NEW CRITERIA OF AUTOMATIC CHOICE OF MODES USED IN MODAL SYNTHESIS METHOD

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Abstract: *Modal synthesis technique is a methodology used for dynamic modeling of great and/or complex structures. In this technique the complete structure is subdivided in substructures and the modal information of each one of them are synthesized to identify dynamic parameters of the complete structure. Results quality gotten through the modal synthesis method depends on the choice of modes of each one of the substructures that will be used to synthesize the complete structure. This paper presents a new criterion of automatic choice of modes based in the energy of the two substructures modes. This energy can be divided in two distinct parcels. One parcel is related to the degrees of freedom of the interface between the substructures and the other one is relative to the internal degrees of freedom. The two parcels of energy and the relation between them are considered. According to the criterion, the modes are compared between itself. Each mode is compared with the others of the proper substructure and with all of the another substructure. From there the selected and removed modes of the two substructures are defined and will be used to synthesize the complete structure. The considered criterion is evaluated in one modal synthesis technique that uses the modal superset of residual flexibility (MSRF). The defined process of choice can be applied in processes of experimental modal synthesis. The methodology is evaluated in discreet mass-stiffness-damping models and finite elements model.*

Keywords: *Modes selection, experimental modal synthesis, dynamic modeling.*

1. Introduction

The modal synthesis technique is a methodology used for the dynamic modeling of great and/or complex structures. The structure is subdivided in many substructures which modal information is used to identify the dynamic parameters of the complete structure. The quality of the results, gotten by the method of modal synthesis, depends of substructures modal bases that will be used to synthesize the original structure.

In the processes of modal synthesis that use the modal superset of residual flexibility, the user could define which modes will be kept and removed from the modal base. These removed modes are used to obtain the modes of residual flexibility that will to enrich the modal base of each substructure. Thus, even remaining a enough number of modes in the base, some modes which would be significant in the identification from original system could be removed. This fact also is observed in the majority of the methods of modal synthesis. In the method of automatic choice, Modes Automatic Choice Criteria (MACC), purposed by Cleudmar, A.(1998), was observed that when the contour energy level of the substructures is small, eigenvalues and eigenvectors more precision was identified. For each isolated substructure, as minor the contour coordinates energy relatively to the internal coordinates energy for one determined mode, minor would be the mode influence in the synthesis of the complete system. This assumption was based on the fact that internal parcels energy in the selected modes are the main responsible of the vibration movement of the complete system, since the relative parcels to the contour of the substructures will be subjected to the imposed coupling conditions.

Then, the comparison between the contour energy parcels and intern of the substructures supplies a quality index between the modes of a given substructure. This assumption is the MACC base. The choice of one determined mode with high contour energy in comparison to internal nodes means low quality in the synthesis final results.

The MACC improved the modal synthesis technique efficiency introducing an automatic process of modal choice. The final convergence of synthesis is function of modal base selection and, generally, that is made manually for the user who must know the structure in study. However, in the work of Cleudmar, 1998, it showed mainly that specific modes bands that would be adjusted to the final process of synthesis were obtained, if the analyzed experimental structure was complex and presented non linearity. Therefore, beyond the effect of the boundary energy it must have complementary procedures aiming at to still more improve the process of modal bases choices that will be kept in the substructures.

This work presents a new modes automatic process based in the modes energy of the two substructures, as in the MACC. However, this energy is divided in two distinct parcels. One is related to the interface degrees of freedom between the substructures and to another one it is mentioned to the internal degrees of freedom. The two parcels of energy and the relation between them are considered. In accordance with the criteria the modes are compared between itself and each mode is compared with another modes of the proper substructure and with all of other modes. From the selected and removed modes are defined there, of the two substructures, that will be used in the synthesis of the complete structure.

2. New criteria of automatic modes choice

The modes choice process of modal synthesis method defines the quality of results and the frequency band that is desired to synthesize. In the synthesis of determined frequency band, better or worse results could be gotten in function of the modes choice.

Despite the MACC getting better modal bases in the many analyzed structures, was observed that no selected modal bases exists and that some cases they would present an improvement in the synthesis, however the method as it was considered could not mount this set. Aiming to improve the precision of these results a new criteria of automatic choice in the modes was developed, determined SMCC (Structuralized modes choice criteria) which also is based in the modes energy, as defined in the MACC.

In this new criteria the internal energy, contour energy and total energy, will be considered. It considers two connected substructures (A) and (B) where u_i are the internal coordinates and u_c are the interface coordinates, as showed in the figure 1.

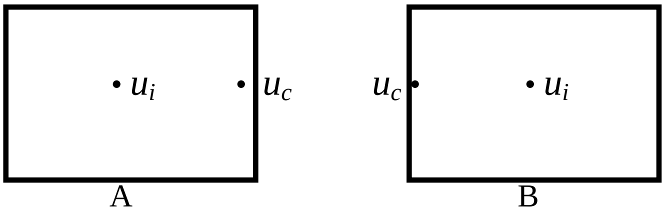


Figure 1 - Connection between two substructures A and B to form the global structure.

The total energy and the internal energy parcels and contour energy parcels are obtained by the Equations from (1) to (6). In these equations the corresponding degrees of freedom to each of interface energy parcel and intern energy parcel are considered separately. In the case of the beam models with finite elements the energy calculation is approached by:

$$Eia(ja) = \frac{Ua(gia, ja)^t . Ma(gia, gia) . Ua(gia, ja) . Wa(ja)^2}{2} \tag{1}$$

$$Eea(ja) = \frac{Ua(gea, ja)^t . Ma(gea, gea) . Ua(gea, ja) . Wa(ja)^2}{2} \tag{2}$$

$$Eta = Eia + Eea \tag{3}$$

$$Eib(jb) = \frac{Ub(gib, jb)^t . Mb(gib, gib) . Ub(gib, jb) . Wb(jb)^2}{2} \tag{4}$$

$$Eeb(jb) = \frac{Ub(geb, jb)^t . Mb(geb, geb) . Ub(geb, jb) . Wb(jb)^2}{2} \tag{5}$$

$$Etb = Eib + Eeb \tag{6}$$

where,

Eia : Interface energy of the substructure A
Eea : Internal energy of the substructure A
Eta : Total energy of the substructure A
Ua : Modes of the substructure A
Wa : Frequency of the substructure A
Ma : Mass matrix of the substructure A

Eib : Interface energy of the substructure B
Eeb : Internal energy of the substructure B
Etb : Total energy of the substructure B
Ub : Modes of the substructure B
Wb : Frequency of the substructure B
Mb : Mass matrix of the substructure A B

gia : DOF interface of the substructure A
gea : DOF internals of the substructure A
ja : j^{th} mode of the substructure A
gib : DOF interface of the substructure B
geb : DOF internals of the substructure B
jb : j^{th} mode of the substructure B

In this work the modal synthesis method of residual flexibility (SMFR), Araújo (1998), was used to evaluate the new criteria SMCC.

Initially, the modal bases will be selected of each substructure through to determination of the internal and contour energies using the Eqs. (1), (2), (4) and (5). Then, the total energies of the two substructures connected will be calculated through Eqs. (3) and (6), considering the possible modal sets kept for each substructure separately. The modal bases of the substructures according to the minimum values are classified with all combinations of total energies.

The result also is defined by summation, separately, of the internal energy, contour energy and the variation of the contour energy.

All the methodology was implemented in a computational code in Matlab environment.

3. Models of numerical simulation

Four models of numerical simulation had been used to evaluate the new criteria of modal bases choice (SMCC), two discrete models mass-stiffness-damping and two numerical models using formularization by finite elements. The models of finite elements had been simulated in a Matlab code.

3.1. Mass-stiffness-damping model

Figure 2 presents one of the discrete models mass-stiffness-damping used in the evaluation of the methodology. This model considered the complete structure subdivided in two symmetrical substructures being that each substructure possessed 5 degrees of freedom. Each substructure had condition of free contour. The discrete masses were all equal being of 1 kg. By the same way, the springs also were all equal with rigidity of 500000 N/m and damping of 1 N.s/m.

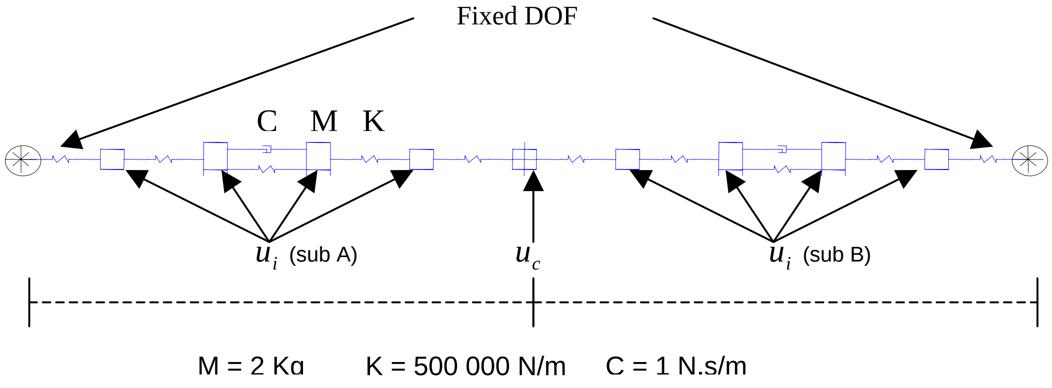


Figure 2. Mass-stiffness-damping model with two substructures of 5 dof each one.

The modal bases of the two substructures and the complete structure had been obtained. The modes of the substructures are used in the synthesis of the complete structure and the original structure to compare with the synthesized modes. Each substructure has 5 modes and, because of that the complete structure has 10 modes. The interface between the substructures has one degree of freedom, with this, in accordance with method SMFR, there will be only one mode of residual flexibility that goes to compose the modal superset of residual flexibility and more (k) flexible modes of each substructures that will be chosen by the new criteria defined (SMCC).

Beyond these 10 degrees of freedom model, other mass-stiffness-damping discrete model with 10 degrees of freedom in each substructure was evaluated. This model is similar to the one in figure 2, however extended to 10 lumped masses, possessing each one of them one degree of freedom.

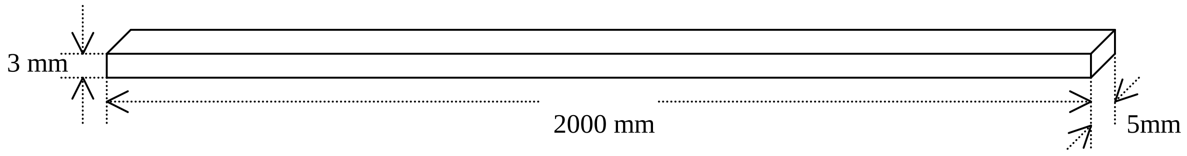
3.2. Model of a clamped-clamped beam

This model is constituted of an aluminum clamped-clamped beam as shown in figure 3. To effect the modal synthesis, the structure was divided in two substructures as shown in the figure 3c and 3d. The dimensions of the beam are shown in the figure 3a and the physical parameters of the finite elements model are shown in table 1.

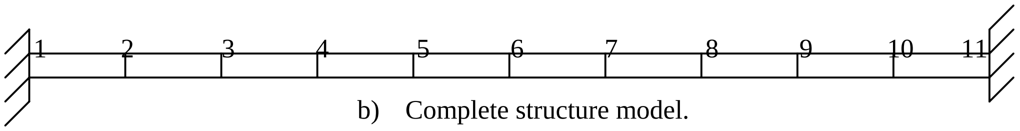
Table 1. Physical properties of the beam material.

Material	**Aluminum**
Specific weight (ρ)	2780 [Kgf/m^3]
Elasticity modulus (E)	7,0 x 10^{10} [N/m^2]
Poisson coefficient (ν)	0,34

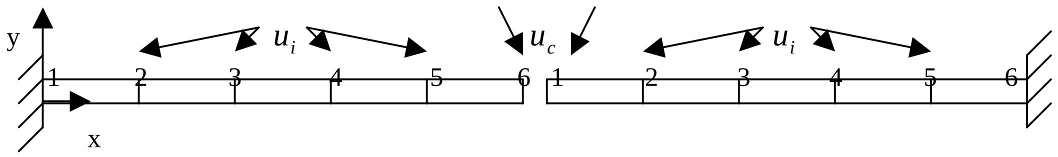
The modal bases used in the modal synthesis had been obtained modeling the substructures for finite elements, considering 5 beam elements giving a total of 6 node. It was used an option of element of Euler-Bernoulli beam considering only the transversal displacement, getting one dof for node. Aiming to get the modal bases of the original system to compare with the results of the synthesized complete structure, it was shaped, also, the original structure for finite elements, using the option of element of Euler-Bernoulli beam. The complete structure was divided in 10 elements of beam with 11 nodes. In figure 3 the schematic finite elements models of the substructures and the complete structure are shown.



a) Aluminum beam.



b) Complete structure model.



c) Substructure model A d) Substructure model B

Figure 3. Clamped-clamped beam model with two substructures of the 5 dof each one.

Each substructure has 5 modes and the complete structure has 10 modes. The interface between the substructures has a degree of freedom, then there was only one mode of residual flexibility that goes to compose the modal superset of residual flexibility more (k) flexible modes for each substructures that can be selected.

4. Results

The figure 4 shows to the relative error in the natural frequencies and the index MAC, Ewins (1984), for all the possible combinations in modes kept for all the analyzed models classified in accordance with the isolated addition of the energy in the internal modes.

Considering the internal energy one noticed that the classification in the kept modes can be made by the lesser value of the addition of all internal energy in the modes kept in the two substructures, as it can be seen in figure 4. In the four analyzed models the choice of the modal base using the SMCC generated good results. In the best condition of assembly of the modal bases, the models with 5 DOF for each substructure, has 4 modes kept for each substructure that results in 8 modes synthesized for the complete structure. The models with 10 DOF for each substructure, has 9 modes kept for each substructure, that results in 18 modes synthesized for the complete structure. Using the new criterion of automatic modes choice, it was possible to select the best modes to be used in the process of modal synthesis.

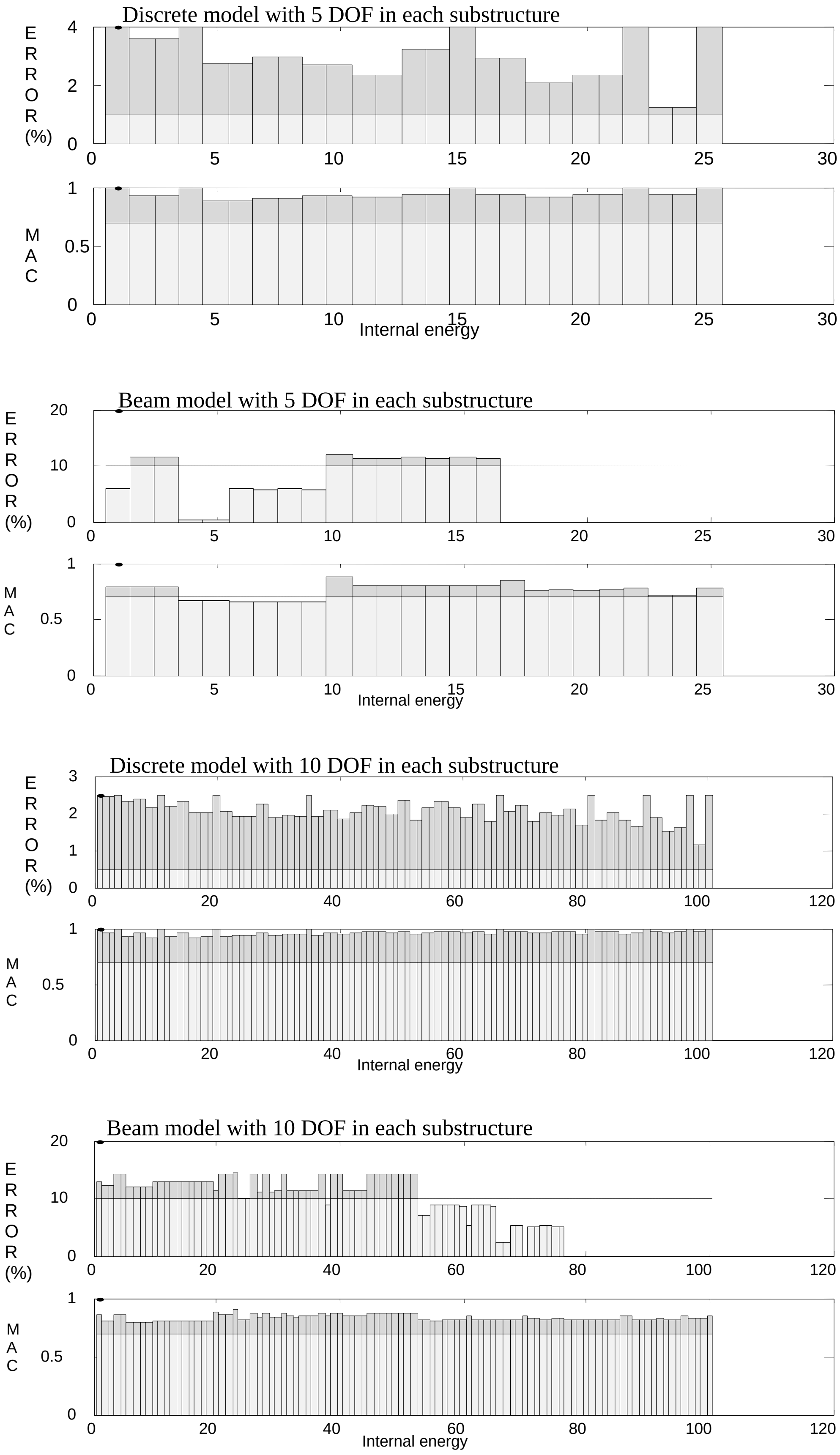


Figure 4. MAC and Relative error (%) in function of the internal energy.

In accordance with the criteria specification was observed that as lesser the addition of the internal node energy in the modes kept in the two better substructures will be better the results of the synthesis, as can be observed in figure 4. Moreover, it was observed that as lesser the variation of the contour energy in each mode of a substructure in relation to the modes of the other, better will be the precision of the results gotten in the modal synthesis. It was verified that the choice in the modes is more necessary for the models with bigger numbers of degrees of freedom. In the case of the beam models the use of the variation of the contour energy generated better resulted.

5. Conclusions

This work is part of a PhD thesis that still in development and has as objective to improve the efficiency of purely experimental modal synthesis methods. In particular a new methodology for the modes automatic choice of the substructures was presented that compose the general structure. It must be detached that the procedure of modal choice is basic for any method of modal synthesis, being, in particular, more critical when the analysis is made with purely experimental data.

In the simulated tests it was observed exactly that the new called criterion of SMCC obtained to always get the best conditions of modal bases in function of one determined amount in modes kept for each substructure, considering a certain noise level in the models. These results had not been presented in this work. Despite the gotten results the method it must be evaluated in measured data experimentally similar to prove its effectiveness. This analysis is in development phase.

6. Acknowledgements

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7. References

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