# STICK-SLIP ANALYSIS IN DRILL STRINGS

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***Abstract.*** *This work attempts to model the nonlinear dynamic behavior of torsional stick-slip vibration in rotating systems in interaction with longitudinal vibration. This phenomenon is found in many drilling processes, frequently in oil well drilling, and the vibrations are coupled through the cutting force at the bit. Vibration in this kind of systems is an important cause of premature failure of the components. The longitudinal vibration was analyzed at ICVOP 2002; now the focus will be on torsional vibration, but considering the coupling to the longitudinal vibration. Stick-slip causes a violent torsional vibration due to the long drill pipe and the consequent large amplitudes and it is necessary to understand the dynamic behavior to take care with some operation parameters. To model the stick-slip phenomenon in oil well drilling it is used a slender rotating vertical column, which represents a simplified model of a drill string. For this torsional column, it is considered that the top is rotating with a constant rotary speed and the bottom end is subject to the cutting force. This force represents the effect between rock and bit and is strongly nonlinear. To solve the equation of motion of the model, the one-dimensional space domain is discretized through finite difference, after, the classic Runge-Kutta method is employed to obtain the time response. The numerical simulations show that the behavior of the system depends strongly of the rotation velocity. When represented in state-space the behavior shows a limit cycle that is typical of systems with dry friction. The coupling with the longitudinal vibration occurs under the action of the weight on bit, which consists of a static load perturbed by a dynamic one. The interaction between these two kinds of vibration over the non linear force that connects the bodies is the goal of this work.*

***Keywords****: Dynamics, Vibration, Stick-Slip, Finite Differences.*

## 1. Introduction

In the oil/gas industry the drilling operation has special importance. Usually, the drilling process is made with a rotary drill-string, therefore, it is necessary to understand the dynamical behavior of it to know the potential operational parameters for the drilling process. The drill-string is a slender structure composed of steel tubes linked together by special joints. In this work it is assumed that the drill-string has two parts, the drill-pipe and the drill-collar. The drill-collar is a short segment of thick walled pipes, this kind of pipes is used to avoid the drill-string buckling and generate an axial force over the formation.

At the free end of the drill-collar is attached a bit, it is used to make the hole in the rock formation. In the present work it is assumed that the bit is of the tri-cone type and is drilling in hard rock formation.

In order to investigate the dynamics of the system, the axial and torsional motion equations for the drill-string are obtained using the classical continuous mechanics, after, they are solved using numerical methods. Finally, the numerical results are compared with those from the related literature.

## 2. Drill String Modeling

### 2.1 Description

The drill-string dynamics is complex because the axial, torsional and lateral vibrations may be present simultaneously and coupled. When the vibrations are severe, resonances, there are dysfunctions in the drill-string that conduces to premature damage and failure of its components. The dysfunctions associated for axial, torsional and lateral vibrations are bit-bounce, stick-slip and whirl motion, respectively. Cunningham (1968) observed, from real drilling process that the bit-bounce is frequent when drilling in hard rock formations with tri-cone bits. Also, the stick-slip dysfunction was pointed out by Dareing (1968) in field observations and Brett (1992) reproduces it in an experimental test rig.

### 2.2 Equation of Motion

For simplicity of the drill-string model, shown in Fig. 1, the motion equations are obtained from a differential element. The axial and torsional displacements are represented by variables $u = u(x,t)$ and $\phi = \phi(x,t)$, respectively. For a differential element, axial and torsional motion equations are represented by the partial differential equations (PDE)

$$EAu'' = \rho\ddot{u} + \gamma_a\dot{u} - \rho g$$
$$GJ\phi'' = \rho J\ddot{\phi} + \gamma_t\dot{\phi} \quad (1)$$

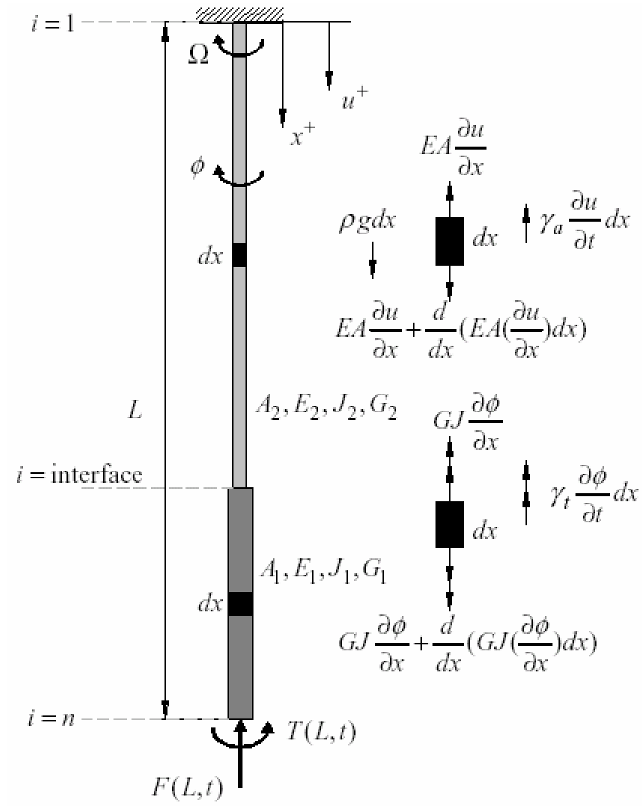

Figure 1. One dimensional drill-string.

where $(\ )' = \frac{\partial}{\partial x}$ and $(\dot{\ }) = \frac{\partial}{\partial t}$ are the space and time derivatives, respectively, and *g* represent the gravity. The physical and geometrical properties of the drill-string are the specify density $\rho = \rho_{steel}$, the elasticity modulus *E* and shear modulus *G*, the transverse section area *A* and the area moment *J*. The interaction between the perforation fluid (drill-mud) and the drill-string are represented by the axial $\gamma_a$ and torsional $\gamma_t$ viscous damping coefficients (in this work they are constants). Actually, the damping coefficients depend of the drill-string oscillations, Spanos et al. (1995), these oscillations influence the drill-mud flow along the drill-string.

**2.3 Boundary Conditions (BC)**

For axial movement, it is assumed an axial force acting at the bit. This force has two components, static and dynamic ones. Usually the static force is called the weight on bit (*WOB*) and represents 80-85% of the drill-collar weight. The dynamic force is a consequence of the axial movement of the bit's cones. For the top end of the drill-string is assumed that it is clamped.

For torsional movement are specified a resistive torque acting at the bit and a constant rotational velocity at the top end. The resistive torque, usually called as torque on bit (*TOB*), is the resistance of the formation to be crushed. Negative TOB were observed in drilling operation that sometimes tends to unscrew the drill-string components, Wolf et al. (1985). For the TOB, in this work it is used a model given by Tucker et al. (1997), this model is function of the bit velocity and a friction function.

$$x = 0 \rightarrow \begin{cases} \text{Axial:} & u(0,t) = 0 \\ \text{Torcional:} & \dot{\phi}(0,t) = \Omega \end{cases} \quad (2)$$

$$x = L \rightarrow \begin{cases} \text{Axial:} & EAu' + F(L,t) = 0 \\ \text{Torcional:} & GJ\phi' + T(L,t) = 0 \end{cases} \tag{3}$$

$$F(L,t) = F_{bit} = WOB + F_0 \sin(3\dot{\phi}_{bit} t)$$

$$T(L,t) = T_{bit} = TOB = \mu F(L,t) f(\dot{\phi}_{bit}); \qquad f(\dot{\phi}_{bit}) = \tanh(\dot{\phi}_{bit}) + \frac{\alpha_1 \dot{\phi}_{bit}}{1 + \alpha_2 \dot{\phi}_{bit}^2}$$

Where, $\dot{\phi}_{bit} = \dot{\phi}(L,t)$ is the bit velocity. The nonlinear continuous function $f(\dot{\phi}_{bit})$ is used to represent the dependence of the *TOB* with the bit velocity, Fig. 2. The value of the constant $\mu$ depends of type of bit used; it is 0.04 for roller cone bits and 0.5 for PDC bits.

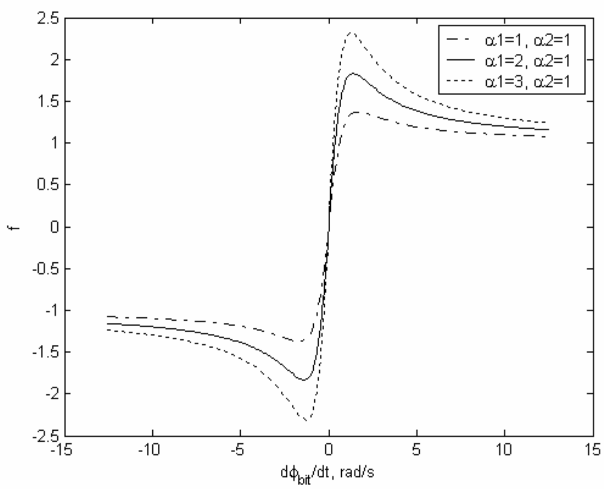


Figure 2. Friction function for the *TOB*.

**3. Numerical Solution**

It is easy to see that the coupling between the axial and torsional movement is in the boundary conditions, therefore we are dealing with nonlinear partial differential equations. Unfortunately the PDE does not have an analytic solution and it is necessary to employ numerical methods. In this work it is used the central finite difference method to discretize the space and the Runge-Kutta integrator for the time solution.
Using central finite difference for spatial coordinate

$$u_i' = \frac{\partial u_i}{\partial x} = \frac{u_{i+1} - u_{i-1}}{2\Delta x} \qquad u_i'' = \frac{\partial^2 u_i}{\partial x^2} = \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2}$$

$$\phi_i' = \frac{\partial \phi_i}{\partial x} = \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x} \qquad \phi_i'' = \frac{\partial^2 \phi_i}{\partial x^2} = \frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{\Delta x^2}$$

Arranging Eq. (1) it follows that

$$\ddot{u}_i = a_0(u_{i+1} - 2u_i + u_{i-1}) - a_1 \dot{u}_i + g \qquad i = 2 \ldots n-1$$
$$\ddot{\phi}_i = b_0(\phi_{i+1} - 2\phi_i + \phi_{i-1}) - b_1 \dot{\phi}_i \qquad i = 2 \ldots n-1$$

where $a_0 = \frac{EA}{\rho \Delta x^2}$, $a_1 = \frac{\gamma_a}{\rho}$, $b_0 = \frac{G}{\rho \Delta x^2}$, $b_1 = \frac{\gamma_t}{J\rho}$, are constants.

For extremes nodes ($i = 1$ and $i = n$) it is necessary to satisfy the boundary conditions, resulting

$$u_1 = 0 \rightarrow \dot{u}_1 = \ddot{u}_1 = 0, \qquad \ddot{u}_n = 2a_0(u_{n-1} - u_n - \frac{2\Delta x^2}{EA} F_{bit}) - a_1\dot{u}_n + g$$

$$\dot{\phi}_1 = \Omega \rightarrow \ddot{\phi}_1 = 0, \qquad \ddot{\phi}_n = 2b_0(\phi_{n-1} - \phi_n - \frac{2\Delta x^2}{GJ} T_{bit}) - b_1\dot{\phi}_n$$

Here, it is necessary to point out that the rotation of the top and the bottom of the drill-string are represented by $\dot{\phi}_1 = \Omega$ and $\dot{\phi}_n = \dot{\phi}_{bit}$ respectively. Finally, the discretized motion equations can be expressed in compact manner for all nodes

$$\begin{cases} \ddot{u}_i = f_i(\dot{u}_j, u_k, t) \\ \ddot{\phi}_i = g_i(\dot{\phi}_j, \phi_k, t) \end{cases}, \qquad i, j, k = 1 \ldots n$$, in matricial form it results

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{f}(\mathbf{x}) \tag{4}$$

Where $\mathbf{x} = \begin{bmatrix} \mathbf{u}^T & \boldsymbol{\varphi}^T & \dot{\mathbf{u}}^T & \dot{\boldsymbol{\varphi}}^T \end{bmatrix}^T \in \mathbb{R}^{4n}$, $\mathbf{u} = [u_i]_{nx1}$, $\boldsymbol{\varphi} = [\phi_i]_{nx1}$, $i = 1 \ldots n$, $\mathbf{x}$ and $\mathbf{A}$ are the vector and matrix state. Moreover, $\mathbf{A}$ and $\mathbf{f}$ represent the linear and non linear part of the system. Finally Eq. (4) is solved using the classical Runge-Kutta method.

## 4. Numerical Results

For numerical simulation the values shown in Table 1 are used, many of they were taken from Dareing et al., (1968).

### 4.1 Axial Vibration

The axial vibration was solved analytically by Dareing et al., (1968). In his work the same drill string configuration is used, where a harmonic displacement for the drill bit is prescribed. In this work we compare our results with those obtained by Dareing. The Fig. 3 shows the analytical and numerical results for axial vibration.

Table 1. Numerical values, [Dareing et al., 1968].

| | | |
|---|---|---|
| **Drill-pipe** | | |
| Length | 2250 | m |
| OD (4.5 in)x ID (3.826 in) | 0.1143x 0.09718 | m |
| Axial damping coefficient | 20.0 | N-s/m^2 |
| Torsional damping coefficient | 0.1 | Nm-s/m |
| **Drill-collar (BHA)** | | |
| Length | 250 | m |
| OD (6.5 in)x ID (2.25 in) | 0.1651x 0.05715 | m |
| Axial damping coefficient | 20.0 | N-s/m^2 |
| Torsional damping coefficient | 0.1 | Nm-s/m |
| BHA Weight | 3.0E+5 | N |
| **Other parameters** | | |
| $\mu$ | 0.04 | |
| $\alpha 1, \alpha 2$ | 2, 1 | |
| Young modulus | 2.1E+11, 8.0E+11 | N/m^2 |
| Steel, mud densidity | 7850.0, 1500.0 | kg/m^3 |

The curves in Fig. 3 represent the axial displacement in drill-pipe/drill-collar interface point. They were obtained varying the prescribed displacement frequency on the bit. The analytical and numerical results show a good agreement for the drill-string natural frequencies. This figure shows that the axial displacement is high for some frequencies that represent the natural frequencies. The curve shows the combined effect of drill-pipe and drill-collar, the drill-pipe is more sensitive to frequency variation because of its longer length and smaller area and it is clear that the resonances occur at 4.5, 8.9, 13, 17.2 … rad/s. The high amplitude at 20, 60, 100 … rad/s are attributed to drill-collar resonance.

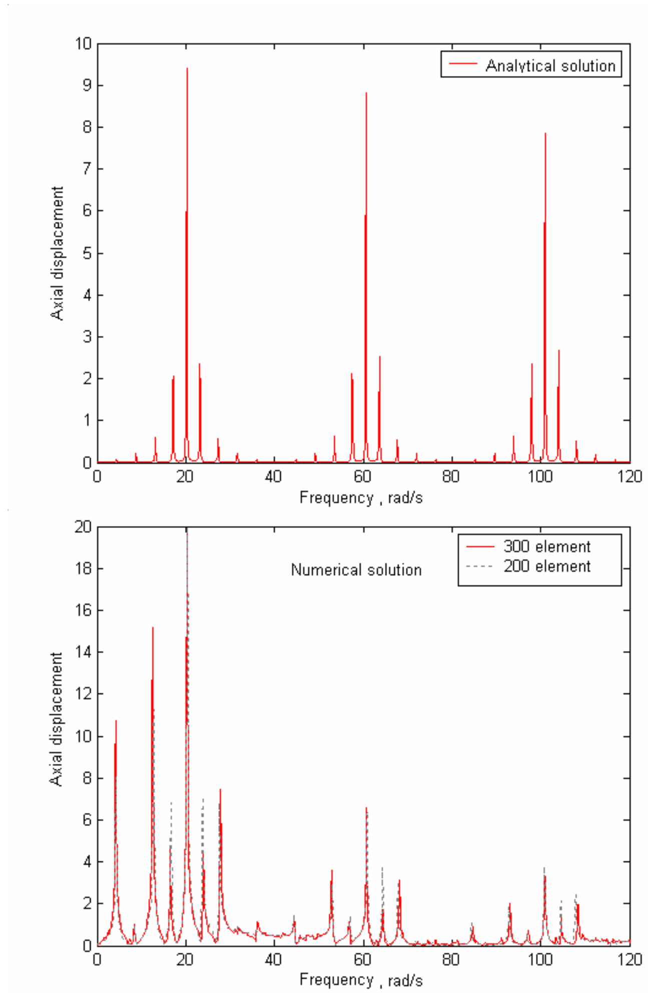


Figure 3. Drill-string axial vibration.

### 4.2 Axial-Torsional Vibration

Oil well drilling is often accompanied by torsional vibration. The basic cause of this vibration is the stick-slip phenomenon, and this type of motion may also excite a severe axial vibration in the drill-collar. The torsional vibration is attributed to drill-string static friction effects, Brett (1992). Briefly, if the static friction coefficient is sufficiently greater than the dynamic friction coefficient, stick-slip can occur. It was shown that the coupling between the axial and torsional vibration occurs in the force acting over the bit. Actually the axial vibration influence the torsional vibration but the inverse is not possible, as it can be concluded looking the boundary condition at the bit.

When drill-string rotation begins, the drill-pipe stores torsional energy until the applied torque exceeds the total static friction torque on the BHA. The BHA then begins to rotate and, since the static friction is higher than the dynamic friction, the stored energy in the drill-pipe is transferred to inertial energy in the BHA. It then can accelerate to a speed 2 or 3 times faster than the steady state rotational speed. This transfer of energy from drill-pipe to BHA can be self-excited and will result in stick-slip vibration. The stick-slip self-excited vibrations generally disappear as the rotary table speed is increased beyond a threshold value. This is shown in the follow figures.

In Fig. 4, it is possible to see the behavior of three different points of the drill-string (top, middle and bit). Simulations are done for $\Omega = 10 \text{ rad/s}$, $WOB = 85\%$ of drill-collar weight $(WOB = 255 \text{ kN})$ and $F_0 = 0$ (there is no dynamic axial force). For these parameters we can note that for steady-state motion, the bit velocity reaches zero values periodically, corresponding to an unstable stick-slip motion. Also, in Fig. 5 the space-phase plot is shown where the curve presents a periodic behavior that is common in systems with dry friction.

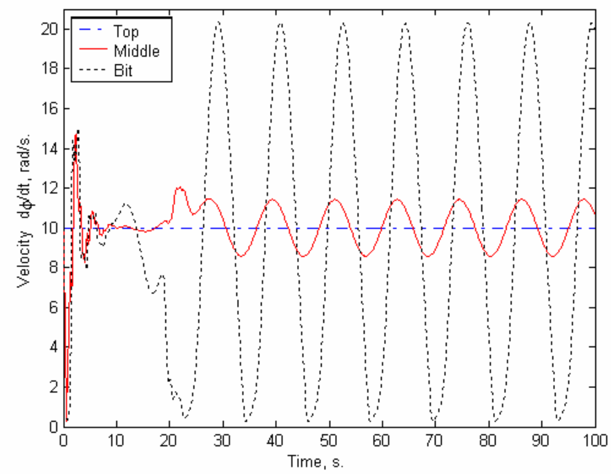


Figure 4. Velocity at three drill-string points for $\Omega = 10$ rad/s ; unstable behavior.

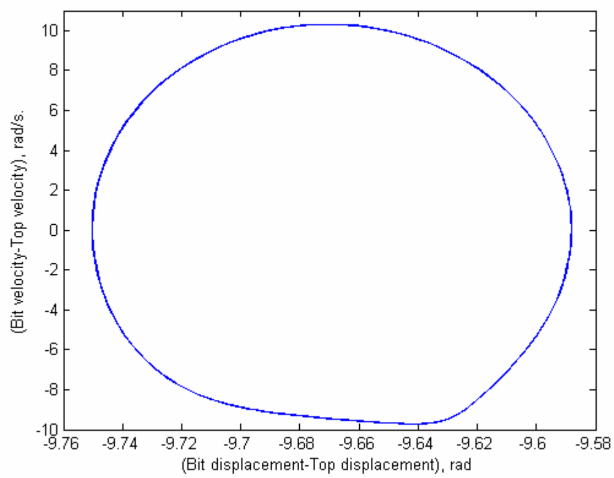


Figure 5. Space phase of the bit motion for $\Omega = 10$ rad/s .

When the rotary table speed is increased, for example to $\Omega = 15$ rad/s keeping the other parameters constant, it is observed that the motion of different drill-string points converge to unique value in steady-state, that is the same of the rotary table. This is a stable behavior of the system, Fig. 6.

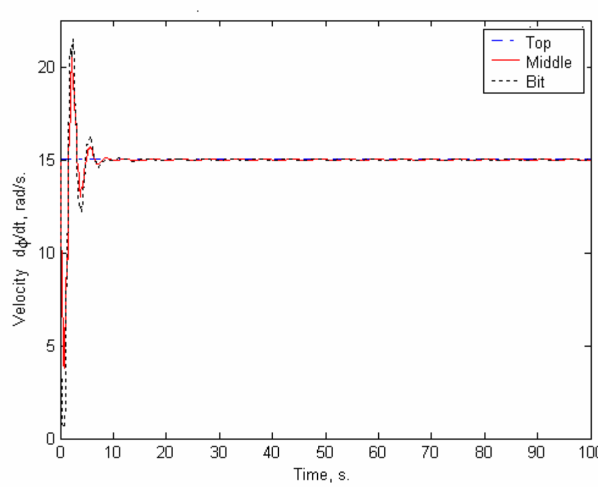


Figure 6. Velocity at three drill-string points for $\Omega = 15$ rad/s ; stable behavior.

Now, let us make a simulation considering that on the bit is acting an axial dynamic force being different from zero $F_{bit} = WOB + F_0 \sin(3\dot{\phi}_{bit} t)$ and $WOB = 255$ kN, $F_0 = 0.15\ WOB$. In Fig. 7 it is observed a fluctuation of the velocity for several drill-string points. This fluctuation is due the axial dynamic force with a frequency three times the bit rotation (typical for tri-cone bits).

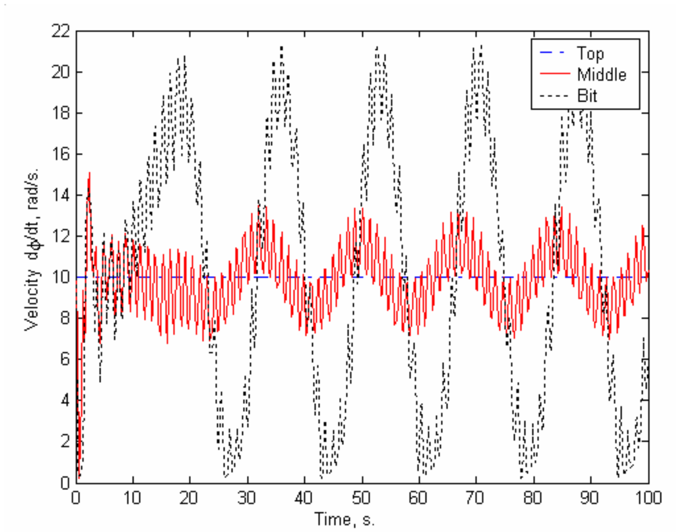


Figure 7. Time plot for $\Omega = 10$ rad/s, $F_{bit} = WOB + 0.15 WOB \sin(3\dot{\phi}_{bit} t)$.

To resume, it is necessary to point out that the stable/unstable behaviors of the drill-strings found above agree qualitatively with those reported by Kreuzer (1996) and Franca (2004).

**5. Conclusions**

We obtained a mathematical model to investigate the axial and torsional vibration of vertical drill-strings. The motion equations are coupled through boundary conditions at the bit. A systematic procedure to solve the PDE numerically was also presented.

We showed that the axial behavior of the drill-string is strongly governed by the BHA dynamics because the BHA vibrations are large if compared with the drill-pipe.

The axial-torsional model developed can describe satisfactorily the stick-slip vibrations, this phenomenon depends basically of the rotary table speed.

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