ANALYSIS AND IDENTIFICATION OF THE DYNAMICS OF VIBRATION'S COMPLEX SYSTEM

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Abstract: *In this work they come the project, the analysis and the obtained results of simulations and experimental of a model developed for identification of the dynamics of complex systems. The analyzed complex system is a system of two degrees of freedom represented by a double pendulum. The identification of the dynamics based on the analysis and interpretation of the relationship between the cinetic energy and potential energy that the the pendulums acquire when they are in movement. The analyses of the energies were compared with the one of the behavior of the system in the domain of the time and in the domain of the frequency using for that RNMCs. The simulations and the experimentations were made with and without additional weights coupled to the pendulums, that it made possible the determination of the variation of the weights on one of the pendulums. The detection of the presence of the two pendulums is basically identified in one of the pendulums.*

Keywords*: Control and Robotics, Vibrations and Acoustics, Complex Systems.*

1. INTRODUCTION

Complex systems, such as the systems of suspension of the motorcycles, suspension of the automobiles, double pendulums or our own body can be modeled mathematically as systems of more than a degree of freedom. Those systems can have so many degrees of freedom how many they are necessary for your modelling. What determines the degrees of freedom of a system it is the number of necessary independent coordinates to specify your dynamics. In a system of double pendulum for instance, the dynamics of the two pendulums can be specified being known your positions, speeds and accelerations represented in two coordinates, that can be coordinated in the axis x or in the axis y, or angular coordinates.

Now some techniques of vibration analysis (or oscillation of pendulums) sophisticated are available for being used in diagnosis and forecast of flaws in complex systems. Among them, it can be mentioned the techniques of Artificial Intelligence represented by Specialist Systems, Networks Artificial Neurais, Fuzzy Logic, etc.

The objective of this work is to develop a system of identification of parameters, characterized by the reduction of oscillations of complex systems, using Network Artificial Neural and Fuzzy Logic. The dynamics of the complex system will be analyzed using information about the kinetic and potential energy of the two pendulums.

2. THE COMPLEX SYSTEM

The equation of movement of a dynamic system of second order can be calculated using the conservation of energy, where the kinetic energy is conserved in the mass in reason of the speed and the potential energy is conserved under the form of effort of elastic deformation or under the work form.

The Fig. (1) show a simplified drawing of a complex system composed of two articulate pendulums in your extremities. Observe that position detector only exists in the pendulum 1.

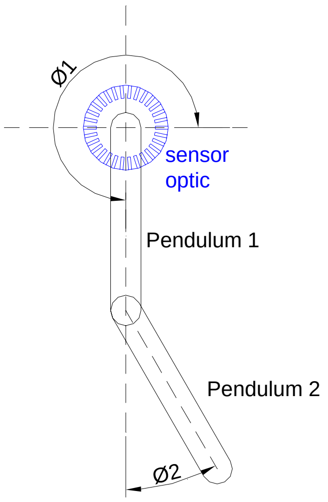


Figure 1. Double Pendulum

In this article we show obtained results of simulations and experimental results of a new method for the identification of the dynamics of a double pendulum. The new method uses the information about the potential energy and the kinetic energy represented respectively by the position and the angular speed of the pendulums. During the simulations and in the experimental phase the two pendulums are positioned in appropriate angles and soon after they are loosened, making them to move in free movement. The simulations were made with and without additional masses (of different values) coupled to the pendulums. The experimental results were obtained starting from the model of a double pendulum in the form presented in Fig. (1).

2.1 Analysis of the complex system

A manipulator's dynamic equation (**Gray,** 1997) can be written in the form of the Eq. (1).

$$\tau = M(\theta)\ddot{\theta} + V(\theta + \dot{\theta}) + G(\theta) \tag{1}$$

where $M(\theta)$ defines the manipulator's mass matrix, $V(\theta + \dot{\theta})$ contains all those terms which have any dependence on joint velocity, and $G(\theta)$ it is a vector of the gravity terms. The Eq. (1) is known as equation of state space because the term $V(\theta + \dot{\theta})$, appears in the equation with position dependence and speed. Each element of $M(\theta)$ and $G(\theta)$ depends of θ, that is the position of all the junctions of the manipulator. Each element of $V(\theta + \dot{\theta})$ depends so much of θ as of $\dot{\theta}$. Separating the several types of terms that appear in the dynamic equations is formed the head office of the manipulator's mass, the vectors centrifuge and of coriolis, and the gravity vector.

The head office of the manipulator's mass $M(\theta)$ is composed of all those terms that multiply $\ddot{\theta}$, and it is a function of θ. Therefore we have:

$$M(\theta)=\begin{bmatrix} l_2^2 m_2+2l_1l_2m_2c_2+l_1^2(m_1+m_2) & l_2^2m_2+l_1l_2m_2c_2 \\ l_2^2m_2+l_1l_2m_2c_2 & l_2^2m_2 \end{bmatrix} \tag{2}$$

The term speed, $V(\theta+\dot{\theta})$, all contain those terms that possess some dependence with the speed. Therefore we have:

$$V(\theta,\dot{\theta})=\begin{bmatrix} -m_2l_1l_2s_2\dot{\theta}_2^2-2m_2l_1l_2s_2\dot{\theta}_1\dot{\theta}_2 \\ m_2l_1l_2s_2\dot{\theta}_1^2 \end{bmatrix} \tag{3}$$

The term $-m_2l_1l_2s_2\dot{\theta}_2^2$ is caused by a force it centrifuges, that is recognized because it depends on the square of the speed. The term as $-2m_2l_1l_2s_2\dot{\theta}_1\dot{\theta}_2$ it is caused by a coriolis force and will always contain the product of two different joint velocities.

The term gravity, $G(\theta)$, contains all those terms in which the gravitational constant, *g*, appears. Therefore we have:

$$G(\theta)=\begin{bmatrix} m_2l_2gc_{12}+(m_1+m_2)l_1gc_1 \\ m_2l_2gc_{12} \end{bmatrix} \tag{4}$$

Following the project methodology of Craig (1986), the expression for the calculation of a manipulator's of two degrees of freedom torque is given in function of the position, velocity and acceleration. That can be written in the way presented in the Eq. (5) and Eq. (6).

$$\begin{aligned} T_1 &= m_2l_2^2(\ddot{\theta}_1+\ddot{\theta}_2)+m_2l_1l_2c_2(2\ddot{\theta}_1+\ddot{\theta}_2)+(m_1+m_2)l_1^2\ddot{\theta}_1-m_2l_1l_2s_2\dot{\theta}_2^2 \\ &-2m_2l_1l_2s_2\dot{\theta}_1\dot{\theta}_2+m_2l_2gc_1+(m_1+m_2)l_1gc_1 \end{aligned} \tag{5}$$

$$T_2=m_2l_1l_2c_2\ddot{\theta}_1+m_2l_1l_2s_2\dot{\theta}_1^2+m_2l_2gc_1+m_2l_2^2(\ddot{\theta}_1+\ddot{\theta}_2) \tag{6}$$

3. SIMULATION OF THE COMPLEX SYSTEM

The simulations were made for the analysis and identification of the parameters of the complex system. During the simulations the model of the dynamics of two pendulums was presented by Craig (1986) (Eq. (5) and Eq. (6)). Besides, the method of Runge-Kutta of fourth order was used for solution of the equations you differentiate of the model.

The results of the simulations are presented through the behavior of the state variables referring to the position and the speed of each pendulum. There will also be presented the signs in the domain of frequency referring to the oscillatory movement of the pendulums.

In the initial simulations additional masses were admitted for each pendulum. Starting from those masses additional masses were added separately in each pendulum or in both. The increased additional masses had different values and they were placed in varied positions along the pendulum.

3.1. Simulation of the pendulums without extra mass additional

The results presented in this section were obtained with the pendulums without additional mass. Fig. (2.a) it presents the pendulums in the initial position (unstable position). The pendulum 1, presented in the red color in Fig. (2.a), it was positioned to 90º with relationship to the rest position.

The pendulum 2, presented in the blue color, it was positioned to -30º in relation to the first pendulum.

Fig. (2.b) shows the traced curves of the state variables (paths) obtained of the movement of the pendulums after they be loosened in the initial position. The wave form in red color, represents the behavior of the state variables (speed in function of the position) observed in the pendulum 1. The wave form in blue color represents the behavior of the variables of state of the pendulum 2. it is Observed that the path of the pendulum 1 presented a form almost spiral, and the path of the pendulum 2 presented a chaotic path.

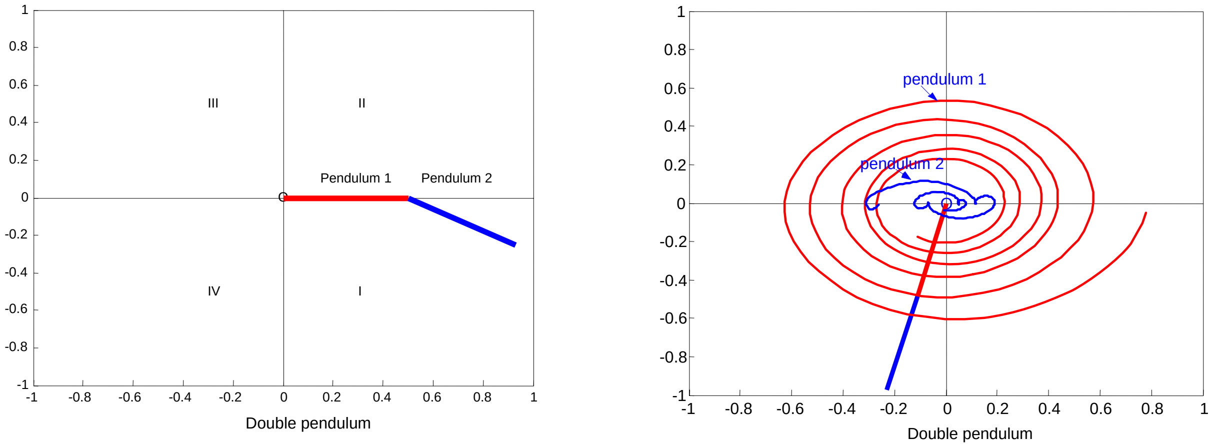


Figure 2. Initial Position (the) and the paths of the pendulums (b)

Fig. (3.a) shows the displacement of the pendulum 1 in function of the time. Fig. (3.b) shows the spectrum of the sign of the pendulum 1 in the domain of the frequency obtained using transformed of Fourier. The spectrum presents a harmonica well evidenced that represents the frequency of the movement of the pendulum 1 (natural frequency - 6 Hz). the frequency that represents the movement of the pendulum 2 (natural frequency - 4 Hz) it is not evidenced in the spectrum of the sign of Fig. (3.b).

Fig. (3.c) shows the displacement of the pendulum 2 in function of the time. In Fig. (3.d) shows the spectrum of the sign of the pendulum 2 in the domain of the frequency obtained using her transformed of Fourier. The spectrum presents two harmonicas that represent the frequencies of the movements of the pendulums 1 and 2 (natural frequencies, fl=6Hz and f2=4Hz), evidenced through the respective picks.

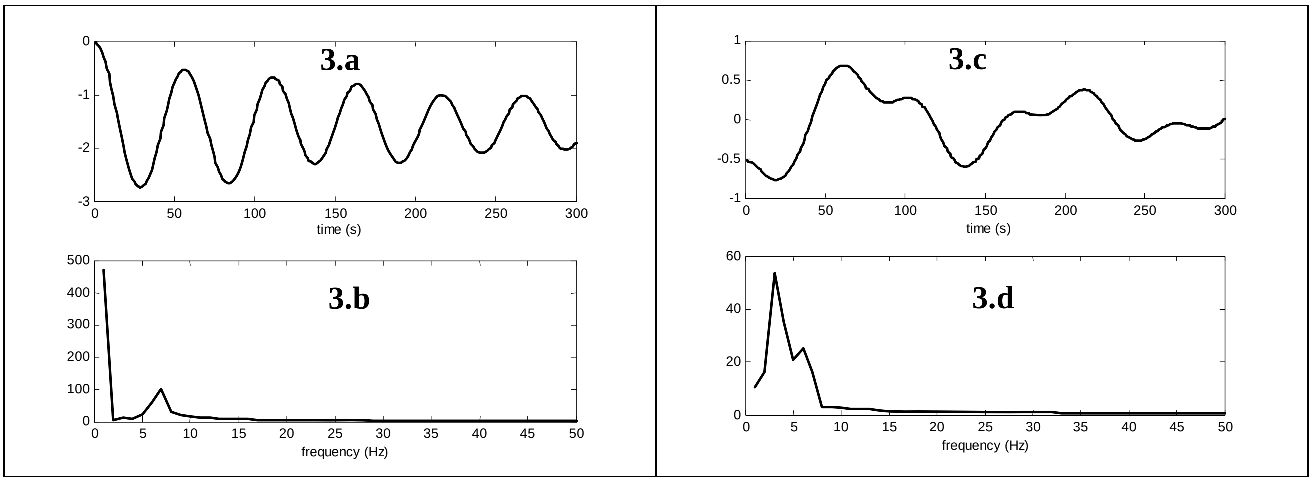


Figure 3. Sign in the domain of the time (a,c) and in the frequency of the pendulums (b,d)

3.2. simulation of the pendulums with additional mass

In that simulation mass was added to the pendulum 2, and observed the behavior of the path of the two pendulums. In Fig. (4) it is observed that the spiral of the movement of the pendulum 1 presents a different form from the one that it was presented in Fig. (3.b) (simulation of the two pendulums without additional mass).

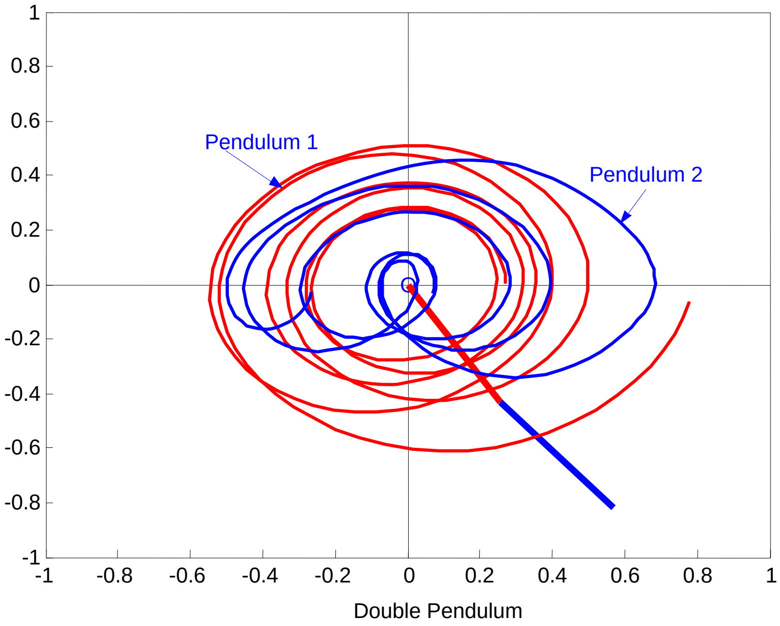


Figure 4. Path of the pendulums with extra weight

Fig. (5.a) shows the displacement of the pendulum 1 in function of the time. Fig. (5.b) shows the obtained spectrum of the harmonic analysis using transformed of Fourier of the movement of the pendulum 1. The spectrum presents a harmonica well evidenced that represents the frequency of the movement of the pendulum 1 (natural frequency - 7 Hz) and a harmonica, faintly evidenced, that represents the movement of the pendulum 2 (natural frequency - 4 Hz).

Fig. (5.c) shows the displacement of the pendulum 2 in function of the time. Fig. (5.d) he/she comes the obtained result of the harmonic analysis using transformed of Fourier of the movement of the pendulum 2. it is Observed that now is possible the visualization of the two harmonicas (f1=7Hz and f2=4Hz), although the second harmonica is still the one that it presents a value more accentuated. with the additional mass.

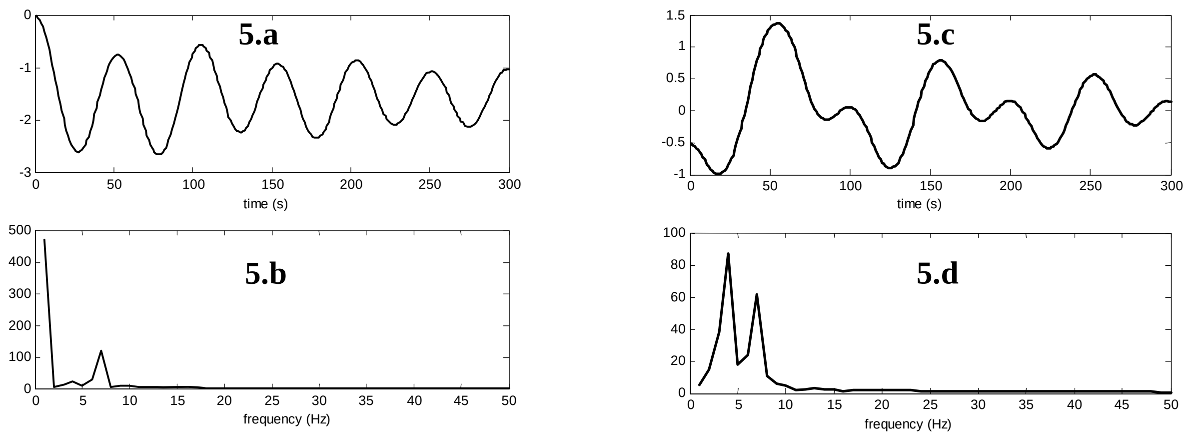
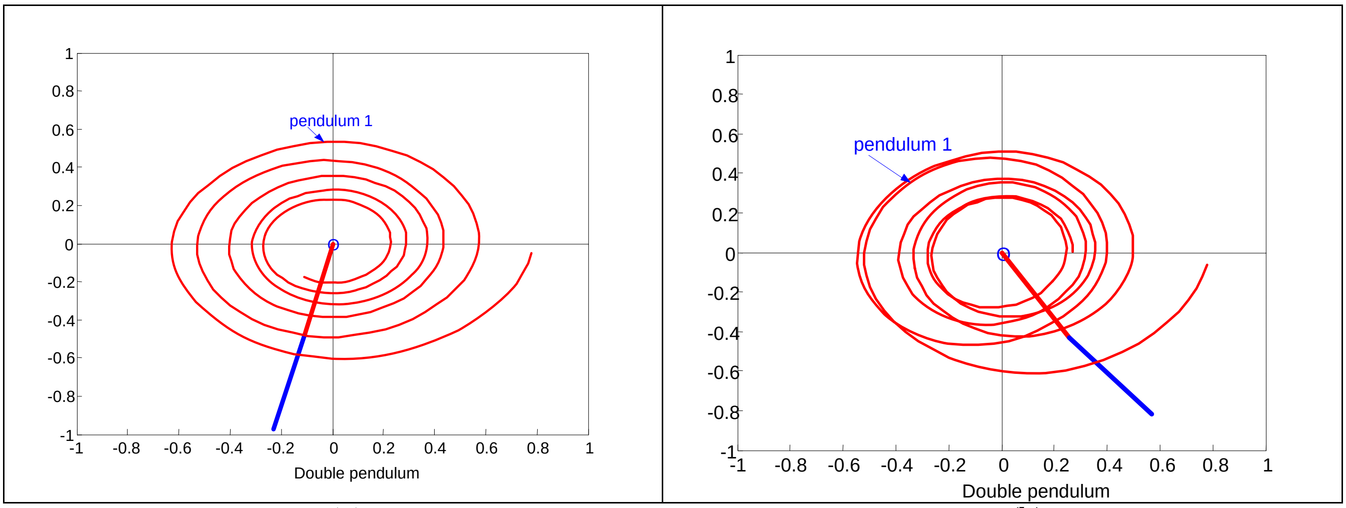


Figure 5. Sign in the domain of the time (a,c) and in the frequency of the pendulums (b,d) with extra weight

4. IDENTIFICATION OF DYNAMICS OF THE TWO PENDULUMS

In the identification of the dynamics of the two pendulums it is observed that the determination of the first harmonica being used the path of the pendulum 1 depends strongly on the weight used in the pendulum 2. Because the curves of the paths of the pendulum 1 (with and without additional mass) indicate the disturbance exercised by the pendulum 2.

In Fig. (6) a comparative one is made among the paths of the pendulums without (Fig. (6.a)) and with additional mass (Fig. (6.b)) in the pendulum 2. In Fig. (6.a), without additional mass, the relative spiral to the movement of the pendulums comes with a geometry pattern. Already Fig. (6.b), with extra weight in the pendulum 2, the relative spiral to the movement comes with deformation in your geometry. In that case, it can be considered that there was a variation in the kinetic and potential energy of the curve of the pendulum 1 that must have been caused by the existence of the other pendulum. And that through the analysis of the path of the pendulum 1 the existence of the extra pendulum can be detected (pendulum 2) in a system with two pendulums.



(a) *(b)*

Figure 6. Comparison among the two paths: (the) without and (b) with additional mass.

Fig. (7) shows the curve of the displacement of the pendulum 1 for different additional masses placed in the pendulum 2. The additional masses were always placed with growing values starting from Fig. (7.a). In Fig. (7.a) it is noticed that the consecutive maximum values of the curve always maintain decreasing not indicating the action of the pendulum 2 on the pendulum 1. In Fig. (7.b) the weight was a little larger than the previous, and even so the decreasing pattern of the maximum values of the curve was maintained, characterizing the non influence of the pendulum 2 on the pendulum 1. In Fig. (7.c) and in Fig. (7.d), where the extra additional mass had larger value that the extra additional mass used for obtaining of Fig. (7.a) and Fig. (7.b), they appear decreasing and growing consecutive maxima, indicating the action of the pendulum 2 on the pendulum 1.

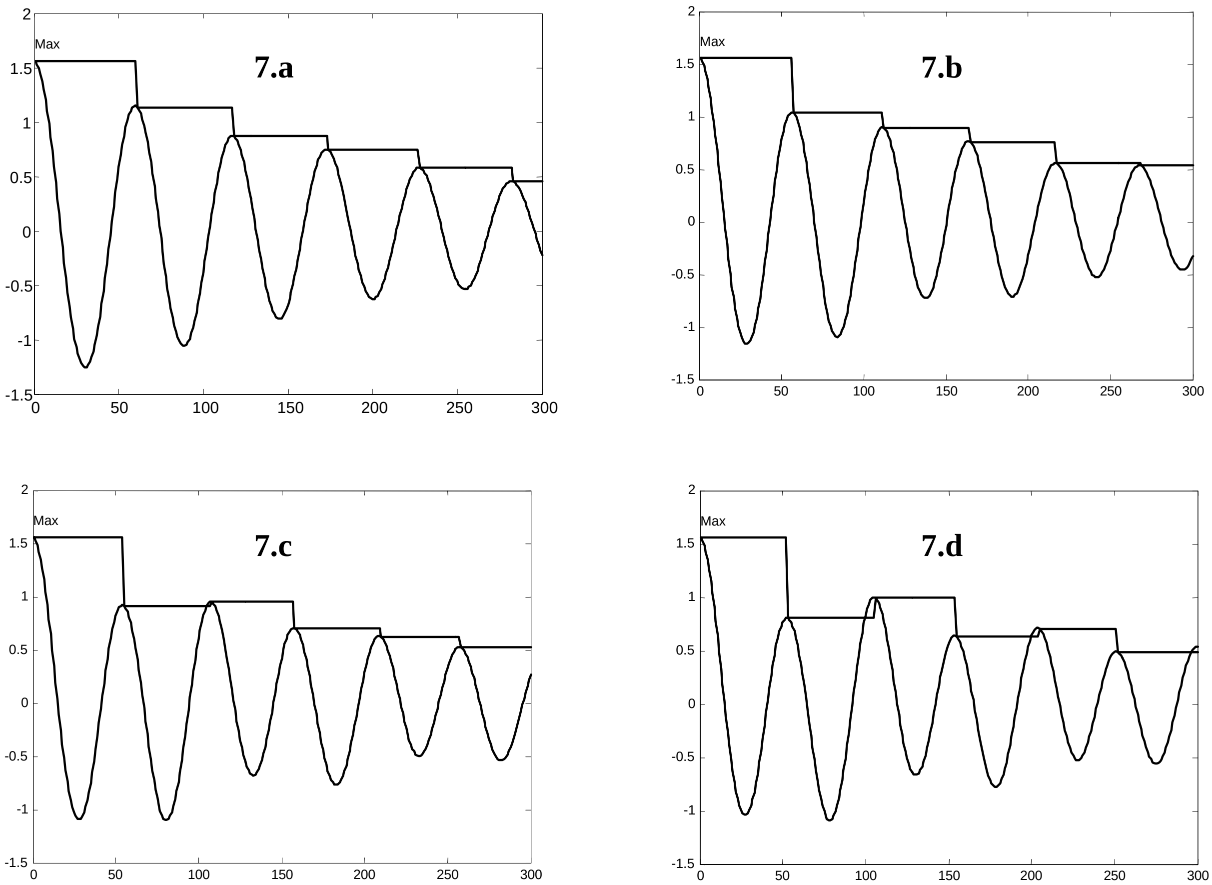


Figure 7. Move oscillatory of the pendulums with extra additional mass differentiated (Tab. (1))

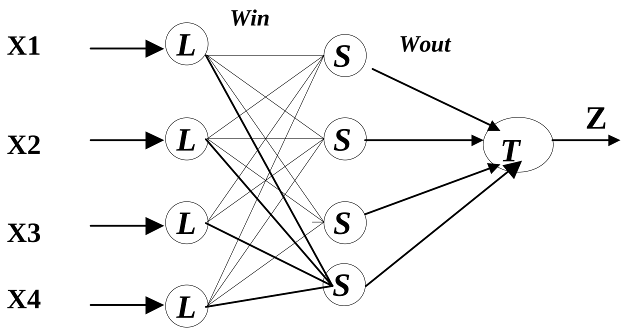
The Table (1) shows the obtained results of simulations for seven values (growing values) of extra additional mass coupled to the pendulum 2. it is Observed that the consecutive values of the maxima for the first two mass additional extras behave in a decreasing way. Starting from the third extra weight the behavior of the consecutive maximum values comes growing and decreasing. The column 2 Pend of Tab. (1) indicates the occurrence of extra weight in the pendulum 2. In the column Objective the value was associated the when the values of the maxima were decreasing, and associated values zero when the maxima were decreasing and growing.

Table 1. Values maxima of the movement of the pendulums for different additional mass

Maxima					2 Pend	Objective	RNMC
Peso 1	1.5575	1.1347	0.8682	0.7402	Não	1.	0.9999
Peso 2	1.5566	1.0374	0.8958	0.7586	Não	1.	0.9953
Peso 3	1.5560	0.9123	0.9490	0.7068	Sim	0.	0.0018
Peso 4	1.5558	0.8027	0.9938	0.6298	Sim	0.	-0.0021
Peso 5	1.5559	0.6980	1.0210	0.5758	Sim	0.	-0.0016
Peso 6	1.5564	0.6254	1.0561	0.5414	Sim	0.	0.0020
Peso 7	1.5572	0.6492	0.2456	0.8902	Sim	0.	0.0000

5. ANALYSIS NEURAL

To proceed, it was decided to use a RNMC (network multiple layer neural) for the identification of the double pendulum using the information contained in Tab. (1). The Fig. (8) to the left, shows used RNMC. RNMC four lineal neurons in the entrance layer (X1, X2, X3 and X4), eight neurons sigmoides in the exit layer, and a neuron of the hyperbolic tangent type in the exit layer (Z). RNMC was trained during 100 iterations using as entrance the seven groups of the four values maxima of Tab. (1). the objective of the training of RNMC (value wanted in the exit of RNMC) were the values presented in the column Objective of Tab. (1). The Fig. (8) to the right shows the variation of the acting index during 100 iterations. The mistakes observed in the training were smaller than 1%. The results presented by RNMC they are in the last column of Tab. (1).



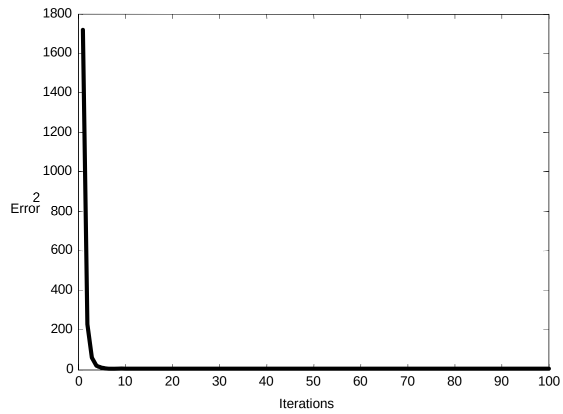
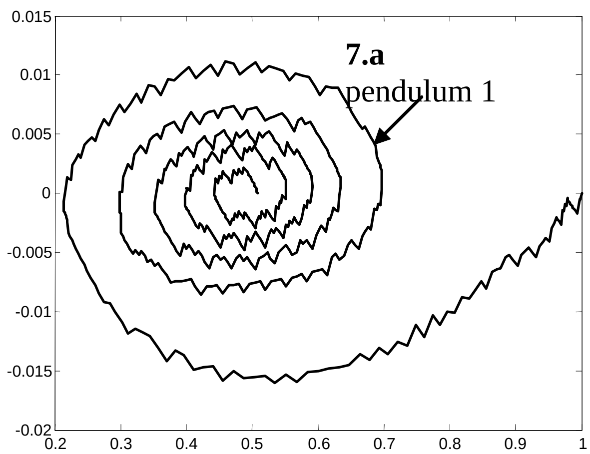


Figure 8. RNMC and the variation of the index of acting of RNMC

6. EXPERIMENTAL DATA

To proceed we shows the results of an experimental test of the double pendulum. The two pendulums had the same mass m=0.01Kg and length L=0.1m and additional mass with m=0.02Kg. we used one photoelectric sensor coupled to the pendulum 1 (according to Fig. (1)) that developed in the atmosphere C++ builder through a program, with communication with the parallel port of the computer, making made the reading of the position of the pendulum. An amplifying sign board was developed for the sensor that also made digital that sign to be read by the computer. After the sign to be collected (800 points) it was again turned into analogical sign and analyzed using Matlab.

The Fig. (7.a) shows the relative spiral to the movement of the pendulum 1 without additional mass in the pendulum 2. it is Observed that the presented spiral is regular.



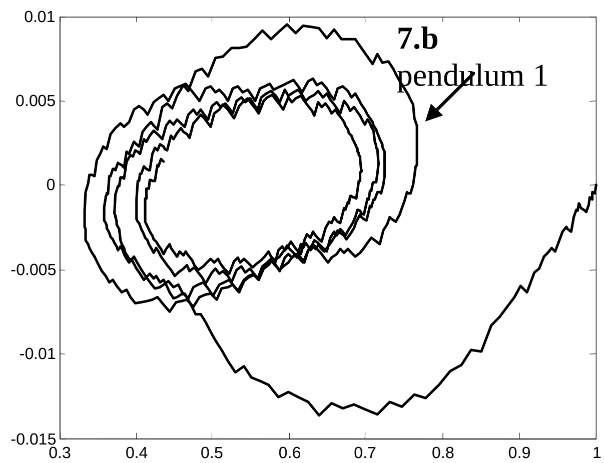


Figure 7. Displacement of the pendulum 1: (a) with and (b) without extra weight

The Fig. (7.b) shows the relative spiral to the movement of the pendulum 1 with additional mass in the pendulum 2. it is Observed that the presented spiral is irregular. Although the graphs of Fig. (7) are with noise, it is possible to verify that the path of the pendulums without extra weight in the pendulum 2 (Fig. (7.a)) it presents a spiral well with a form held. Already in Fig. (7.b) the spiral comes quite deformed with relationship the previous, characterizing the influence of the pendulum 2 perfectly on the pendulum 1.

The Fig. (8.a) shows the minimum values of the displacement obtained with the pendulum 2 without additional mass, the minima are in the growing order as expected. The Fig. (8.b) shows the minimum values of the displacement obtained with the pendulum 2 with additional mass, the minima are in the growing and decreasing order indicating the presence of the extra weight in the pendulum 2.

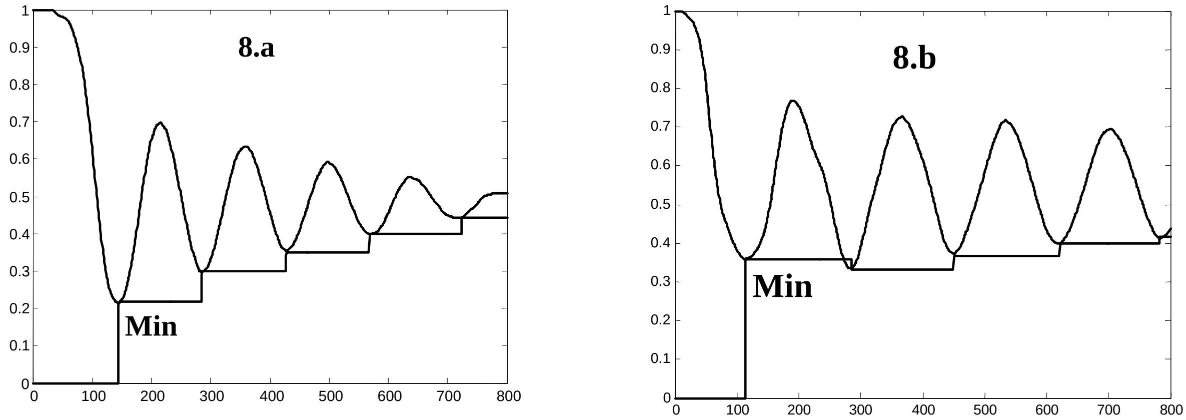


Figure 8. Minima in the displacement: (a) of the pendulum 1 and (b) of the pendulum 2.

7. CONCLUSION

A system was presented capable to detect the existence of two pendulums and capable to identify the dynamics of the pendulums using additional masses conveniently coupled in the simulations and in the experimental test.

Starting from results of simulations it was observed that to detect the existence of two pendulums:

1) for small oscillations of the pendulum 2 it can be necessary the use of big additional mass (in relationship the masses of the pendulums);

2) the graph of the speed in function of the displacement (variables of states) it was shown suitable for the detection of the dynamics of the new pendulum;

3) the curve of the maxima (or of the minima) of the displacement values shows as an appropriate form for the determination of the dynamics of the two pendulums;

4) RNMC was shown appropriate the analysis of the values of the maxima (or of the minima) of the displacement of the pendulum 1 for the detection of the dynamics of the pendulums.

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