

COMPARISON BETWEEN CONVENTIONAL FLIGHT TEST MANEUVERS FOR AIRCRAFT AERODYNAMIC DERIVATIVES ESTIMATION

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Abstract.*This paper is focused on comparison of conventional flight test maneuvers for aerodynamic parameters estimation. Concerning the a priori knowledge of the flight testing aircraft, the analysis is done through the spectrum analysis of the inputs and through the computation of the Cramer-Rao lower bound matrix for the parameters estimation for each applied maneuver. The comparisons between the different flight test maneuvers concern the resultant parametric estimation reliability. The objective is to exhibit the fundamental importance of the excitation signal for the system identification entire process. Furthermore, it is desired to show that the approach for the parametric identification reliability assessment through the Fisher's Information Matrix computation could be used as planning, development and flight test maneuver optimization criterion.*

Keywords: *parametric identification, sensor calibration, flight path reconstruction*

1. Introduction

The aircraft aerodynamic derivatives estimation through flight test data analysis is on of the most important tools for aircraft development and certification. The aircraft parameter estimation procedure is composed by four main elements equally important: the estimation and optimization method, the flight test data acquisition system, the aircraft dynamic models and the flight test maneuvers, also called excitation signals. The aircraft parameter estimation success depends on these four elements.

The adequate flight test maneuvers specification is very important in order to guarantee the entire parameter estimation procedure reliability. The specification of the aerodynamic parameter estimation flight test maneuver must increase the efficiency of the parameter estimation algorithms and must provide the access of better identificability levels for the parameters. The main goal is to develop an excitation signal that excites as high as possible the dynamic system modes in order to provide high sensitivity of the output equations with respect to the system parameters, while operational, flight safety and mathematical constraints are respected.

This work deals with the specification of conventional flight test maneuvers. The conventional flight test maneuvers considered are the doublet, the 2-1-1 and the 3-2-1-1. These maneuvers are well known as excitation signals used for aircraft parameter estimation and will be specified concerning the input auto-spectrum, or the input power spectrum density (PSD). The PSD of the conventional excitation signals will be maximized on the system modes in order to provide the maximum excitation of the system variables and maximize de output sensitivity to the system parameters.

The flight test campaign that provide data for the present analysis took place as a part of cooperation between the Centre for Aeronautical Studies - UFMG (CEA-UFMG) and the Technological Institute of Aeronautics - CTA. The goal of this cooperation is to start flight testing activities for parameter estimation of light aircraft in Brazil. The aircraft flown is two side-by-side seat research aircraft, the CEA-205 CB.9 Curumim, developed and built by CEA-UFMG. Two flight data acquisition systems were used in parallel, one of them developed by CEA, and a system identification tool, currently in development, were testified on real flight test. During the flights, several flight test maneuvers were performed, aiming statistical basis for comparisons between dynamics excitation.

From the results of the conventional flight test maneuvers comparison, it is clear that for the system identification process reliability improvement it is strongly recommended that excitation signals specification must be treated on a case-by-case basis, concerning the a priori information available about the system dynamics on analysis.

2. The output-error method and the maximum likelihood criterion for parametric estimation

The parametric estimation can be carried out through different methods, such as: equation-error, filter-error and output-error. In this study, the output-error was applied. Unlike equation-error methods, the output-error ones are not asymptotically biased in the presence of measurement noise. In the other hand, the output-error estimators do not allow for process noise, what can be accomplished by filter-error methods, which support the both measurement noise and process noise (Maine, Iliff; 1985) (Maine, Iliff; 1986).

The basic idea of the output-error method is to compare the outputs of the mathematical model with the real aircraft flight test response while both are submitted to the same excitation signal inputs. The difference between the model output and the aircraft response is called the output-error. This error compose the cost function, which is minimized with respect to the system parameters to be estimated.

The maximum likelihood criterion, also called MLE (Jategaonkar, 2001), is used in this study for the parametric estimation. The MLE is the most commonly used technique for estimation parameters from dynamic flight test data. Considering that the aircraft dynamics can be expressed by:

$$\dot{x}(t) = f(x(t), u(t), \theta) \tag{1}$$

where x and u are the state vector and the control vector, respectively. In addition:

$$y(t) = h(x(t), u(t), \theta) \tag{2}$$

$$y_m(i) = y(i) + \upsilon(i) \tag{3}$$

where y is the output equations vector and y_m is the measurement vector, which is assumed to be polluted with zero mean Gaussian measurement noise, described by:

$$E\{\upsilon(i)\} = 0 \tag{4}$$

$$E\left\{\upsilon(i)\,\upsilon^T(j)\right\} = R \cdot \delta_{ij} \tag{5}$$

Under reasonable assumptions and considering the distribution of the measurement noise, it is possible to determine the conditional probability $p(Y|\theta)$ for each set of parameters θ, where Y denotes the measurement variables, as follows:

$$p(Y|\theta) = \left[(2\pi)^{-\frac{q}{2}} |R|^{-\frac{1}{2}}\right]^N \exp\left\{\sum_{i=1}^{N} -\frac{1}{2}\left[y_m(i) - y(i)\right]^T R^{-1} \left[y_m(i) - y(i)\right]\right\} \tag{6}$$

The maximum likelihood estimation is such that the parameter vector θ is estimated while $p(Y|\theta)$ is maximized. Regarding computational purposes, instead of maximize $p(Y|\theta)$ it is convenient to minimize $-L(p(Y|\theta))$, that is equivalent. Besides, assuming that R is constant, it is possible to compose the cost function as follows:

$$J(\theta) = \frac{1}{2}\sum_{i=1}^{N} \left[y_m(i) - y(i)\right]^T R^{-1} \left[y_m(i) - y(i)\right] \tag{7}$$

The cost function in Eq. (7) can be minimized by several methods, such as Gauss-Newton, Levenberg-Marquadt, Nelder-Mead. In this work, the Gauss-Newton method was used.

3. Parametric Estimation covariance by Fisher's Information Matrix and its relationship to the excitation signals

In this section, will be discussed the theoretical parametric estimation covariance assessment through the Fisher's Information Matrix. Analyzing closely the Cramer-Rao inequality, considering the existence of $p\left(Y|\theta\right)$ and that the MLE is asymptotically unbiased, it is possible to demonstrate that the lower bound for parametric estimation covariance can be obtained from the Fisher's Information Matrix (Balakrishnan, 1968) (Goodwin, Payne; 1977) (Iliff, Maine; 1981).

$$\text{cov}\left(\hat{\theta}\right) \geq M\left(\theta\right)^{-1} \tag{8}$$

where *M* is the Fisher's Information Matrix, which by definition is:

$$M(\theta) \equiv E\left\{\left(\frac{\partial \ln\left[p(Y|\theta)\right]}{\partial\theta}\right)^T \left(\frac{\partial \ln\left[p(Y|\theta)\right]}{\partial\theta}\right)|\theta\right\} \tag{9}$$

With reasonable assumptions, it is possible to prove that the MLE is asymptotically unbiased. What means that the parametric estimation error approaches 0 when the time interval analyzed become longer. In addition, the equality on the Eq. (8) holds when the estimator is an efficient estimator. It is important to point out that is possible to show that, under mild regularity conditions, the Cramer-Rao inequality approaches the equality when the parametric estimation is a MLE estimation.

The Fisher's Information Matrix compute the flight test data information content. At this point, it is important to show an alternate expression for the information matrix. For the system described by Eq.(1) - (5), combining Eq. (7), and keeping in mind that $y_m(i)$ is a realization of a random vector sequence, and is therefore not a function of θ, it is possible to alternatively express the information matrix as follows:

$$M = \left[\sum_{i=1}^{N} \frac{\partial y\left(i\right)}{\partial\theta}^T R^{-1}\frac{\partial y\left(i\right)}{\partial\theta}\right] \tag{10}$$

This expression for the information matrix shows that the information content of the flight test data is a function of both the sensitivities of the output equations with respect to the system parameters and the measurement noise covariance matrix, R. In this way, becomes clear that the output equations sensitivity to the system parameters must be maximized in order to provide the minimal covariance of the parametric estimation.

Considering that the system described in Eq. (1) - (3) can be represented by the following small perturbations dynamic model (Morelli, 1990):

$$\dot{x} = A\left(\theta\right)x\left(t\right) + \left(\theta\right)u\left(t\right) \tag{11}$$

$$x\left(0\right) = 0 \tag{12}$$

$$y\left(t\right) = C\left(\theta\right)x\left(t\right) + D\left(\theta\right)u\left(t\right) \tag{13}$$

$$y_m = y\left(i\right) + \upsilon\left(i\right) \tag{14}$$

the sensitivity matrix is:

$$S_i = \frac{\partial y\left(i\right)}{\partial\theta} \tag{15}$$

and can be computed by:

$$\frac{d}{dt}\left[\frac{\partial x}{\partial\theta_k}\right] = A\frac{\partial x}{\partial\theta_k} + \frac{\partial A}{\partial\theta_k}x + \frac{\partial B}{\partial\theta_k}u \tag{16}$$

$$\frac{\partial x}{\partial\theta_k}\left(0\right) = 0 \tag{17}$$

$$\left[\frac{\partial y}{\partial\theta_k}\right] = C\frac{\partial x}{\partial\theta_k} + \frac{\partial C}{\partial\theta_k}x + \frac{\partial D}{\partial\theta_k}u \tag{18}$$

where $k = 1, 2, 3, ..., n_\theta$, and n_θ is the number of parameter of the system. The control input vector u, also called excitation signal vector, or simply flight test maneuver, influences the output sensitivity both directly as a forcing function and indirectly through the states. It is clear that the information matrix, and therefore the Cramer Rao bounds, depends on the applied excitation signal.

4. Spectral analysis of excitation signals

The present work deals with a parametric estimation flight test maneuvers specification technique that is based on the spectral content of the excitation signal. More precisely, the power spectral density (PSD) of the inputs are analyzed and the maneuvers are specified by shaping the inputs in a way to increase the PSD of the signals in the range of the natural modes of the dynamic system of interest. This translate into exciting the dynamic system modes such that the output sensitivities to the system parameter are high.

The parameter estimation flight test maneuvers specification, therefore, will be as efficiency as better be the a priori knowledge of the dynamic system. The sensitivity of this technique to the a priori model indicates its robustness. In the next section, more details about the PSD analyzes for the excitation signals specification will be shown.

5. Case study results

The case study considered in this work is the parametric estimation of the longitudinal short period of the CEA-205 CB.9 Curumim aircraft. The aircraft short period is the high frequency and high damped longitudinal dynamics that involves the variation of angle-of-attack and pitch rate without excitation of flight velocity and altitude. The mathematical model assumed is the linearized small perturbation dynamic model, described with respect to the short period dimensional aerodynamic derivatives:

$$\dot{x} = \begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha & 1+Z_q \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta e} \\ M_{\delta e} \end{bmatrix} \delta e \tag{19}$$

where α and q are the state variables angle-of-attack and pitch rate, and δe is the input variable elevator deflection. The output equations are:

$$y = \begin{bmatrix} \alpha_m \\ q_m \\ a_z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ Z_\alpha \frac{V_0}{g} & Z_q \frac{V_0}{g} \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ Z_{\delta e} \frac{V_0}{g} \end{bmatrix} \delta e \tag{20}$$

where α_m, q_m and a_z are the measured perturbation of angle-of-attack, pitch rate and vertical acceleration, respectively.

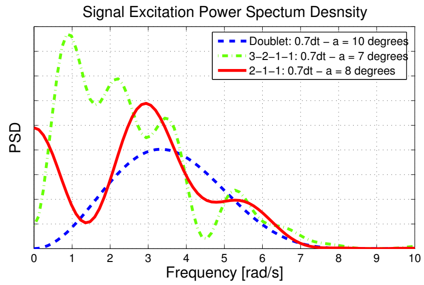
The a priori short period model of the flight testing aircraft, obtained from engineering knowledge and from previous flight tests, indicates the mode natural frequency of 3.31 rad/s. The flight conditions selected was 4500 ft of pressure altitude and 70 mph of true air speed. The parametric estimation flight test maneuvers, therefore, will be specified in the way that the shaping of the input signals provides the maximum excitation signals PSD in the frequencies near 3.31 rad/s and that the flight test runs be performed at 4500 ft and 70 mph.

5.1 Flight test methodology and planning

The effort presented in this work is dedicated to the analysis and understanding of the phenomenons around the specification, execution, and data reduction of different parametric estimation flight test maneuvers. Therefore, in search of adequate concluding remarks and statistical basis, for each flight test maneuver that has being studied, several flight test runs were performed.

In this paper, the results of conventional flight test maneuvers analysis are shown. The conventional flight test maneuvers that have being considered are the doublet (or 1-1), the 2-1-1 and the 3-2-1-1. These maneuvers are bang-bang excitations and are composed by square waves. The specification of these maneuvers for flight testing consists in choose how long will be the square waves and choose the signal amplitude. The determination of the square waves period is done through the δt, which denotes how long the unitary pulse is. Concerning the 2-1-1, for example, this signal is composed by two δt of $\pm a$ (signal amplitude), followed by one δt of $\mp a$ and one δt of $\pm a$. The signals shaping, therefore, are defined by δt. In addition, δt provides the PSD adjustment in order to maximize the excitation of the dynamic mode of interest.

The δt for the doublet, for the 2-1-1 and for the 3-2-1-1 that maximizes the signals PSD near 3.31 rad/s is 0.7 s. Concerning the flight test hazard analysis and its minimization procedures for flight test risk level attenuation, it was determined that the flight test maneuvers for short period excitation must respect $\pm$ 0.6 g of vertical acceleration excursion, it is the main constraint imposed to the procedure. By previous planning simulations concerning the short period a priori dynamic model it was adjusted the excitation signals amplitude a in order to respect the constraints. Firstly, the doublet simulation was analyzed and was observe that a 10 degrees 0.7 δt doublet elevator deflection provided the vertical acceleration output response excursion of approximately $\pm$ 0.6 g, respecting the vertical acceleration constraint. In the same way, were selected the 8 degrees 0.7 δt 2-1-1 and the 7 degrees 0.7 δt 3-2-1-1, which also respected the imposed constraint. The power spectrum densities of these inputs are shown in Fig. 1. The respectively covariances computed by information matrix for each maneuver are also shown in Fig. 1.



Absolute parameter covariance by Fisher's Information Matrix

Parameter		Doublet	2-1-1	3-2-1-1
Param_1	Z_α	0.0407	0.0119	0.0089
Param_2	Z_q	0.0269	0.0092	0.0068
Param_3	$Z_{\delta e}$	0.0399	0.0096	0.0078
Param_4	M_α	0.7333	0.2420	0.1710
Param_5	M_q	0.0235	0.0080	0.0072
Param_6	$M_{\delta e}$	0.0411	0.0204	0.0204

Figure 1. Power spectrum density of the specified excitation signals and the respectively covariances computed by Fisher's Information Matrix

The signals PSD's and the absolute parameter covariances computed by the information matrix are consistent. The 3-2-1-1 and the 2-1-1 introduces more power in the frequencies near the frequency of interest, therefore the parameter covariances applying the 3-2-1-1 and the 2-1-1 are smaller than applying the doublet.In addition, the band width of the 2-1-1 and 3-2-1-1 PSD's are larger near 3 rad/s, what increase the robustness of these maneuvers concerning that the a priori knowledge of the short period dynamics is not very reliable. The planning simulation time responses are shown in Fig.2 - Fig.4.

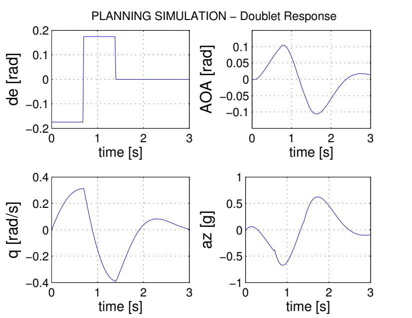


Figure 2. Doublet planning simulation

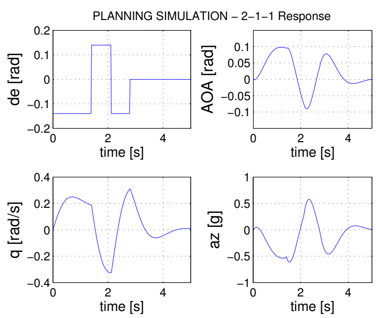


Figure 3. 211 planning simulation

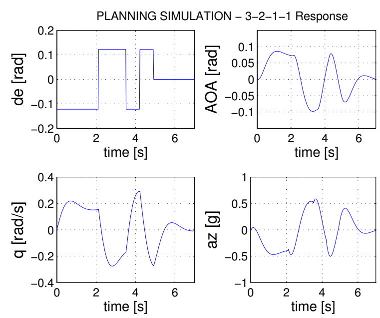


Figure 4. 3211 planning simulation

It was planned that the flight tests should be done on calm air in order to avoid convective turbulence that results in process noise. In addition, 40 runs for each maneuver that were being studied were specified for flight testing. Resulting in more than 120 flight test maneuvers applied just for conventional signal excitation analysis, aiming adequate statistical basis for comparison. For the statistical analysis, just the 20 best parametric estimation results of each maneuver were selected. The flight test input and the aircraft flight test output responses for these 20 best runs of each maneuver are shown in Fig.6 - Fig.8.

All applied inputs were performed directly by the pilot. The time and amplitude were controlled by the pilot through a visual elevator deflection measurement device installed on the left seat of the aircraft. It is being assumed that the uncertainty of the ideal realization of the input sequences by the pilot is a part of the qualitative analysis of the specified maneuvers. As more complex the inputs are, more difficult its implementation by the pilot will be. Besides, it is important to point out that was observed some degradation on the data for the 3-2-1-1. It could be mostly due to the increase in the process noise, which was also observed during the 3-2-1-1 flight test runs, but some portion of this degradation may be also due to the increase in complexity presented by the 3-2-1-1, what make its repeatable implementation more arduous.

5.2 Parametric estimation results

The 20 best parametric results for each flight test maneuver were select for analysis. The analysis were done by adjusting Gaussian normal probability density function to each parametric estimation result. The PDF of each parameter estimation distribution was compared with respect to the different maneuvers applied. Figure 9 to Figure 14 show the histograms, the PDF's and the scatters of the estimations separated by parameter, for the doublet, for the 2-1-1 and for the 3-2-1-1.

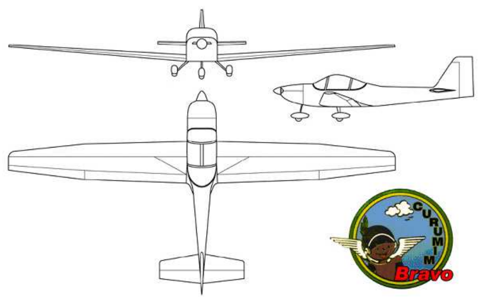
Firstly, it is important to point out that the bias between the resultant estimations distribution mean and the a priori parameter value is assumed to be due to the relative poor knowledge about the aerodynamic characteristics in the stage of flight test planning and maneuver specification. It is suitable, therefore, to consider that the best results are those that the scattered around a mean value is the lowest. Besides, this mean value can be ponder as the best estimated value for each parameter.

It is possible to observe that the PDF of the 2-1-1 and 3-2-1-1 are more accentuated for all parameters, indicating lower scattered and higher reliability of these estimations. By comparing the doublet with the 3-2-1-1 and 2-1-1, it is possible to say that the results are consistent with the previous analysis of the excitation signals, which shows that the 3-2-1-1 and the 2-1-1 signals concentrate more power near the short period natural frequency than the doublet. The same happens when the analysis is made by the parameter estimation covariances computed by the information matrix, Fig.1.

Another important point to discuss is that in the flight test planning it was expected that the better results would become from the 3-2-1-1 maneuver. For M_α, M_q and $M_\delta e$ the results of the 2-1-1 were better than the results os the 3-2-1-1. It is clearly observed that the 3-2-1-1 data was polluted by process noise. In addition, it is credited to the increase in the complexity of the input signal the decrease in the ideal pilot input repeatability over the 40 3-2-1-1 flight test runs. These aspects may degrade the results such that the 2-1-1 maneuver presented better results than the 3-2-1-1 maneuver.

5.3 Aircraft description

The CEA-205 CB.9 Curumim is a light aircraft developed and made by CEA-UFMG between 1989 and 1992. The Curumim entered into service in 1993 as experimental and research aircraft. The three view and the main characteristics of this aircraft is shown in Fig.5.



CEA-205 CB.9 Curumim Characteristics		
Wing area	16.40	m^2
Wing span	14.00	m
Length	7.40	m
Mean aerodynamic chord	1.22	m
Aspect ratio	11.95	-
Weight *	560.00	kgf
Load factor limits (555 kgf)	+2.9/-1.5	g
Stall speed (clean, 555 kgf)	120.00	km/h
Maneuver speed	77.00	km/h
VNE	160.00	km/h

* (2 occupants, 70 lts fuel and flight test equipment)

Figure 5. Three View of the CEA-205 CB.9 Curumim Aircraft

6. Concluding remarks

Comparison between conventional doublet, 2-1-1 and 3-2-1-1 parametric estimation flight test maneuvers was performed in this work. In addition, some aspects of technique that provides useful methods for flight test campaigns intended for aircraft aerodynamic derivatives estimation planning and execution was presented.

The most obvious conclusion obtained from this experiment is that the aircraft aerodynamic derivatives estimation procedure is very sensitivity to the applied excitation signals. It became clear that the 3-2-1-1 and the 2-1-1 flight test maneuvers provided high accuracy than the doublet maneuver for almost all parameters, except for those that the identificability is very high, such as M_α, and that the PSD specification technique can be used for easily flight test planning. In the other hand, was observed that the increase in the input signal complexity can cause reduction of the implementation pilot capability, specially in experiments that require high repeatability. The specification of high accuracy parametric estimation flight test maneuvers,however, is intended just to reduce the necessity of flight test recurrence and to increase the flight test campaign advantages. In this way, the authors are encouraged in applying more advanced parametric estimation flight test maneuvers development techniques, such as techniques that directly minimizes the Cramer-Rao lower bounds by shaping square waves bang-bang signals.

Furthermore, this work shows a flight test maneuver specification technique that provides a fulfilled methodology for stability and control derivatives estimation flight test campaign planning and execution. This methodology allow the direct communication between the engineering requirements for estimation results and flight test operational analysis, including flight test hazard analysis, specially for hazards minimizing procedures. This technique can become a strong tool and can play an important hole for increase the effectiveness on operational envelope expansion, automatic pilot, high angle-off-attack and control system design flight test campaign, among other applications from small experimental airplanes and jets, to large transport jets and high performance aircraft.

7. Acknowledgements

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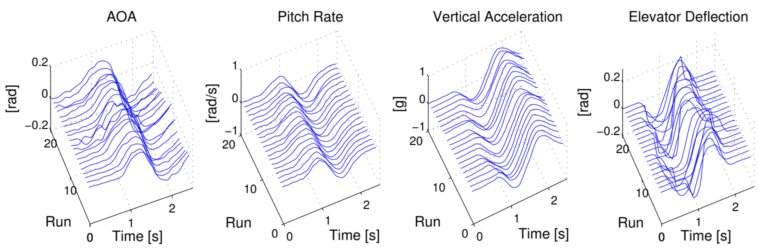

Figure 6. Time Responses of Doublet Flight Test Runs

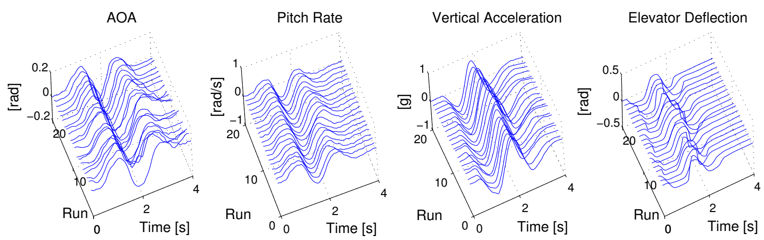

Figure 7. Time Responses of 2-1-1 Flight Test Runs

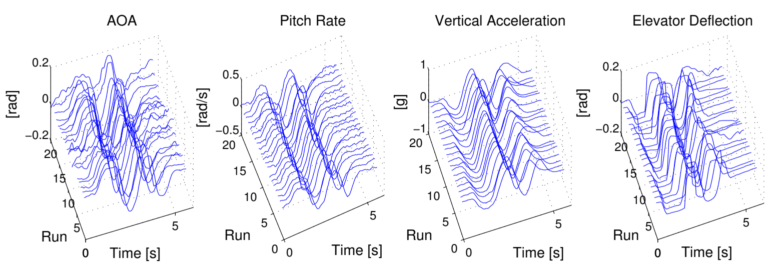

Figure 8. Time Responses of 3-2-1-1 Flight Test Runs

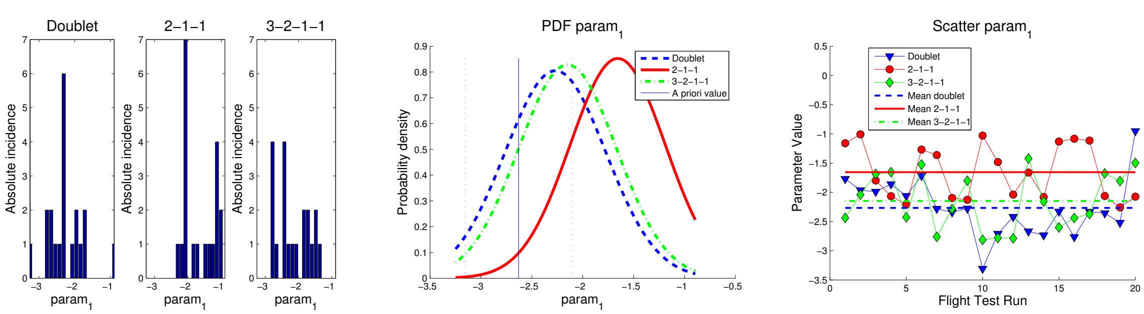

Figure 9. Parametric estimation results for Z_{α} - $Param_1$

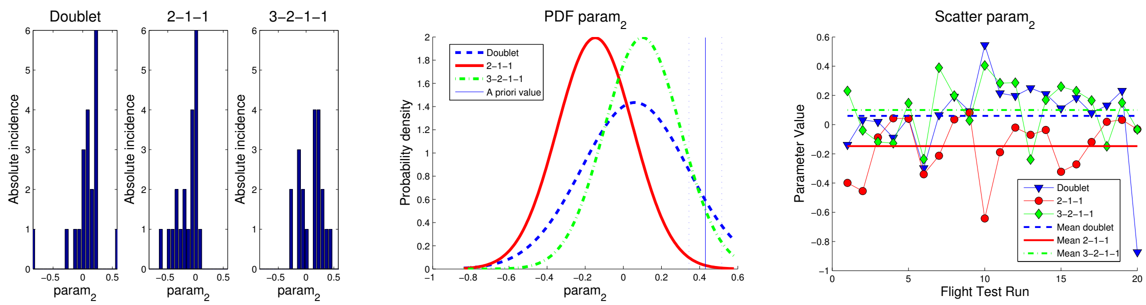

Figure 10. Parametric estimation results for Z_q - $Param_2$

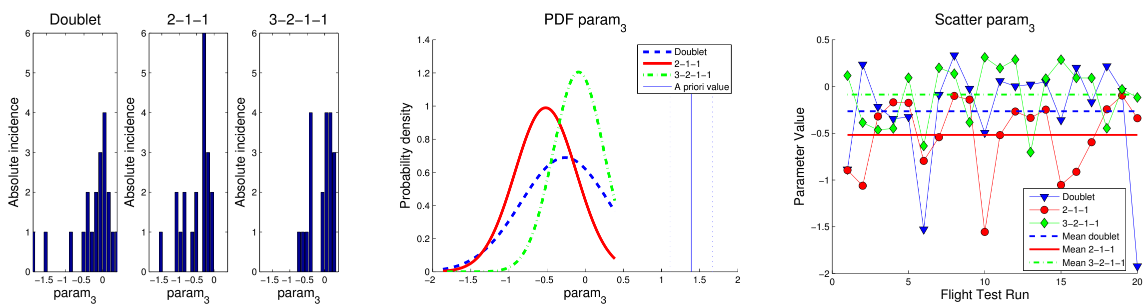

Figure 11. Parametric estimation results for $Z_{\delta e}$ - $Param_3$

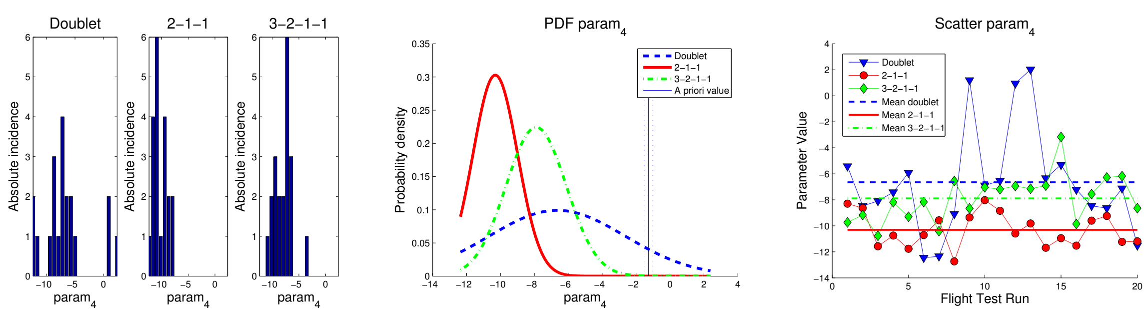

Figure 12. Parametric estimation results for M_α - $Param_4$

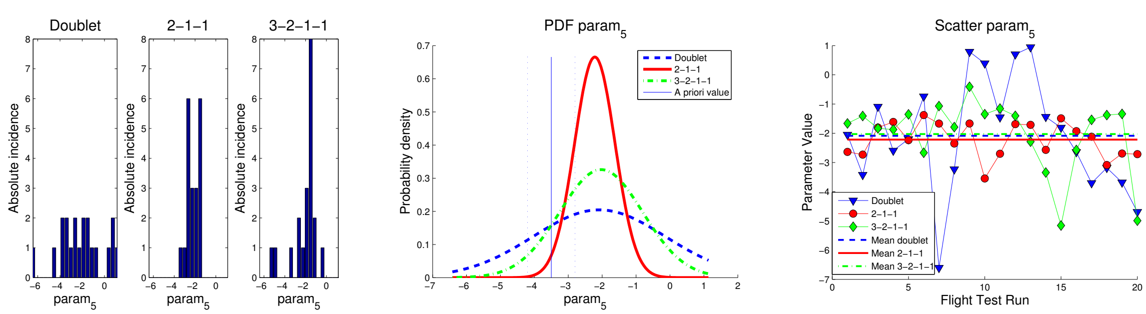

Figure 13. Parametric estimation results for M_q - $Param_5$

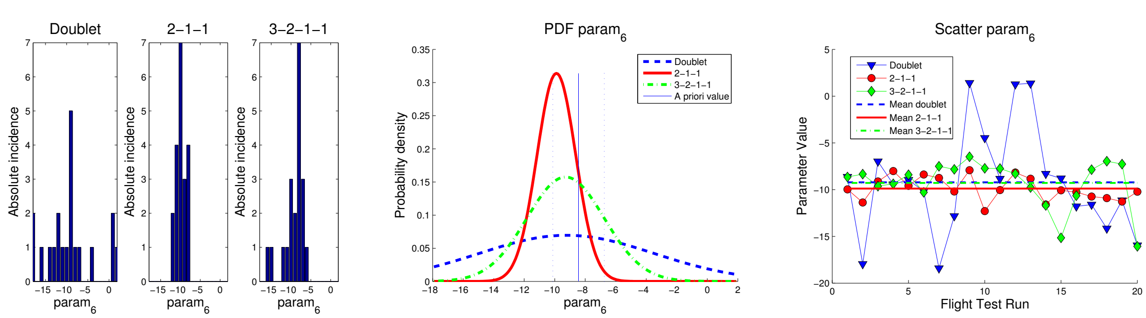

Figure 14. Parametric estimation results for $M_{\delta e}$ - $Param_6$