

# METHODOLOGY FOR THE DETERMINATION OF THE OPTIMUM DIE ANGLE IN WIRE DRAWING

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**Abstract.** *The present work analyzes the optimum geometric drawing die parameters through the friction factor determination. Considering a unitary  $\Delta$  parameter, it was possible to determine the friction factor and the optimum die angle for some reductions from a reduced number of tests. A die set was used with half die angles varying from 1° to 9°. The material was the commercially pure copper. The results are compared to those ones in literature, allowing the validation of the method and of the results.*

**Keywords:** *friction factor, wire drawing, optimum die angle.*

## 1. Introduction

The necessary work for bar drawing,  $W_a$ , includes not only the ideal work for homogeneous deformation,  $W_i$ , but also the effects of die/material friction and of the redundant strains (Hosford and Caddell, 1993). Figure 1 displays the evolution of the deformation of an element of a bar during the drawing process. Appreciable shearing strains, associated with velocity discontinuities, occur both at the entrance and exit regions (regions I and II, respectively). The phenomenon is evaluated through the redundant work,  $W_r$  (Caddell and Atkins, 1968; Johnson and Rowe, 1968), defined as the energy dissipated per unit volume of the material, in order to complete the redundant strains (Campos and Cetlin, 1994). The redundant strains are usually connected to the geometric parameter  $\Delta$ , defined by the ratio  $c/d$  (Hill and Tupper, 1948), according to Fig. 2. Values of  $\Delta$  equal to one minimize the shearing strains, and the redundant work tends to zero (Green and Hill, 1952). A surface element also undergoes shearing strains caused by die/material friction, as illustrated in Fig. 1, which are associated with the shearing work per unit volume,  $W_f$ . Both  $W_r$  and  $W_f$  cause strains above those strictly necessary for deforming the bar, and the final product is stronger, harder and less ductile than if it had been strained only homogeneously, as in pure tension. (Cetlin, 1987; Cetlin and Marcos, 1987; Sadok et al, 1994a,b,c).

The total drawing work,  $W_a$ , is thus evaluated as the sum of the three above mentioned work contributions, according to Eq. (1):

$$W_a = W_r + W_f + W_i \quad (1)$$

The evaluation of the total work per unit volume was performed by Avitzur, through the Upper Bound Method (Avitzur, 1979, 1989). His results are described by Eq. (2), covering the work for the homogeneous deformation, the redundant deformation and the friction:

$$\frac{\sigma_t}{\bar{Y}} = 2f(\alpha) \ln\left(\frac{R_i}{R_f}\right) + \frac{2}{\sqrt{3}} \left[ \frac{\alpha}{\sin^2(\alpha)} - \cot(\alpha) + m \cot(\alpha) \ln\left(\frac{R_i}{R_f}\right) + m \frac{L}{R_f} \right] \quad (2)$$

In Equation (2),  $\sigma_t$  is the drawing stress;  $\bar{Y}$  is the average flow stress of the material;  $\alpha$  is the die semi-angle in radians;  $f(\alpha)$  is given by Eq. (3);  $R_i$  is the radius of the material at the die entrance, whereas  $R_f$  is the same radius at the die exit;  $m$  is the friction factor and  $L$  is the length of the die bearing.

$$f(\alpha) = \frac{1}{\sin^2 \alpha} \left\{ 1 - \cos \alpha \sqrt{1 - \frac{11}{12} \sin^2 \alpha} + \frac{1}{\sqrt{11.12}} \cdot \ln \left[ \frac{1 + \sqrt{\frac{11}{12}}}{\sqrt{\frac{11}{12} \cos \alpha + \sqrt{1 - \frac{11}{12} \sin^2 \alpha}}} \right] \right\} \quad (3)$$

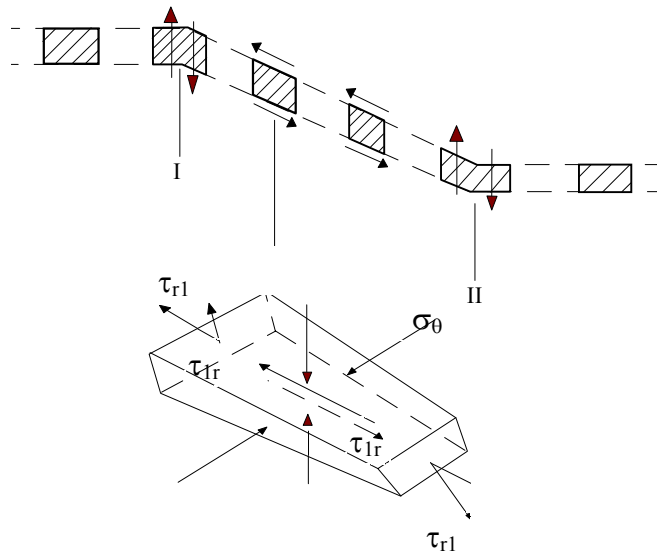


Figure 1: Schematic representation of the redundant strain in bar drawing.

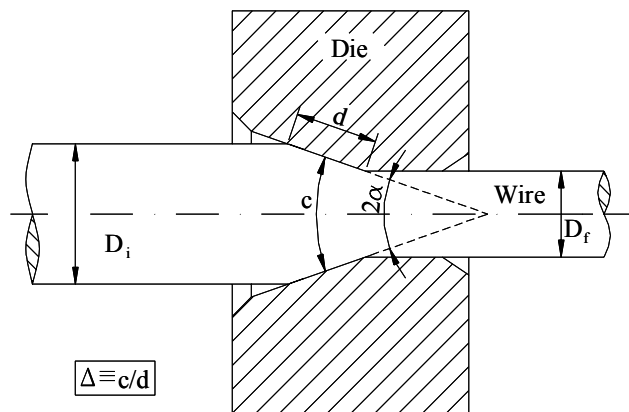


Figure 2: Basic geometric parameters of a drawing die.

Figure 3 represents qualitatively the influence of the die semi-angle on the total work for drawing, as well as on its three components (work for homogeneous deformation, for redundant deformation and for friction). For a given reduction of area,  $r$ , the contact length between the material and the die increases as the die semi-angle,  $\alpha$ , is lowered. Since the compressive stress between the die and the material is not changed, the force along this interface should

increase as  $\alpha$  is lowered, connected to the increase in the contact area. If the friction factor,  $m$ , is relatively constant, the friction work,  $W_f$ , increases as  $\alpha$  decreases. On the other hand, for a given reduction of area,  $r$ , the redundant work,  $W_r$ , grows higher as the die semi-angle is increased. The work for homogeneous deformation,  $W_i$ , depends only on  $r$ , and is not affected by  $\alpha$ . Considering the behavior of the three work components, the total work,  $W_a$ , displays a minimum value associated with a certain die semi-angle, which is called the “optimum” die angle,  $\alpha^*$  (Hosford and Caddell, 1993). Figure 4 shows the experimental values of the drawing stress for various die semi-angles, according to the results from Wistreich (1955, 1958). It can be observed that the optimum die angle increases as the reduction of area in the pass increases. It has also been shown that higher friction leads to higher  $\alpha^*$  (Majta et al, 1992).

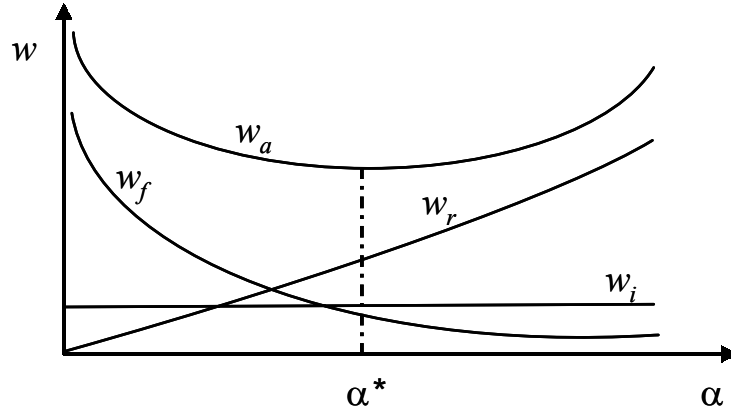


Figure 3. Schematic illustration of the influence of the die semi-angle,  $\alpha$ , on the total drawing work,  $W_a$ , on the work for homogeneous deformation,  $W_i$ , on the friction work,  $W_f$ , and on the redundant work,  $W_r$ , during bar drawing.

The differentiation of Eq. (2) in relation to the die semi-angle  $\alpha$ , which is then taken as zero, leads to an analytical expression for the optimum semi-angle,  $\alpha^*$ , given by Eq. (4):

$$\alpha^* \approx \sqrt{\frac{3}{2} m \ln(R_i / R_f)} \quad (4)$$

According to Eq. (4),  $\alpha^*$  depends only on the friction factor,  $m$ , and on the reduction of area. The adoption of  $\alpha^*$  allows the minimization of the drawing force or the maximization of the reduction of area, for a given set of parameters.

The literature presents several methods for the evaluation of the friction factor (Batalha, 1995; Baker and Wright, 1990, 1992a,b). These methods are based on various hypotheses, and none of them allows the independent determination of  $m$ . Some authors consider that  $m$  as a property linked to the material being drawn, the die material, and the lubricant, and that it would be independent of the tooling geometry and processing speed (Wilson, 1979). Baker and Wright (1993) have shown, on the other hand, that  $m$  decreases as the drawing speed increases and the die semi-angle is lowered. The effect of speed is linked to the loss of lubrication caused by the heating at the die/material interface. The effect of the die semi-angle is associated with lubricant drag into the die/material interface.

The objective of the present paper is the determination of the optimum die semi-angle through Eq. (4). The values of the friction factor,  $m$ , will be determined for two die semi-angles: from 2 to 9°, considering a  $\Delta$  parameter equal to 1. The comparison of the present results with those from Wistreich (1955, 1958) involved the drawing of annealed copper bars.

## 2. Methodology

The initial material was 100 cold drawn copper bars, with a diameter of 6.35 mm and 300 mm long, with the surface finish and roughness typical of cold drawing. The material was vacuum annealed at 550 °C for 100 min. The uniformity of properties of the various annealed bars was checked through Vickers Hardness measurements, leading to a value of  $HV = 46.0 \pm 2.0$ . The final surface roughness of the bars was  $Ra = 0.36 \pm 0.03 \mu m$ .

The annealed copper bars were cleaned with sulfuric ether and then lubricated with a molybdenum disulfide paste before drawing. The dies were tungsten carbide ones, with an entrance diameter of 7.30 mm, bearing length,  $L$ , equal to half of the initial diameter of the material ( $D_i = 6.35$  mm) and exit diameters calculated in such a way as to keep the  $\Delta$  parameter equal to 1 for the range of die semi-angles from 2 to 9° (Cetlin, 1984), according to Eq. (5):

$$D_f = \frac{D_i(\Delta - \alpha)}{(\Delta + \alpha)} \quad (5)$$

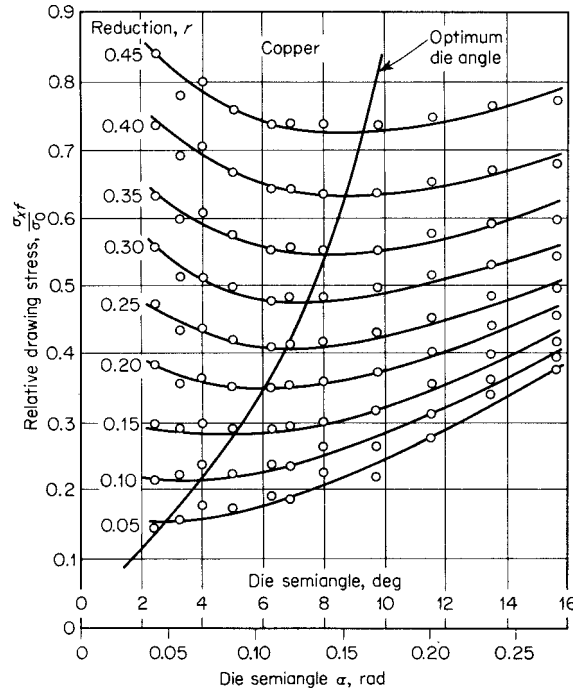


Figure 4. Experimental results for the drawing stress for various semi-angles of the dies (Wistreich, 1955).

The experimental values of  $\alpha$  and  $L$  were measured in a Contourgraph apparatus, from the Drawing Mill of Companhia Siderúrgica Belgo-Mineira. The exit diameter of the dies was evaluated through the diameter of the drawn bars, utilizing a micrometer allowing a precision of 0.001 mm.

The true stress-true strain curve for the material was determined through a tensile test performed in an INSTRON Universal Testing Machine. The data after necking included Bridgman's correction. (Dieter, 1981). The evaluation of the average flow stress,  $\bar{Y}$ , of copper in the various drawing passes involved two different procedures: one for drawing strains below the tensile uniform strain of the material and the other for drawing strains above this value.

Considering drawing strains below the tensile uniform strain,  $\bar{Y}$  was determined utilizing Eq. (6), derived from the trapeze rule.

$$\bar{Y} = \int \frac{1}{2} \Delta x [\sigma(\varepsilon_0) + 2\sigma(\varepsilon_1) + 2\sigma(\varepsilon_2) + \dots + 2\sigma(\varepsilon_{n-1}) + \sigma(\varepsilon_n)] \quad (6)$$

$$\text{Where } \varepsilon = 2 \ln \left( \frac{D_i}{D_f} \right) \text{ e } \Delta x = \varepsilon/n.$$

For higher drawing strains, the smaller number of experimental points did not allow the numerical evaluation of  $\bar{Y}$ . In this case, an analytical procedure was followed, employing Eq. (7):

$$\bar{Y} = \int \frac{1}{2} \Delta x [\sigma(\varepsilon_0) + 2\sigma(\varepsilon_1) + 2\sigma(\varepsilon_2) + \dots + 2\sigma(\varepsilon_{n-1}) + \sigma(\varepsilon_n)] + \int_{\varepsilon_u}^{\varepsilon} \sigma(\varepsilon) d\varepsilon \quad (7)$$

The above mentioned methods for the evaluation of the average flow stress,  $\bar{Y}$ , are based on the tensile test, and their results are reasonable only when the  $\Delta$  parameter is unitary. This minimizes the redundant work during drawing, and the strain resulting from this type of processing becomes similar to that from tension (Rauch, 1992, Lopes et al, 1999).

The calculation of the drawing stress,  $\sigma_f$ , was performed based on the measurements of drawing loads in controlled drawing passes in an INSTRON Universal Testing Machine. The load data as a function of crosshead displacement were collected and processed through a spreadsheet. The data utilized for the calculations of the drawing stress correspond to the steady state region, where the drawing load is approximately constant (see Fig. 5). The drawing stress is the drawing load divided by the final cross section area of the drawn bar.

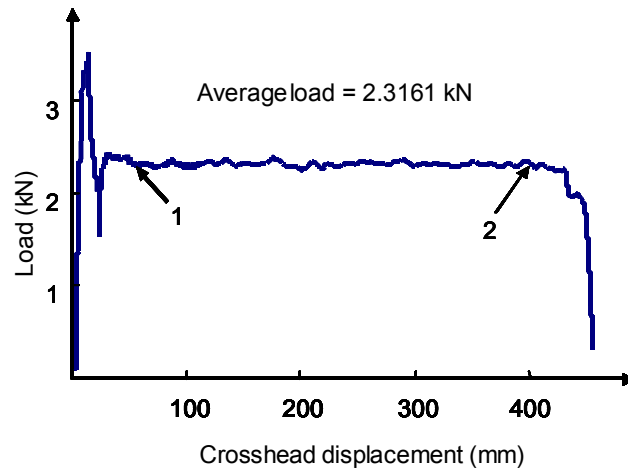


Figure 5. Experimental load-displacement graph (die semi-angle 3°).

### 3. Results and Discussion

The experimental data were obtained for die semi-angles varying from 2 to 9° and always for a value of  $\Delta = 1$ . The friction factor was then calculated through Eq. (2). Utilizing the Fig. 4 from Wistreich (1955, 1958), the optimum die semi-angle and reduction of area were determined by interpolation; the results are presented in Fig. 6. The values of the friction factor,  $m$ , as a function of die semi-angle,  $\alpha$ , for the present results, as well as those from Wistreich (1955,1958) are presented in Fig. 7.

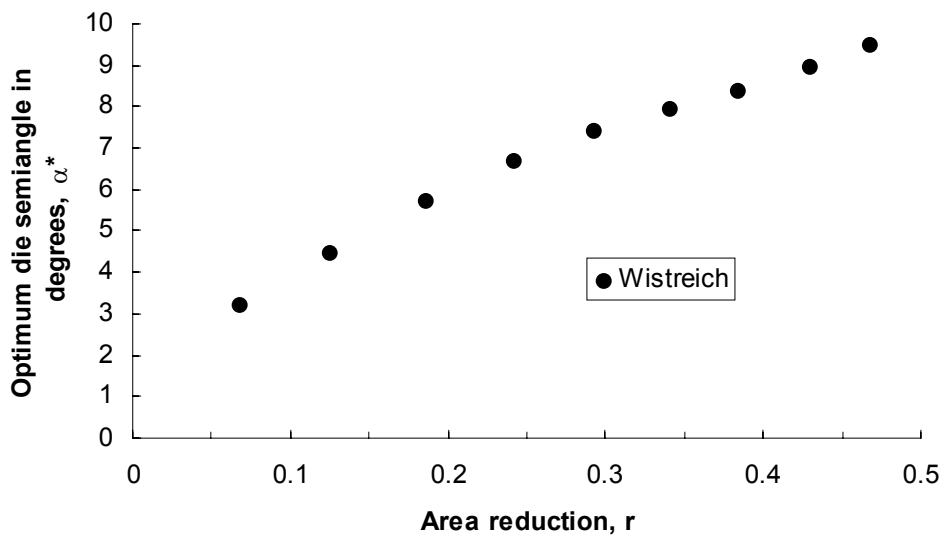


Figure 6: Optimum die semi-angle as a function of reduction of area, obtained through an interpolation from the results by Wistreich (1955,1958).

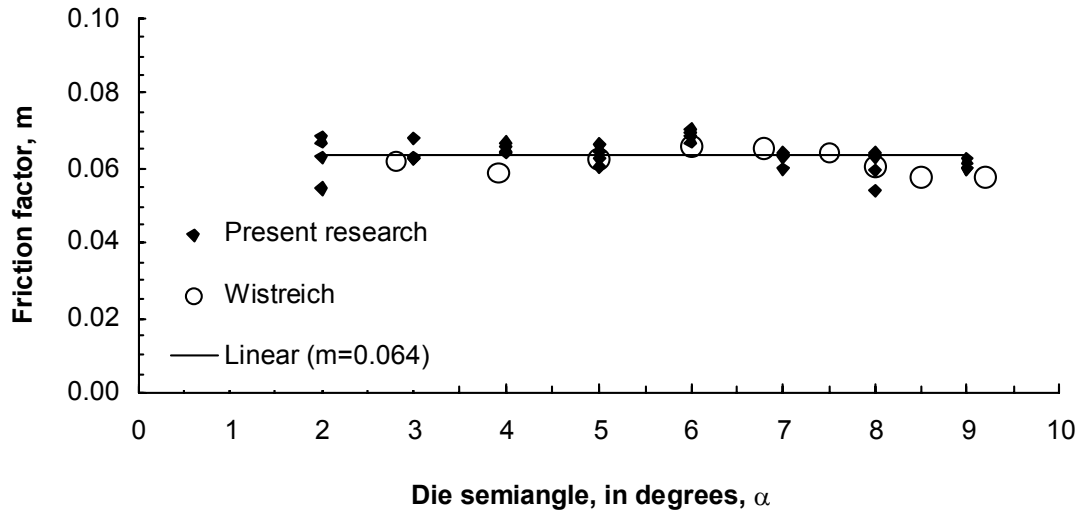


Figure 7. Friction factor versus die semi-angle for the present experimental results and for the results from Wistreich (1955,1958).

Figure 7 shows that the friction factor was constant for the various die semi-angles, with an average value of 0.064 and standard deviation of 0.004. This is consistent with the results from Wistreich (1955,1958), where the average friction factor was 0.062 and the standard deviation was 0.003. Wistreich had to use a large number of experiments in order to evaluate the friction factor for various die semi-angles, from Fig. 4. The present method allows such a determination from a lower number of tests. The present results agree with those from Wilson (1979), who considers that the friction factor is a function of the material under processing, of the tool material and of the lubricant, and is independent of the tooling geometry. On the other hand, our results do not agree with those from Baker e Wright (1992a,b), who consider that the lubricant and the die material have a significant influence on the friction factor, and that lower die semi-angles lower the friction because of increased lubricant drag. It should be remembered, however, that the present experimental drawings were performed under low speed conditions, where such lubricant drag effects should not be important.

Considering now the calculated average friction factor, the values of optimum die semi-angle were determined as a function of the reduction of area, as shown in Fig. 8. This is obviously based on the constancy of the friction factor as die semi-angle is varied. If this were not true, the situation would be substantially more complex. Figure 8 shows that there is an agreement between the present results and those based on Wistreich's method (1955, 1958). In addition to this fact, it confirms the validity of Avitzur's upper bound approach for axisymmetric drawing. Figure 8 shows the best drawing conditions, from the point of view of drawing stress. Figure 8 also allows the evaluation of the  $\Delta$ , for the various optimum situations. Figure 9 shows these values of  $\Delta$ ; it is clear that for low optimum die semi-angles the values of  $\Delta$  area above 2, which means that the strain heterogeneity in the drawn bar will be substantial. Low values of  $\Delta$  are reached only for high optimum die semi-angles (around  $10^\circ$ ), which, however, are associated with intolerably high reductions of area (about 0.5).

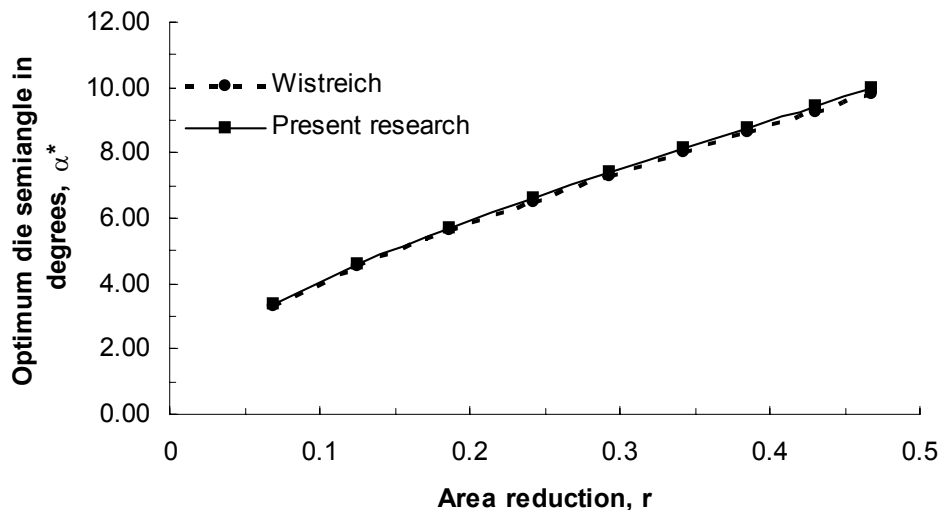


Figure 8. Optimum die semi-angle as a function of reduction of area.

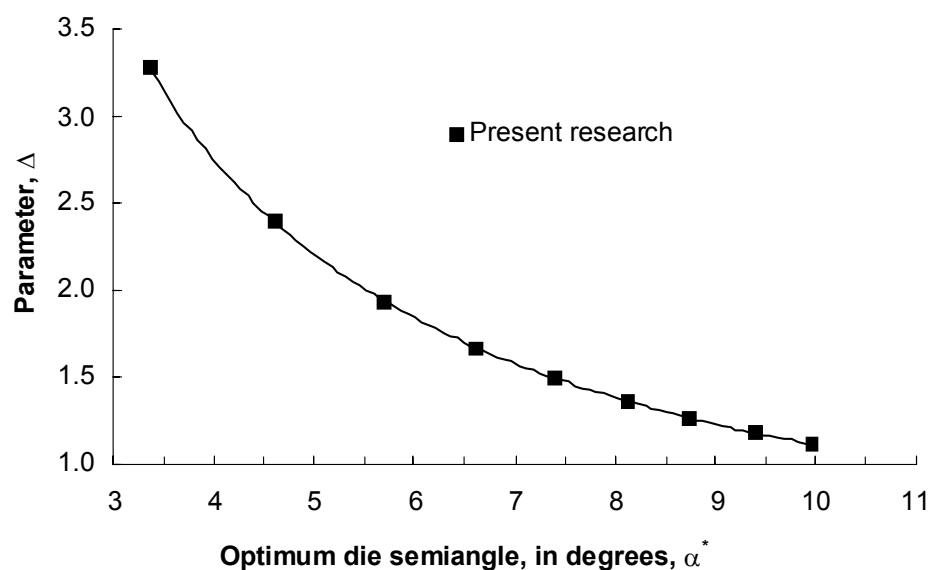


Figura 9. Evolution of the  $\Delta$  parameter with the optimum die semi-angle,  $\alpha^*$ .

#### 4. Conclusions

The methodology proposed in the present paper for the evaluation of the friction factor and optimum die semi-angle led to adequate values.

The friction factor is independent of the die semi-angle, for low drawing speeds.

The drawing of bars through dies displaying the optimum die semi-angle will usually lead to substantial strain heterogeneity in the drawn bar.

#### 5. Acknowledgements

The authors are thankful to Belgo-Bekaert for supplying the drawing dies, and to CNPq, CAPES e FAPEMIG for the financial support of this research.

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