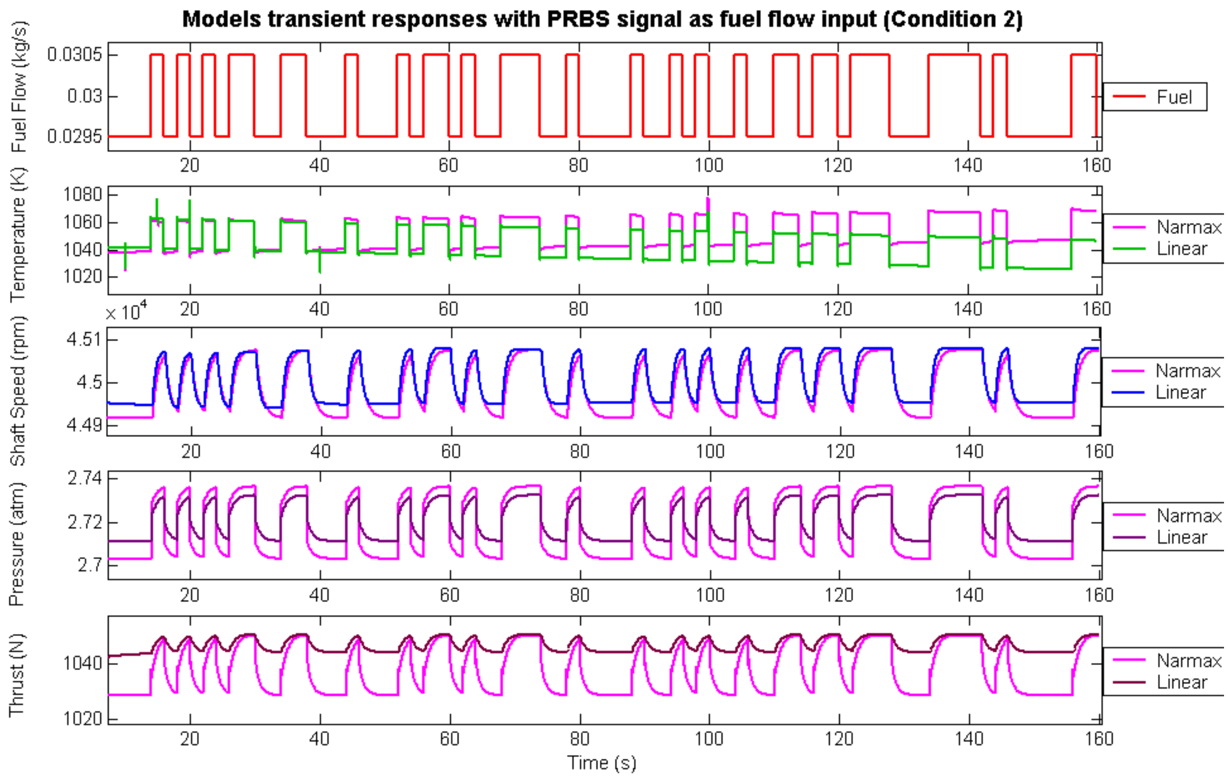

Figure 5 Transient responses of linear and non-linear models identification using PRBS signals on Condition 2.

The errors between the variables in both conditions can be observed: in Fig. 6

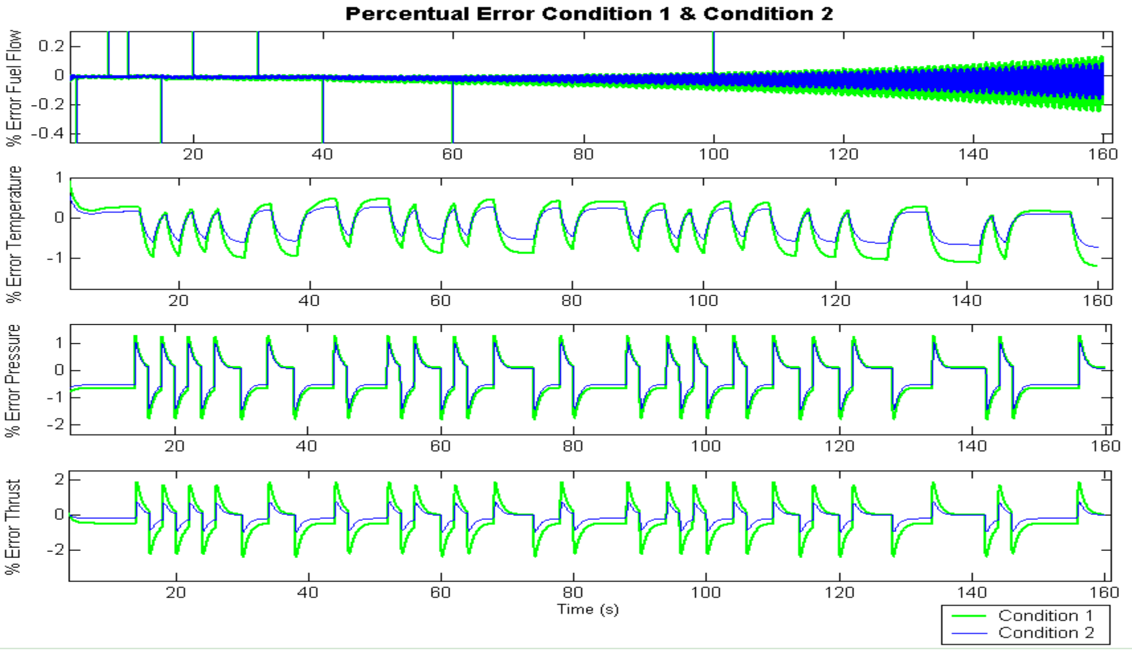

Figure 6 Errors between models (linear and non-linear) in both conditions.

4. Conclusion

A single spool gas turbine identification was established in this paper aiming to detect the main linear and non-linear behavior concerning to transient characteristics. From the main characteristics of a gas turbine, four of them were identified, providing linear and non-linear models. In this case study, the different variations in acceleration and slow down range, considering both kind of models in both of the conditions, are different in almost of the variables identified. This fact is due to the operation temperature, which influences the efficiency in both conditions. As the temperature raises, the efficiency increases and the engine responses become faster. So, the linearization can be easier and the error decreases as the operation point approaches the gas turbine design point. The errors between models in both conditions is about ±0,1% or more to the temperature as the time increases, ±0.5% to shaft speed, ± 1% to compressor discharge pressure and about ±0,5% to thrust. These errors between models are important to the transient analysis because they can tell if the linear model can be used for a control implementation. By the exhibited results, a good linearization can be obtained, with a minimum error and a control can be implemented. To these models improvement, a posterior analysis of the frequency response function in future works will provide a way to estimate a controller system to this gas turbine model, and a general analysis about the entire system will be realized.

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