WAVE SLOSHING INSIDE A 2D RESERVOIR

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Abstract. *The present work is related to the study of sloshing in a reservoir modelled as a 2-D impermeable box, with a flat horizontal bottom and two vertical walls, spaced by a distance d. The solution method is based on solving an integral equation that arises from Cauchy's integral theorem for functions of a complex variable. The nonlinear unsteady free surface flow is numerically simulated by means of a boundary-integral method. We suppose two distinct initial conditions: a sinusoidal standing wave profile and a flat free surface, both with total mean depth h. The aspect ratio h/L and the frequency of oscillations ω are the parameters of the problem. A lateral and/or vertical harmonic motion is imposed at the boundaries of the reservoir. Interesting features of the free surface are obtained and discussed.*

Keywords: *Free surface flows, nonlinear sloshing, boundary integral method.*

1. Introduction

Free surface waves can be generated by means of an imposed movement on a partially filled reservoir. Violent wave impacts inside confined spaces have been observed on vehicles transporting liquid substances, subject to large accelerations/decelerations. In such cases wave reflection on the walls induces hydro-dynamical forces, which may cause hazardous destabilizing effects. This fluid/structure interaction may cause the container's rupture, instability and loss of maneuverability on vehicles loaded with liquids. That is the case of fuel tanks in trucks, aircraft, spacecraft and ship, which in certain circumstances are subject to harmonic loads of high amplitude, and may reach resonant modes. Bredmose *et al.* (2003) observed that two very different types of response may coexist in a confined tank: a violent, brief impact of the liquid on the container wall and long, lasting/large amplitude sloshing motions. Ockendon & Ockendon (1973) find for a 2D box of length πL and depth *hL*, undergoing harmonic horizontal oscillations, a discrete spectrum of frequencies:

$$\omega = [gk \tanh(kh/L)]^{1/2}, \qquad (1)$$

where *g* is the acceleration of gravity and *k* is the wavenumber. Resonance occurs, when the fluid is forced to oscillate at these frequencies.

The generation of patterns of steep, standing waves is an important feature observed in sloshing motions. If certain driving frequencies are imposed, vertical accelerations in a container induces the growth of standing waves, also known as Faraday waves. Bredmose *et al.* (2003) report numerically and experimentally the formation of “table-top” breaking waves when studying steep, breaking, Faraday waves. Longuet-Higgins (2001) shows that vertical jets may result from high amplitude standing waves; that is known as the “bazooka” effect. The free surface flows mentioned above have gravity as its main restoring mechanism. On a smaller scale, however, surface tension may significantly affect wave properties. Small scale ripples, or capillary waves, occur due to external accelerations imposed on the reservoir. Billingham (2001) finds that, under zero gravity conditions, periodic and chaotic solutions and solutions where the topology of the fluid changes, either through self-intersection or pinch off, are possible.

The present work is concerned with free surface waves produced on a container, shaped as a 2D box, which is partially filled with liquid (see figure 1). The box is made of two vertical walls, set at a distance *L* apart. There may be an impermeable horizontal bottom set at a mean distance *h* from the free surface, or the water may be deep. Displacements of the free surface are measured by the distance from a rest state in which the surface is flat. Cartesian coordinates are defined by setting the *x*-axis in the undisturbed surface with the *y*-axis vertically upwards so that the fluid occupies the half plane $y \le 0$ when at rest. The free surface is defined as $y = \eta(x,t)$.

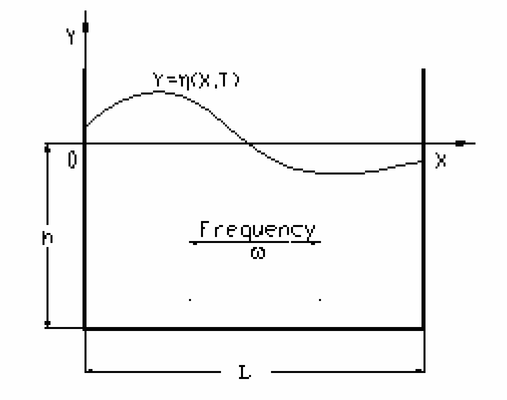


Figure 1. Fluid domain geometry.

2. Governing equation and boundary conditions

We assume an inviscid and incompressible free surface flow with the velocity field $\vec{u}(x,y,t)$ being given by the gradient of a full velocity potential $\Phi(x,y,t)$, which satisfies Laplace's equation in the fluid domain,

$$\nabla^2 \Phi = 0 . \tag{1}$$

The flow is supposed to be irrotational away from the free surface. The oscillations imposed at the box are introduced in the model by decomposing Φ into a regular part ϕ (due to surface waves) and a perturbed part $\bar{\phi}$ (due to a harmonic external forcing).

The free surface´s kinematic and dynamic boundary conditions are given by,

$$\frac{D\vec{r}}{Dt} = \nabla \Phi , \tag{2}$$

$$\frac{D\Phi}{Dt} = \frac{1}{2} | \nabla \Phi |^2 - \frac{p}{\rho} - gy . \tag{3}$$

$\vec{r}(x,y,t)$ is the position vector; *y* is the elevation of the free surface above the undisturbed water level; *g* is the acceleration due to gravity (acting vertically downwards); ρ is the fluid density; *p* is the pressure on the exterior side of the surface. At the vertical walls and at the bottom, the boundary condition is of the Neumann type i.e. both walls and bottom are supposed to be rigid and impermeable. In all the computed cases presented in this paper surface tension effects are neglected.

To complete the boundary value problem an initial condition for the free surface is chosen,

$$\eta(x) = \eta_0(x), \qquad \Phi(x,\eta) = \Phi_0(x,\eta_0), \tag{4}$$

for $t = 0$. Attention is directed to two initial conditions: a sinusoidal standing wave profile and a flat free surface, both with total mean depth *h*.

3. Fully nonlinear boundary integral solver

The boundary value problem is solved using an adapted version of the fully nonlinear potential flow program developed by Dold & Peregrine (1986). The method consists of applying a boundary-integral method to a free surface flow problem, which reduces significantly the computational demand for the calculation of the fluid motion, since only surface properties are evaluated. The numerical method solves an integral equation that arises from Cauchy's integral theorem for functions of a complex variable. The original numerical scheme was modified by Teles da Silva *et al.* (2003) for the inclusion of a lateral and/or vertical harmonic motion at the boundaries of the reservoir.

The calculation of the free surface velocity $\nabla\phi$ becomes relatively simple when applying Cauchy's integral theorem. If we take $z = x + iy$ as the complex equivalent of the position vector $\vec{r} = (x,y)$ for a certain time *t*, ϕ is an analytic function of *z*. The wave complex potential gradient is defined as,

$$q = \frac{\partial \phi}{\partial x} - i\frac{\partial \phi}{\partial y}, \tag{5}$$

which is also an analytic function of *z*. On the boundary, *z* is treated as a function of the parameter ξ and time *t*. Similarly, taking $Z(\xi,t)$ as the complex equivalent of the surface profile vector $\vec{R}=\left(x(\xi,t),y(\xi,t)\right)$, *q* can be defined in terms of the tangential and normal gradients of ϕ at the surface,

$$\overline{q} = \frac{\partial Z}{\partial n_1}\left(\frac{\partial \phi}{\partial n_1} + i\frac{\partial \phi}{\partial n_2}\right). \tag{6}$$

We assume that the surface contour *C* that surrounds the fluid domain is smooth, then applying Cauchy's integral theorem leads to,

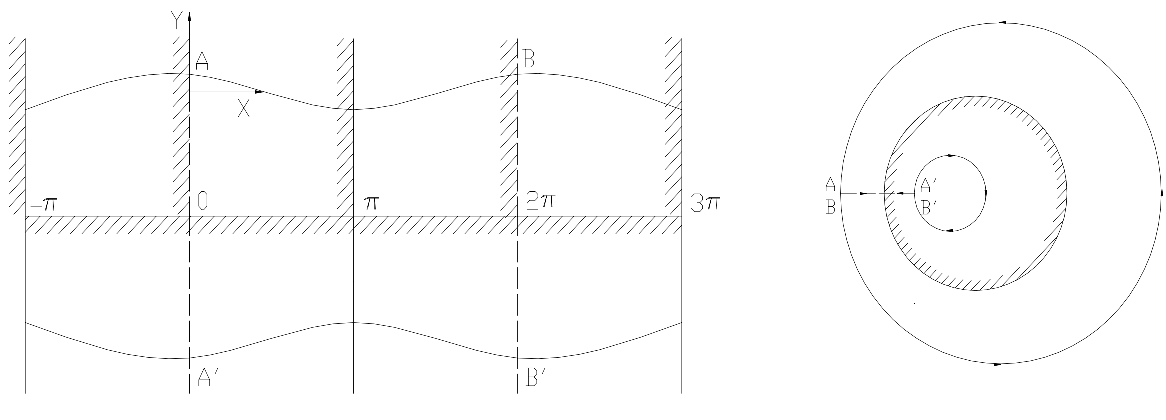
$$\frac{\partial \phi}{\partial n_2} = \frac{1}{\pi}\oint_C \Im\left(\frac{\partial Z/\partial n_1}{Z'-Z}\right)\frac{\partial \phi'}{\partial n_2}dn_1' + \frac{1}{\pi}\oint_C \Re\left(\frac{\partial Z/\partial n_1}{Z'-Z}\right)\frac{\partial \phi'}{\partial n_1}dn_1', \tag{7}$$

in which $\partial \phi / \partial n_2$ can be determined since $\partial \phi / \partial n_1$ can be calculated directly. The arclength n_1' is a scalar variable, which increases in an anticlockwise sense, around the closed contour *C*. The primed variables *Z'*, $\partial \phi' / \partial n_1$ and $\partial \phi' / \partial n_2$ are evaluated at points on the surface corresponding to n_1'.

For the purpose of evaluating $\partial \phi / \partial n_2$ when the surface is periodic in *x*, the "infinite" fluid surface is transformed into a finite closed contour via a conformal mapping of the form,

$$\varsigma(\xi,t) = e^{-iZ(\xi,t)}. \tag{8}$$

No generality is lost in assuming time and space dimensions to be suitably scaled by choosing a certain length *L* to make this period exactly 2π. Figure (2) shows schematically the *z* and ς-planes with a periodic wave and its corresponding reflections onto the bed and vertical walls.



(a) (b)

Figure 2. (a) A sketch of the *z*-plane with a periodic wave and its reflection onto the bed. (b) The corresponding ς -plane obtained via conformal mapping.

The harmonic external forcing is imposed by taking the equivalent perturbed potential velocity $\bar{\phi}$ of the horizontal and vertical accelerations, namely α_x and α_y ,

$$\alpha_x = \alpha_{x_0} \sin(kx + \omega t), \tag{9}$$

$$\alpha_y = \alpha_{y_0} \sin(\omega t), \tag{10}$$

where α_{x_0} and α_{y_0} are the amplitudes of the accelerations, k is the wavenumber and ω is the frequency of oscillations.

Basically the method of solution consists of the following stages. Initially the full potential Φ is known on the surface for each time step. The potential $\bar{\phi}$ due to the harmonic external forcing is also defined and subtracted from the surface value of Φ, such that the remaining surface wave potential ϕ can be used with Cauchy's integral theorem to calculate the velocity $\nabla\phi$ on the free surface. Then the potential $\bar{\phi}$ is added back in and corresponding "total" velocities are evaluated. The inclusion of $\bar{\phi}$ requires the computation of its partial derivatives with respect to x, y and t up to the third derivative, since the time-step criterion uses a Taylor series expansion truncated at the sixth power. Once an accurate converged solution is obtained for the full velocity potential Φ on the free surface, the cycle can begin again. Such stages are repeated until either the final time is reached, or the algorithm breaks down. Full details can be found in Dold (1992) and Moreira (2001).

4. Numerical results

At first, we investigate the free surface flow generated by a sinusoidal wave as an initial condition, with the reservoir at rest ($\omega = 0$). For small wave amplitudes ($A_0 < 0.2$), a gentle standing wave profile is obtained, which agrees with linear theory. For higher wave amplitudes ($A_0 \geq 0.2$), steep unsteady waves appear in the numerical computations. Figure 3 shows the unsteady evolution of a sinusoidal wave with amplitude 0.3 as the initial condition. In such case a wave run-up is observed with a magnitude of $0.58L$, which is approximately twice the initial wave amplitude (see figure 3b). Secondary wave modes are developed after reflection (figures 3c and 3d).

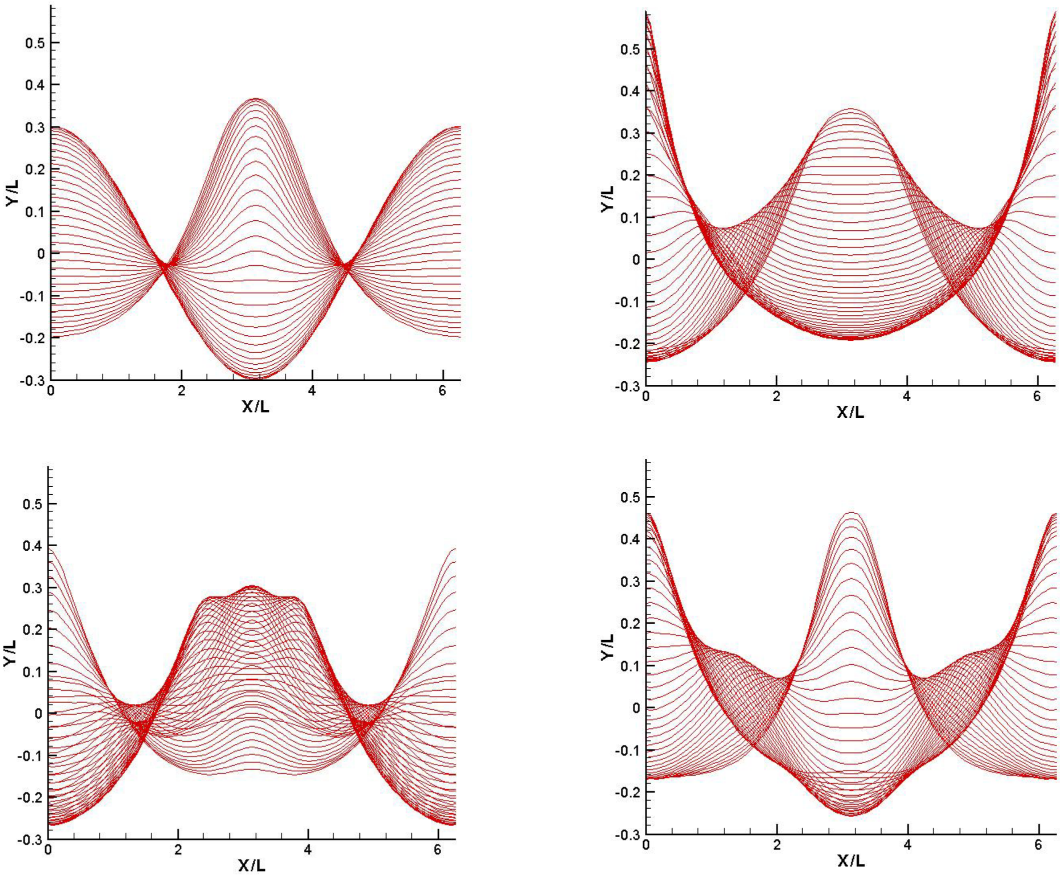
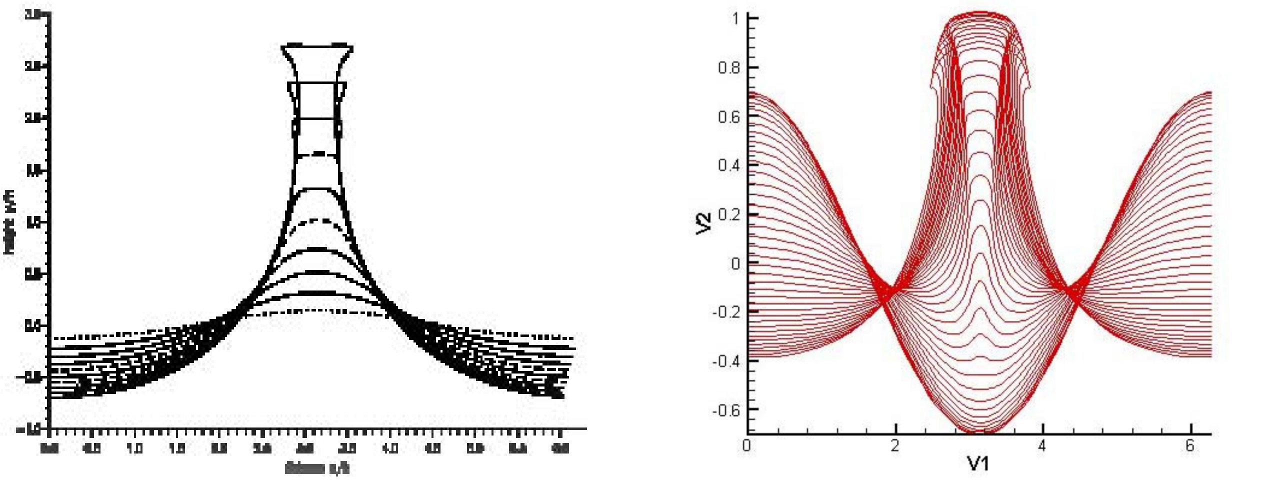


Figure 3: Evolution of non-stationary waves due to a sinusoidal wave with amplitude 0.3 as the initial condition.

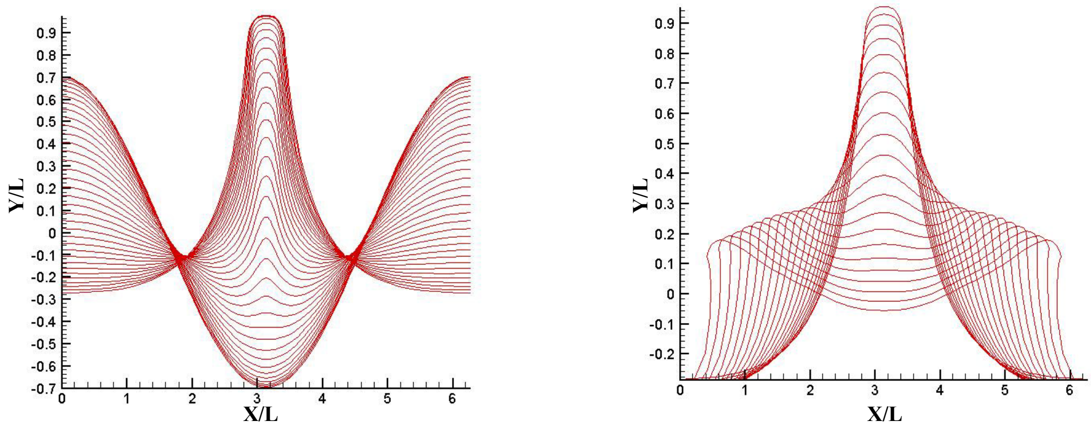
Bredmose *et al.* (2003) show the existence of nonlinear waves, called "table-top" waves, in experiments and numerical simulations. Figure (4a) shows the maximum heights for initial wave amplitudes varying from *0.1* up to *1.0*. A transition between a linear, gentle, wave solution to a fully nonlinear, steep, wave profile is observed. Figure (4b) shows the nonlinear evolution of a wave of initial amplitude 0.7.



(a) (b)

Figure 4: (a) Maximum heights for initial wave amplitudes varying from *0.1* up to *1.0*. (b) Nonlinear evolution of a wave of initial amplitude *0.7*.

The nonlinear results presented in figure (4b) are strongly modified by a vertical oscillatory movement imposed on the reservoir. The harmonic motion satisfies equation (10) in the fluid domain with α_0 = *- 0.5*, *k = 1* and *ω = 1*. Figure 5 shows the corresponding free surface flow. The "table-top" formation is postponed, with two wave fronts propagating in opposite directions (see figure 5b).



(a) (b)

Figure 5: Nonlinear evolution of a wave of initial amplitude 0.7 with a vertical oscillatory movement imposed on the reservoir.

5. Preliminary experimental results

An experimental set up is developed, aiming to observe the free surface behaviour due to an imposed harmonic motion on the container. An aquarium measuring 40 x 25 x 20 cm^3 is installed on a planer at the workshop of the Mechanical Engineering Department, Fluminense Federal University. The images are collected through a Nikon digital camera with 25 frames per second. Only horizontal motions are considered in this set of experiments. Figure 6 shows the evolution of a bore to the moment of impact. The camera is positioned in an inertial reference frame. Even for small displacements of the reservoir, an energetic splash can be observed. For bigger scale experiments, see for example http://www.jiscmail.ac.uk/files/COZONE/.

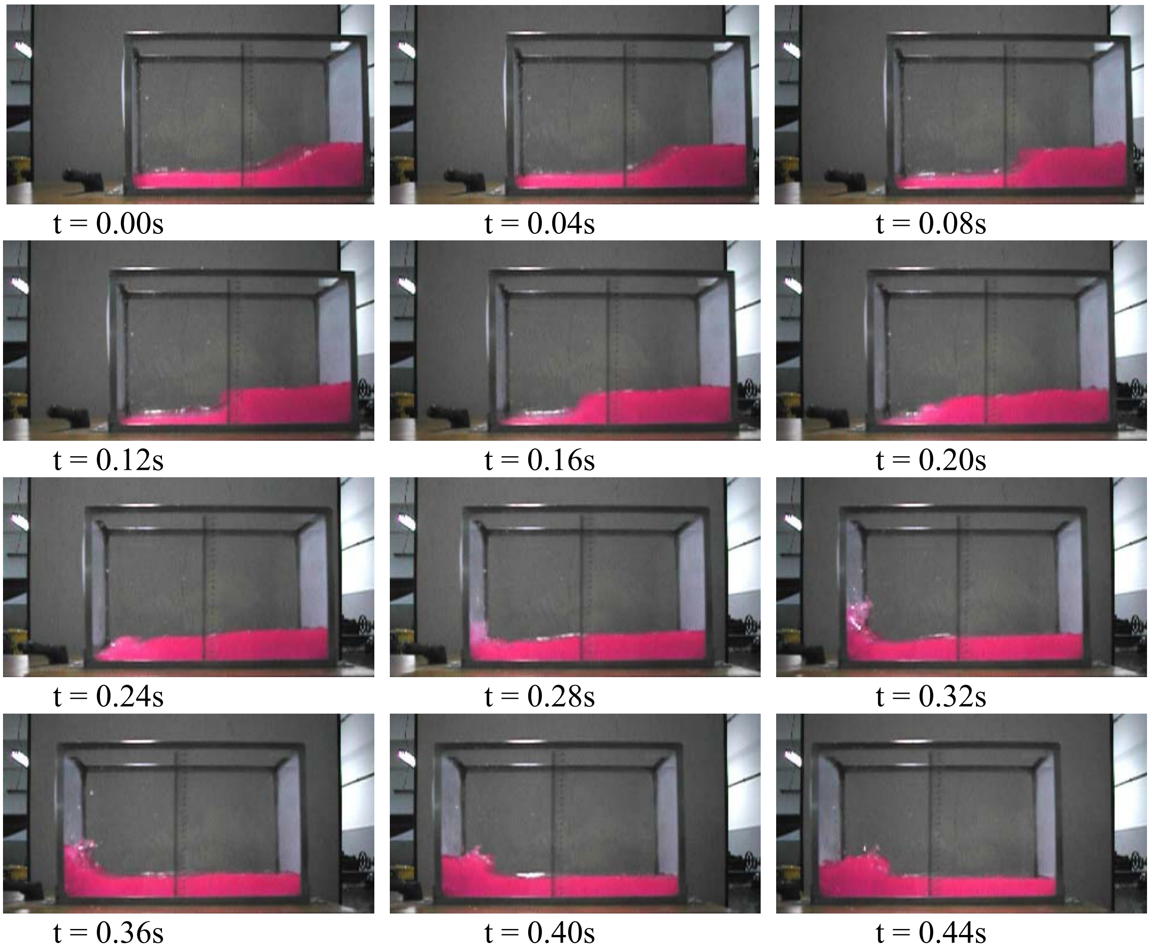


Figure 6: Evolution of a non-stationary wave with a flat free surface initial condition.

6. Summary

The efficient and precise algorithm developed by Dold & Peregrine (1986) to compute free surface flows has been successfully extended to simulate fully nonlinear sloshing in a 2D box. Preliminary results show that under certain conditions nonlinearity may become important even for smallish wave amplitudes.

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