NUMERICAL STUDY OF NATURAL CONVECTION IN A PARTIALLY OPEN ENVIRONMENT WITH A HEAT GENERATING SOURCE

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Abstract. This paper discusses a numerical study of the thermal and fluid dynamic behavior of air in two-dimensional, laminar, natural convection in an enclosure with different aspect ratios of H/W, 1 and 2, where H is height and W is width. The enclosure has an opening in the cooled right-hand vertical wall. Natural convection is induced simultaneously by the difference in temperature between the vertical walls, represented by a Rayleigh number (Ra_e), and by an internal source of heat, also represented by a Rayleigh number (Ra_i). The internal heat source is placed in different positions on the lower horizontal wall and occupies almost 1% of the total volume of the environment. Numerical simulations were performed for several external Rayleigh numbers (Ra_e) in the range of 10^3 to 10^6, while the intensity of the two effects was evaluated through the relation $R=Ra_i/Ra_e$ in the range of 400 to 2500.

Keywords: natural convection, finite volume method, internal energy source, SIMPLEC

1. Introduction

The numerical modeling of convective heat transfer has been an area of great interest in recent years due to its wide application in engineering. Compared with the experimental method, numerical analysis may provide a more direct way to enhance/reduce heat transfer effectively so as to improve the performance or optimize the structure of thermal devices.

Buoyancy-driven heat transfer and fluid flow from heated plates are a problem in the cooling of electronic devices. The insertion position of these devices, generated by the heat, into an enclosed or partially enclosed space is important due to the efficiency of heat transfer. Also, the force of gravity is the most effective parameter in natural convection heat transfer.

The literature reports on numerical investigations into natural convection heat transfer and flow in an enclosure without a heat source under different boundary conditions. These reports include investigations by Davis (1983), Ostrach (1988) and Hortmann *et al.* (1990), who engaged in numerical studies of enclosed square spaces with two adiabatic and two heated walls having different Rayleigh numbers. However, in practice, there are different geometries and general contour conditions. The internal heat source affects the characteristics of the heat transfer associated with the flow field. Natural convection caused only by a distributed volumetric heat source in a two-dimensional enclosure has been widely examined by Chu and Churchill (1976), Khalilollahi and Sammakia (1986), Keyhani *et al.* (1988), Farouk (1988), Ho and Chang (1994), Deng and Tang (2002) and Oztop *et al.* (2004).

Numerous investigations have been made of natural convection caused only by external heating in a partially-open space; however, few results have been reported for natural convection caused simultaneously both by the difference in temperature on the external walls and by the heat-generating source, although problems of this type, which are often important, need to be studied in order to understand the complex behavior of natural convection flow and heat transfer.

Xia and Zhou (1992) numerically investigated a partially open square space with an internal source of heat and different temperatures on the external vertical walls. Reinehr *et al*. (2002) studied transient natural convection in a rectangular environments, with an aspect ratio of $H/W = 2$, with an internal heat source in varying positions and dependent on the Rayleigh number. Brito *et al*. (2003) studied mixed convection in a square partially open space with a rectangular heat-generating body at its center.

This work consisted of a numerical investigation of natural convection in a steady state flow in a rectangular environment with different aspect ratios of width (W) and height (H), i.e., $1 \leq H/W \leq 2$, comparing the results obtained with the available literature. The environments have an opening in the cooled right-hand vertical wall, the left-hand vertical wall is heated and the upper and lower walls are adiabatic. The natural convection is induced simultaneously by the difference in temperature between the vertical walls, and is represented by a Rayleigh number (Ra_e) and by an internal heat-generating source represented by a Rayleigh number (Ra_i) occupying approximately 1% of the enclosed volume. The position of the heat-generating source varies along the lower horizontal wall. Values of 10^3 to 10^6 are used for the Rayleigh number (Ra_e), while the $R = Ra_i/Ra_e$ relation is estimated to range from 400 to 2500, and the influence on the flow patterns is analyzed and discussed based on the distribution of temperatures and streamlines. The Boussinesq approach is used and a Prandtl number is kept stable. The Finite Volume Method is used here with the Power-Law and SIMPLEC schemes.

2. Problem Description and Mathematical Modeling

The flow was modeled using the equations of continuity, Navier-Stokes and energy conservation, and several simplifications were adopted, i.e., laminar flow, Newtonian and incompressible fluid, a two-dimensional system, and a steady state flow. The domain was filled with an incompressible Boussinesq fluid, which satisfies $\rho = \rho_{ref} [1 - \beta(T - T_{ref})]$, with β being the thermal expansion coefficient, and the subscript *ref* denoting the reference state. All the other physical properties, μ, k and c_p, were considered constant and the effects of compressibility and viscous dissipation were neglected. Thus, the governing time-independent Navier–Stokes equations in dimensional form are:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{1}$$

$$\frac{\partial(\rho uu)}{\partial x} + \frac{\partial(\rho vu)}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left[\mu \frac{\partial u}{\partial x}\right] + \frac{\partial}{\partial y}\left[\mu \frac{\partial u}{\partial y}\right] \tag{2}$$

$$\frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho vv)}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left[\mu \frac{\partial v}{\partial x}\right] + \frac{\partial}{\partial y}\left[\mu \frac{\partial v}{\partial y}\right] + g\rho_{ref}\beta(T - T_{ref}) \tag{3}$$

$$\frac{\partial(\rho uT)}{\partial x} + \frac{\partial(\rho vT)}{\partial y} = \frac{\partial}{\partial x}\left[\frac{k}{c_p}\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{k}{c_p}\frac{\partial T}{\partial y}\right] + \frac{q}{c_p} \tag{4}$$

where x and y are, respectively, horizontal and vertical coordinates, u and v are, respectively, horizontal and vertical velocity component, sp is the pressure, T is the temperature, μ is the viscosity, ρ is the density, k is the thermal conductivity and c_p is the isobaric specific heat of the fluid, q is the volumetric heat generation rate for the source, whose value is obtained through Eq. (9). The subscript *ref* refers to the values for the temperature of 293 K.

Eqs. (1) through (4) are solved in terms of the dependent variables p, u, v and T. However, the fluid dynamic results are presented in terms of the stream function (ψ) through the isotherms of the non-dimensional temperature (θ). These variables are described by:

$$u = \partial\Psi/\partial y \tag{5}$$

$$v = -\partial\Psi/\partial x \tag{6}$$

$$\theta = \frac{T - T_C}{T_H - T_C} \tag{7}$$

The air flow is investigated in a rectangular environment with width, W, and height, H, heated on the left-hand vertical wall by the temperature, T_H, and cooled on the upper half of the right-hand vertical wall by the temperature, T_C. These two walls are kept at those temperatures while the horizontal walls are adiabatic. The diagram of the two-dimensional physical system and the coordinates are illustrated in Figure 1. The lower half of the right-hand vertical wall is open and in contact with the air outside the enclosure. The lower horizontal wall has a heat-generating source placed in three different positions.

In Reinehr *et al.*'s study (2002), the height of the heat-generating source was double that of its width, occupying 1% of the total volume of the environment. In that study, 1600 volumes were placed in the enclosure, so the heat-generating source occupied 16 volumes. However, in our study, the heating source possessed 15 volumes, with 3 nodal points in the width and 5 in the height.

The boundary conditions used were:

i) for the velocities u = v = 0 in all walls;
ii) for $x = 0$ and $0 \le y \le H$ then $T = T_H$;

$$\text{iii) for } x = L \begin{cases} \text{if } 0 \le y \le H/2 \begin{cases} \text{if u} > 0 \text{ then } \dfrac{\partial \mathrm{T}}{\partial \mathrm{x}} = 0 \\ \text{if u} \le 0 \text{ then T} = \mathrm{T_{ref}} \\ p = p_{atm} \\ v = 0 \end{cases} \\ \text{if } y > H/2 \quad \{ T = T_C \end{cases}$$

$$\text{iv) for y = 0 and } 0 \le x \le W \text{ then } \frac{\partial T}{\partial y} = 0$$

$$\text{v) for y = H and } 0 \le x \le W \text{ then } \frac{\partial T}{\partial y} = 0$$

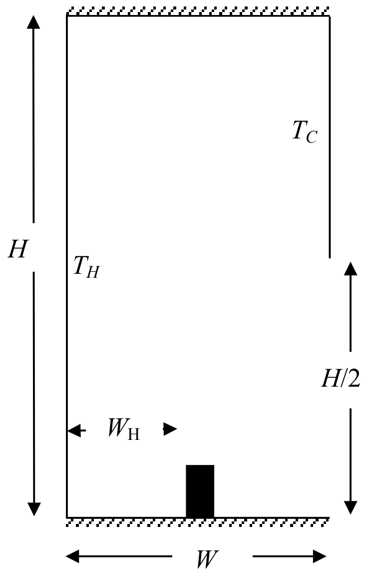


Figure 1. Geometry employed.

where $T_C = T_{ref} = 293$ K is the cold temperature determined for the upper half of the right-hand vertical wall and T_H is the heat temperature determined for the left-hand vertical wall, obtained through Eq. (8), i.e., from the Rayleigh number based on the difference in temperature between the vertical walls:

$$Ra_e = \frac{g\beta(T_H - T_C)H^3}{\nu\alpha} \tag{8}$$

while the Rayleigh number based on the volumetric heat generation rate is defined as:

$$Ra_i = \frac{g\beta q H^5}{\nu\alpha k} \tag{9}$$

where g is the gravitational acceleration, and ν, α and $\beta = 1/T_{ref}$ are, respectively, the fluid's kinematic viscosity, thermal diffusivity and thermal expansion coefficient. The fluid considered inside the enclosure is the atmospheric air using Prandtl's number, Pr = $\nu/\alpha = 0.72$.

Rayleigh's number is represented by the difference in temperature between the vertical walls, $10^3 \le Ra_e \le 10^6$. The intensity of the heat generated by the heating source is represented by the Rayleigh number (Ra_i), which is based on the volumetric heat generation rate. The influence of the intensity of the two Rayleigh numbers is evaluated through the relation presented in Eq. (10):

$$R = \frac{Ra_i}{Ra_e} \tag{10}$$

where $400 \leq R \leq 2500$.

The differential equations represented by Eqs. (1) to (4), together with their respective boundary conditions, were solved using the Finite Volume Method (FVM) described by Patankar (1980). In this method, the solution domain was divided into small finite control volumes. The differential equations were integrated to each of those control volumes. This integration resulted in algebraic equations which, when solved, simultaneously or separately supplied the components of pressure and velocity. The Power-Law interpolation was adopted to calculate the diffusive and convective terms. The SIMPLEC (*Semi IMPlicit Linked Equations Consistent*) algorithm was used to couple pressure and velocity.

The discretized equations were solved iteratively, using the line-by-line method known as the Thomas algorithm, or TDMA (*TriDiagonal Matrix Algorithm*). To accelerate the convergence, a sub-relaxation of 0.5 was applied to all the variables, u, v, p and T, except for the results given in Table 1, where no sub-relaxation was used on the pressure (Patankar, 1980). The solution domain was covered with a uniform grid which is used in all numerical simulations and consisted of 40 x 40 control volumes. The variables were stored in staggered mode. The numerical solution was considered convergent when the highest absolute number of mass conservation was less than 10^{-10}. Grid refinement not was applied.

3. Results and Discussion

Figures 2 to 8 show the streamlines and isotherms for different Ra_e. The isotherms are presented for dimensionless temperature, as described in Eq. (7), while the streamlines are given by Eq. (11):

$$\psi_{a\dim} = \frac{\psi}{\alpha} \tag{11}$$

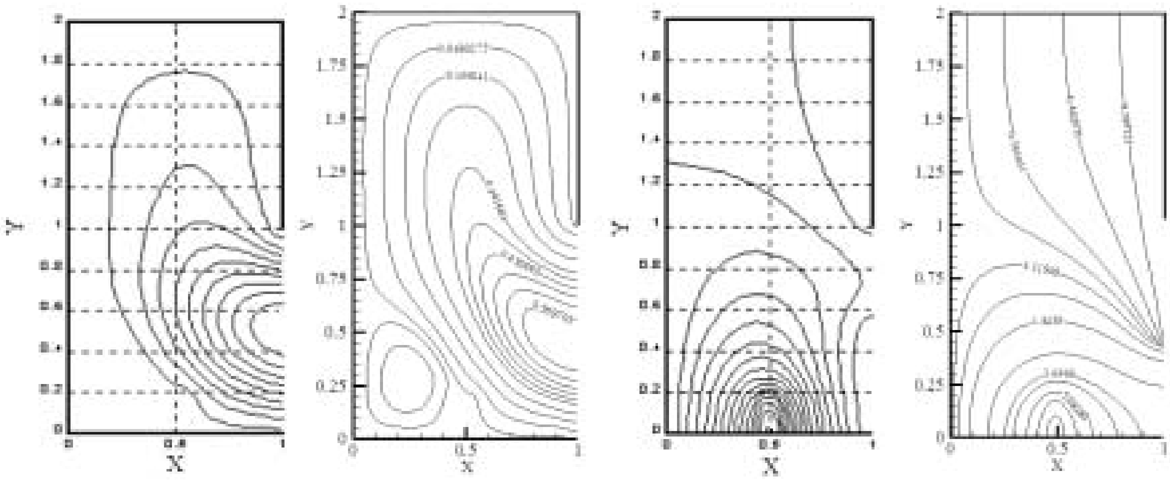
Figures 2, 3 and 4 present the fluid dynamic behavior of the air in the enclosure. Note that the increase in temperature differences between the vertical walls (Ra_e number) affected the fluid dynamic behavior by augmenting the intensity of the flow in the space. This behavior was observed for the various values of R employed, but is illustrated here only for $R = 2500$ and in the different positions of the heat-generating source on the lower horizontal wall.

These figures show some of the isotherms obtained for $Ra_e = 10^3$, 10^4 and 10^5, respectively. The recirculation region is seen to increase with as the Ra_e number increases, although it does not reach the entire height of the enclosure, contrary to what was observed by Xia and Zhou (1992), who used an aspect ratio of $H/W = 1$. Hence, in our study, the higher buoyancy induced by Ra_e was not overcome by the buoyancy induced by the effect of the heat source, represented by Ra_i or R, a fact that was also observed by Reinehr *et al.* (2002). The isotherms obtained here exhibited differences in relation to those obtained by Reinehr *et al*. (2002), in whose study the left vertical wall of the enclosure had one isotherm touching the wall, forming an angle of inclination, which was impossible in our study because the wall was not adiabatic and the inclination of the isotherms was 90°.

The air's fluid dynamic behavior was only slightly affected by the variation in the position of the heat-generating source. The most significant influence was reflected in the intensity of the eddy formed between the heating source and the left vertical wall, particularly in positions $W_q/W = 0.5$ and 0.75.

Figures 5 and 7 show the streamlines for different Ra_e and $R = 2500$ with an aspect ratio of $H/W = 1$. These figures indicate that the difference between temperatures on the vertical walls, in terms of the fluid dynamic behavior, causes the intensity of the flow to increase in the enclosure, even when the maximum dimensionless temperature in the enclosure decreases as the Ra_e fixing R increases, as illustrated in Figures 6 and 8. With a higher Ra_e, however, the influence of the heating source on the flow is minimal, as indicated in Figure 5.

In Figure 7, when $Ra_e = 10^3$ and $R = 2500$, note that the heating source controls the flow and the transfer of heat in the partially open environment, and that the external heat exerts a negligible influence. The intensity of the counterclockwise roll occupies nearly 30% of the environment and is induced by the buoyancy of the heat-generating source when R is high and Ra_e is low.



Reinehr *et al.* (2002) | Present work | Reinehr *et al.* (2002) | Present work

Figure 2. Streamlines and isotherms, respectively, for $Ra_e = 10^3$, $R = 2500$, $W_H/W = 0.5$ and $H/W = 2$.

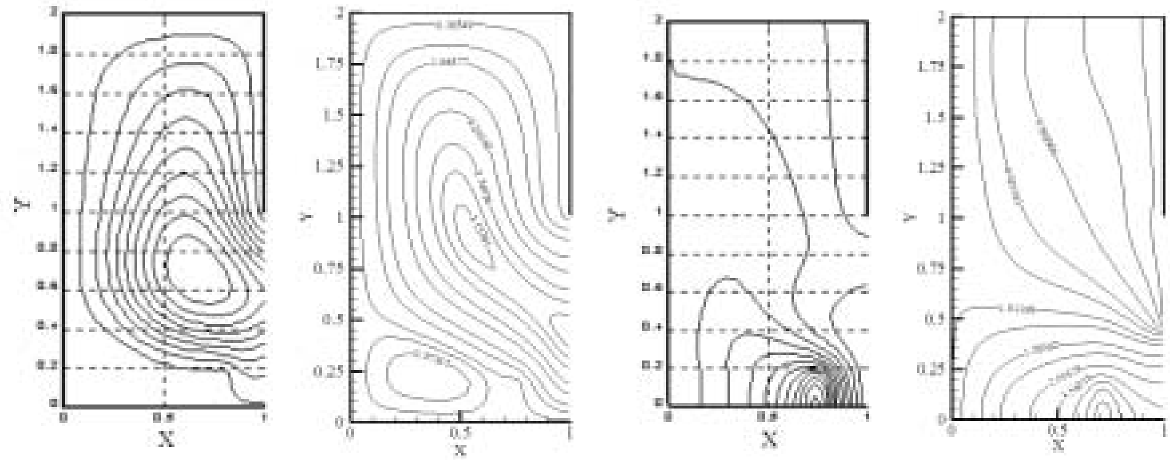


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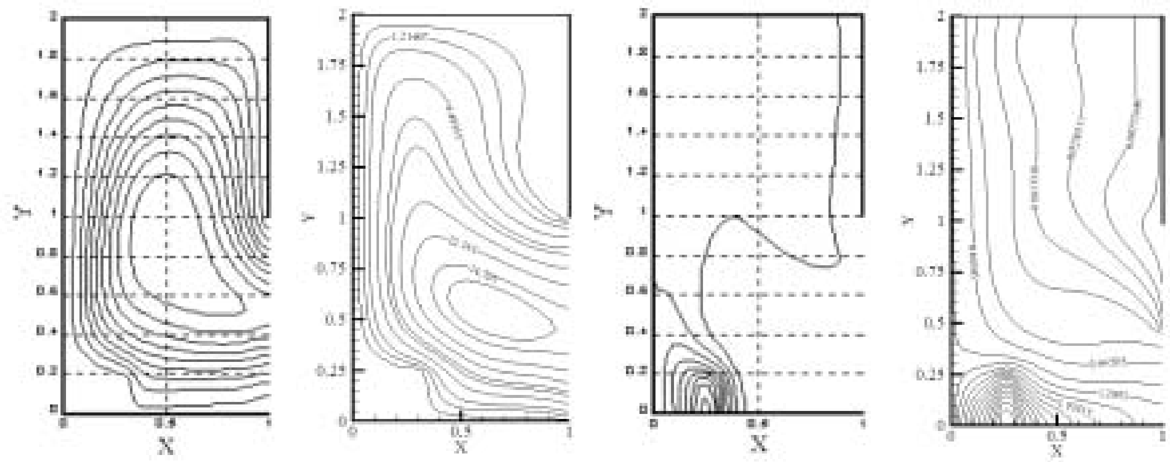
Figure 3. Streamlines and isotherms, respectively, for $Ra_e = 10^4$, $R = 2500$, $W_H/W = 0.75$ and $H/W = 2$.



Reinehr *et al*. (2002) | Present work | Reinehr *et al*. (2002) | Present work

Figure 4. Streamlines and isotherms, respectively, for $Ra_e = 10^5$, $R = 2500$, $W_H/W = 0.25$ and $H/W = 2$.

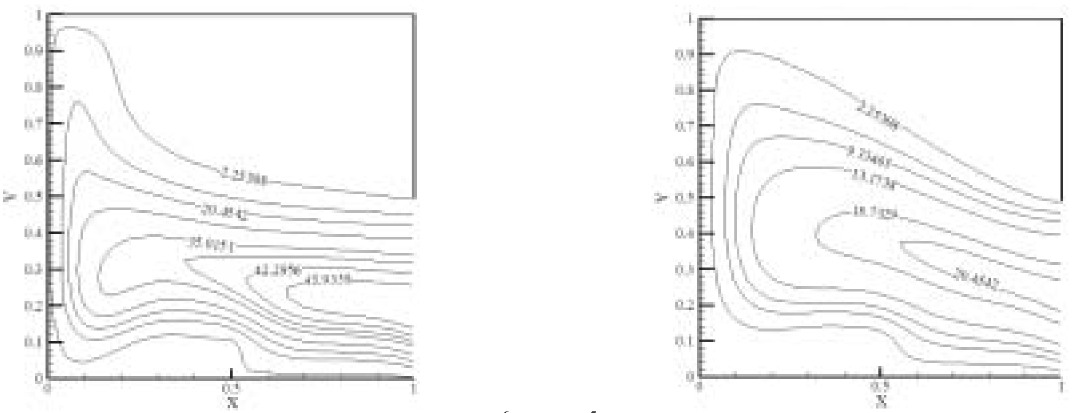


Figure 5. Streamlines for $Ra_e = 10^6$ and 10^5, $R = 2500$, $W_H/W = 0.5$ and $H/W = 1$.

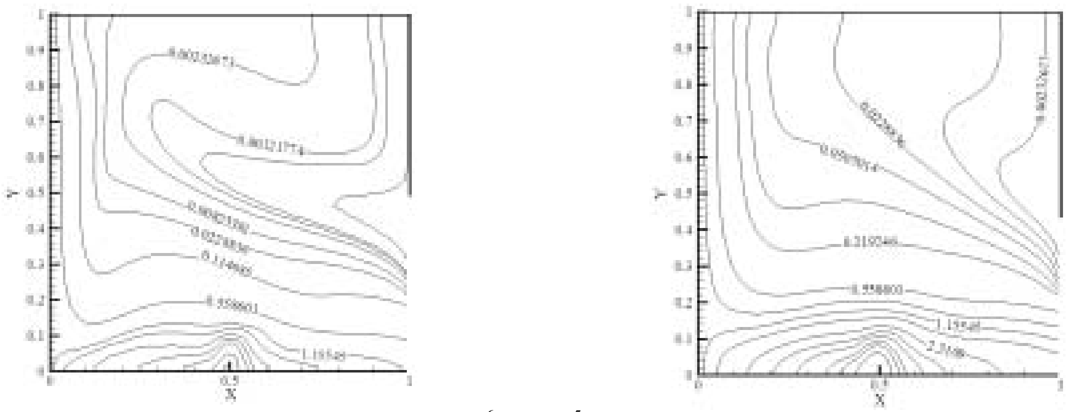


Figure 6. Isotherms for $Ra_e = 10^6$ and 10^5; $R = 2500$, $W_H/W = 0.5$ and $H/W = 1$.

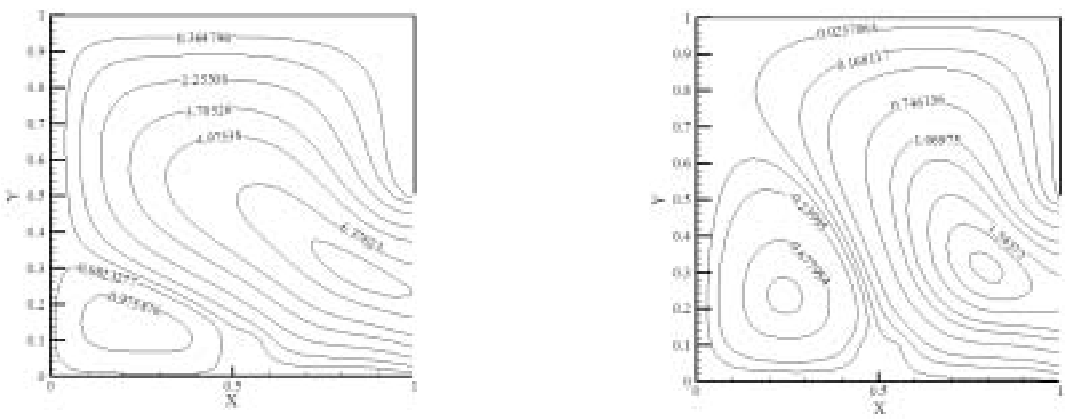


Figure 7. Streamlines for $Ra_e = 10^4$ and 10^3, $R = 2500$, $W_H/W = 0.5$ and $H/W = 1$.

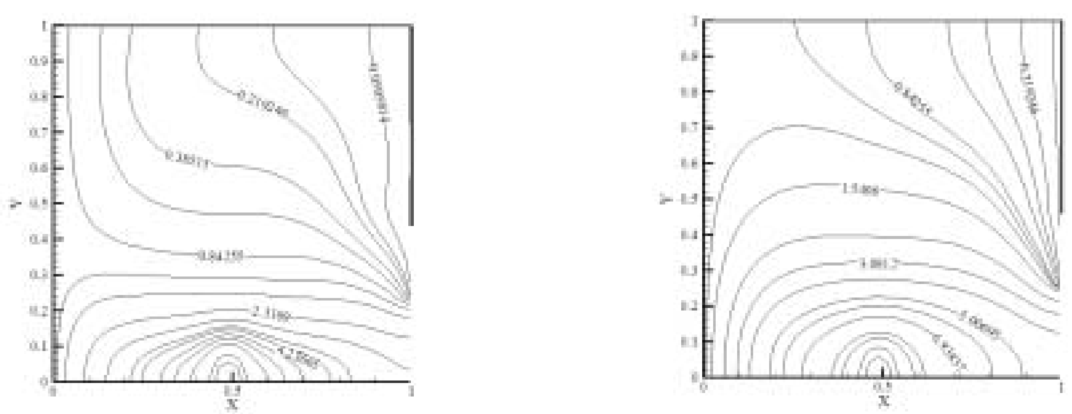


Figure 8. Isotherms for $Ra_e = 10^4$ and 10^3, $R = 2500$, $W_H/W = 0.5$ and $H/W = 1$.

The results obtained for the maximum dimensionless temperature were compared with those of Xia and Zhou (1992) and Reinehr *et al.* (2002), in our study of the thermal behavior of a square enclosed environment, i.e., $H/W = 1$. Table 1 shows these results with the heating source located centrally on the bottom horizontal wall, i.e., in the position $W_q/W = 0.5$. Several different flow patterns occur, with changing Ra_e or R. A comparative analysis of the values of Table 1 indicates that the relative deviations of our work compared with the results of Xia and Zhou (1992) are situated in the range of $0.005 \leq \theta_{max} \leq 0.036$ while, for Reinehr *et al.* (2002), this range is $0 \leq \theta_{max} \leq 0.055$, indicating a good agreement between the results. The relative deviation is calculated using $\theta_{PW/}\theta_{OA}$ - 1, where the subscript *PW* refers to the present work and *OA* represents other authors.

Table 1. Maximum dimensionless temperature for $H/W = 1$ and position of the heat-generating source $W_H/W = 0.5$.

Authors	Xia and Zhou (1992)	Reinehr *et al.* (2002)	Present work
$R = 400$, $Ra_e = 10^5$	1.80	1.82	1.82
$R = 1000$, $Ra_e = 10^5$	3.90	4.07	3.88
$R = 2500$, $Ra_e = 10^3$	15.40	15.69	15.06
$R = 2500$, $Ra_e = 10^4$	10.90	11.50	11.50
$R = 2500$, $Ra_e = 10^5$	8.30	8.46	8.56
$R = 2500$, $Ra_e = 10^6$	6.10	-	6.16

The maximum dimensionless temperature rises considerably with the increase in the relation R, with Ra_e remaining stable, as indicated in Tables 2 to 4, regardless of the position of the heating source and the aspect ratio *H/W*. The maximum value of the dimensionless temperature is consistently inside the heating source. The maximum dimensionless temperature drops gradually as the Ra_e increases when R is kept stable. In certain situations, particularly when the Ra_e number is low in combination with higher values of the R relation, the mechanism of heat transfer by diffusion contributes significantly to the enclosure, as indicated by the isotherms in Figure 3.

In Tables 2 to 4, note the slight difference between the results obtained here and those of Reinehr *et al.* (2002). This difference may be due to the size of the heating source, as well as the criterion of convergence, neither of which are clearly specified in Reinehr et al.'s paper.

Note that the position of the heating source interferes in the values of the maximum dimensionless temperature, although they cannot be precisely quantified. Most of the values of maximum dimensionless temperature presented in Tables 2 to 4 occur when the heating source is located centrally. This behavior is attributed to the position beside the opening of the enclosure, so that the heating source is in contact with cold air, while in the position beside the heated wall, the difference in temperature between the vertical walls and the heating source produce a buoyancy effect.

A comparison of Tables 1 and 3 reveals that the dimensionless temperature increases with a lower aspect ratio, *H/W*, i.e., for $R = 2500$ and various Ra_e numbers presented in these tables. The highest increase in temperature was obtained with $H/W = 1$ and with the lowest Ra_e ($= 10^3$) number, because the heating source controls the flow and heat transfer for high values of R and low values of Ra_e and for low heights. If the enclosure is high, the heating source loses its effect due to the incoming flow of cold air into the enclosure.

Table 2. Maximum dimensionless temperature for $H/W = 2$ and position of the heat generation source $W_H/W = 0.25$.

	Ra_e				
R	10^3	10^4	10^5	10^6	Authors
400	1.767	1.512	1.210	1.000	Reinehr *et al.* (2002)
	1.667	1.697	1.459	1.262	Present work
1000	3.322	2.830	2.302	1.762	Reinehr *et al.* (2002)
	2.902	2.819	2.382	2.017	Present work
2500	7.156	5.926	4.653	3.188	Reinehr *et al.* (2002)
	6.832	5.602	4.610	3.668	Present work

Table 3. Maximum dimensionless temperature for $H/W = 2$ and position of the heat generation source $W_H/W = 0.5$.

	Ra_e				
R	10^3	10^4	10^5	10^6	Authors
400	1.738	1.439	1.053	1.000	Reinehr *et al.* (2002)
	1.770	1.654	1.373	1.165	Present work
1000	3.596	2.998	2.315	1.653	Reinehr *et al.* (2002)
	3.388	3.127	2.475	1.905	Present work
2500	8.053	6.445	4.665	3.164	Reinehr *et al.* (2002)
	7.390	6.370	4.900	3.690	Present work

Table 4. Maximum dimensionless temperature for $H/W = 2$ and position of the heat generation source $W_H/W = 0.75$.

R	Ra_e				Authors
	10^3	10^4	10^5	10^6	
400	1.306	1.132	1.000	1.000	Reinehr *et al.* (2002)
	1.751	1.730	1.356	1.078	Present work
1000	2.900	2.623	2.188	1.500	Reinehr *et al.* (2002)
	3.691	3.159	2.319	1.787	Present work
2500	6.832	6.130	4.760	3.041	Reinehr *et al.* (2002)
	7.885	6.334	4.762	3.485	Present work

4. Conclusions

This work consisted of a numerical investigation of natural convection in rectangular enclosures for different width (W) and height (H) ratios. The enclosures had an opening in the right-hand cooled vertical wall. The left-hand vertical wall was heated and the upper and lower walls were adiabatic. The natural convection was represented by an external Rayleigh number (Ra_e) and by an internal heat-generating source represented by an internal Rayleigh number (Ra_i) which occupied 1% of the total volume of the enclosure. The position of the heating source varied along the lower horizontal wall.

Based on an analysis of the results, we found that the enclosure exerts an influence on the effects of natural convection generated by temperature differences between the vertical walls, Rayleigh number (Ra_e), and by the heating source, Rayleigh number (Ra_i). An increase of the Ra_e number, i.e., of the difference between the temperatures of the vertical walls, caused the maximum dimensionless temperature in the internal enclosure to drop dramatically, with a constant R relation. In contrast, an increase in the R relation caused the maximum dimensionless temperature inside the enclosure to increase, with a constant R relation, in all the heating source positions analyzed here.

In the two aforementioned situations, the intensity of the flow inside the enclosure was found to increase, although the increase in the Ra_e number caused a stronger effect than the increase in the R relation. The position of the heating source exerted little influence on the air's fluid dynamic behavior and on the maximum temperature inside the enclosure. The H/W aspect ratio significantly influenced the results obtained for dimensionless temperature. Lower enclosures presented higher temperatures, while high ones displayed lower temperatures for the same configuration. The opening was found to be advantageous for the flow and heat transfer in the enclosure, although its characteristics were found to be complex and changed according to the location of the heating source, and the Ra_i and Ra_e numbers.

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