DYNAMIC ANALYSIS OF LAZY-WAVE STEEL RISERS

Rafael Loureiro Tanaka
Department of Mechanical Engineering
University of São Paulo, São Paulo, Brazil
rafael.tanaka@poli.usp.br

Clóvis de Arruda Martins
Department of Mechanical Engineering
University of São Paulo, São Paulo, Brazil
cmartins@usp.br

Fernanda Cristina de Moraes Takafuji
Department of Mechanical Engineering
University of São Paulo, São Paulo, Brazil
fernanda.takafuji@poli.usp.br

Lauro Massao Yamada da Silveira
Department of Mechanical Engineering
University of São Paulo, São Paulo, Brazil
lauro.silveira@poli.usp.br

Rosianita Balena
Department of Mechanical Engineering
University of São Paulo, São Paulo, Brazil
rosianita.balena@poli.usp.br

Abstract.*This paper discuss the influence of parameters such as pipe segment lengths and floater characteristics (diameter, pitch and length) in the dynamic response of a Steel Riser with given inner diameter, thickness and material, broadening the still narrow knowledge base on 'lazy-wave' risers, emphasizing its advantages over the simpler catenary solution and aiding designers in the search for the best configuration.*
For this matter an algorithm which makes automatic simulations taking as parameters the range and number of divisions of the previously mentioned variables was developed.This work will focus on the dynamic problem, which takes into account sea currents, platform offsets and waves.
In the dynamic simulation, a frequency domain model with damping linearization and finite element discretization of the resulting problem is implemented. The results are then corrected using a boundary layer technique.
A quick derivation of the model will be presented, as well as a short discussion on its advantages and limitations. Results such as top tension, top angle, TDP curvature, TDP excursion and wave behavior among others will be also presented and discussed, with the intention of drawing general conclusions on the influence of each parameter on the riser's overall dynamic response and a selection of which results to look at when trying to pick the best configurations among several candidates.

Keywords: Lazy-Wave, Steel Risers, Dynamic Analysis

1. Introduction

Offshore oil exploitation imposes great technological challenges, creating a regular development need, especially in a country trying to be self-sufficient in oil production and whose reserves are mostly submerged in great depth regions. Risers are key parts in oil production, since they are responsible for the physical connection between the oil field and the floating production unit. Therefore, they must be very well studied, especially regarding the configurations and stresses they are subject to.

The most common and currently most used configuration is the catenary, in which the riser hangs from the platform subject to his own weight and sea current, until the ground is reached. Although the simplicity of this configuration has great advantages, the tension on the top of the riser grows with the depth, causing great problems in ultra deep waters (deeper than 2000 m). In these cases, the use of another configuration, called lazy-wave, seems to be more attractive. In this configuration, an intermediate flotation is introduced, helping support the riser and reducing its top tension. Moreover, it also minimizes the transference of movement from the top to the region located after the flotation, reducing the riser movement in the region next to the TDP.

The existence of this configuration type multiplies the number of possibilities for each case. While for a catenary the main decisions to be taken are the type of riser and its length, in a lazy-wave configuration it is also necessary to select the type and the amount of floaters or buoys to be used and their position in the riser. The addition of these elements can solve the already described problems (fatigue at top and TDP), but can also create others, as great variations of curvature or points where the effective tension is very low, or even dynamic compression may occur. As additional difficulty, the environmental conditions vary during the year, requiring simulations of the riser response subject to several currents, waves and movements of the platform. Thus, as the same riser must satisfy design requirements under many different conditions, the design of a single riser becomes a very complex task, requiring a lot of human and computational resources.

In order to increase the knowledge base about the lazy-wave configurations, the effect of some parameters on the behavior of a selected riser was studied, looking for behavior patterns that guide the designer in getting a feasible configuration more quickly, reducing the necessary effort in the first design cycle.

The selection of the dynamic load cases used for this analysis will also be discussed, addressing the magnitude of the difference among the cases combining centenary currents with decenary waves or decenary currents with centenary waves and the ones which combines centenary currents with centenary waves. The aim of this discussion is to investigate whether the load cases combining centenary currents with centenary waves are so worse than the others as to justify the simulation of two times more cases.

2. Model

2.1 Introduction

The presented model was originally developed for free hanging risers (Santos, 2003), and was later extended in order to include lazy-wave configurations. The model was developed excluding torsional and flexional rigidity because of their local effect (Pesce, 1997 and Aranha, 1992). On the other hand, due to the lack of time for the riser to adapt its configuration geometrically to the motion of the floating unit, axial rigidity becomes important as the riser finds a instantaneous equilibrium position by deforming axially. Even though flexional rigidity is ignored at first, it is later introduced by means of a boundary layer technique, to recover motion and boundary conditions at the TDP. Only the cases where a portion of the riser is laid on the ground (as the cases of simple catenary and lazy-wave) are treated. In fact, the model hereby presented represents only the suspended part, which goes from the TDP – touchdown point to the top. In the present model, the supported part of the riser will be treated as an addition to the local rigidity. The interaction between the riser and the seabed is still discussed in specialized literature and a consensus was not reached, only some recommendations (Det Norske Veritas, 2001).

The TDP is considered fixed and articulated. This approach simplifies the dynamic problem (Martins, 2000) and only affects the local dynamics of the riser, not affecting its global dynamics.

The dynamic problem can be defined as solving the displacements, forces and moments given the three-dimensional static configuration of the riser and the dynamic loads applied to it. As the dynamic problem will be treated as a disturbance of the static problem, total variables will be defined as the addition of a static and a dynamic parcel.

2.2. Applied Loads

The loads per unit length acting on the riser will also be separated in static and dynamic components. The static components include gravitational, hydrostatic and sea current loads.

The dynamic components include hydrodynamic loads, modeled through the classic Morison formulation, which causes the system damping, the loads caused by the waves and the inertial loads, which are constituted of two parts, one modeled according to the D'Alembert principle and the other proportional to the volume of fluid dislocated by the structure and to the coefficient of added mass. It is worth noticing that as the length of the riser is much bigger than its diameter, the rotational inertia of the element is not taken into account, which is a usual simplification in slender beam theory.

2.3 Dynamic Problem Equations

The following basic equations were treated in the dynamic model development: constitutive equation, force balance, moment balance and geometric compatibility. Being the dynamic parcel of the displacements and angles of much lesser order of magnitude than the static, second order products of these were discarded in the deduction of the equation system that describes the problem.

2.4 Friction

One of the non-linearities to be removed in order to solve the dynamic problem in the frequency domain is the friction with the soil. In this model this is done by replacing the portion of the cable laid on the seabed with a set of linear springs connected to the TDP.

The spring constants are calculated in a quasi-static manner, applying to the TDP of the static equilibrium configuration a force increment which causes a change to the TDP position. The spring constants are then obtained dividing the force increment by the displacement change it caused. These constants later become a part of the system's rigidity matrix.

2.5 Boundary Conditions

The first imposed boundary condition is an articulated top. The TDP is articulated and can move in the seabed plan, linked to a set of springs as discussed in section 2.4.

2.6 Dynamic Model Discretization

Applying the virtual work principle, and imposing null virtual work in any moment, a continuous form equation is obtained. The resolution will be made in discrete domain, using finite element method.

The transformation from the continuous to the discrete system requires form functions to be chosen. In the model only first order differentials of the displacements appear, so the form functions should be continuous. They interpolate linearly the displacements from neighbor nodes for points that are not nodes.

As a result of such discretization, the following matrix system is obtained:

$$[M]\{\Delta\ddot{u}\}+[C]\{\Delta\dot{u}\}+\left([G]+[\overline{K}]\right)\{\Delta u\}=\{P\}e^{iwt}$$

where $[M]$ is the mass or inertia matrix, $[G]$ is the geometric rigidity matrix, $[\overline{K}]$ is the elastic rigidity matrix (already including local rigidity due to the part on the seabed), $[C]$ is the damping matrix, $\{P\}$ is the wave loading vector, $\{\Delta u\}=\left\{\{\Delta x\}\,\{\Delta y\}\,\{\Delta z\}\right\}^t$ are nodal displacements and w is the wave frequency.

2.7 Frequency Domain Solution

As the obtained system is linear with harmonic excitation, the solution can be obtained in frequency domain writing the nodal displacements in the form $\{\Delta u\}=\{\Delta u_0\}e^{iwt}$.

Then the equation can be rewritten:

$$-w^2[M]\{\Delta u_0\}e^{iwt}+iw[C]\{\Delta u_0\}e^{iwt}+\left([G]+[\overline{K}]\right)\{\Delta u_0\}e^{iwt}=\{P\}e^{iwt}\quad \text{or,}$$

$$[D]\{\Delta u_0\}=\{P\}$$

Solving this linear system, the $\{\Delta u_0\}$ vector is obtained.

2.8 Damping Linearization

The damping is a major source of non-linearity in the present problem. In order to use a frequency domain method, the damping must be linearized using one of the many techniques available (Patel,1995). The technique hereby presented is based on matching the dissipated energy in a cycle for both linear and non-linear drag formulations.

According to Morison's formulation, the normal component of the drag force per length unit is:

$$f_n=\frac{1}{2}\rho_{water}Dc_{d,n}|v_n|v_n$$

where v_n is the normal component of the riser velocity relative to the fluid. As both the movement of the riser and of the fluid are generated mainly by the effect of the waves, which can be approximated as harmonic, the following approximation can be made:

$$v_n=wA_0\,\text{sen}(wt)$$

where w is the relative movement frequency and A_0 its amplitude. Doing so, Morison's formulation is rewritten:

$$f_n=-\frac{1}{2}\rho_{water}Dc_{d,n}w^2A_0^2|\text{sen}(wt)|\text{sen}(wt)$$

However, to solve the dynamic problem in the frequency domain this non-linearity must be removed. To do so, a linear expression of the drag force is written:

$$\bar{f}_n=-\frac{1}{2}\rho_{water}D\bar{c}_{d,n}w^2A_0^2\,\text{sen}(wt)$$

where $\bar{c}_{d,n}$ is a modified drag coefficient chosen so that this linear equation gives values close to the non-linear form. The criterion used in this selection was the dissipated energy in a cycle, which is imposed as equal for both formulations. Integrating from 0 to $T = \frac{2\pi}{w}$ and dividing the results, a relation between $c_{d,n}$ and $\bar{c}_{d,n}$ is obtained:

$$\frac{\bar{c}_{d,n}}{c_{d,n}} = \frac{\int_0^T |\sin(wt)| \sin^2(wt)\,dt}{\int_0^T \sin^2(wt)\,dt} = \frac{8}{3\pi}$$

The linearized Morison equation then becomes:

$$\bar{f}_n = -\frac{4}{3\pi} \rho_{water} D c_{d,n} w A_0 v_n$$

With this linear equation the dynamic problem can be solved if the amplitude A_0 is known. However, in the real situation it must be estimated, which is done by means of an iterative process described below.

The initial A_0 value throughout the line is the amplitude of the wave. The domain frequency problem is then solved and the velocities and amplitudes of all the points obtained. With these, the new relative velocities are calculated and a new damping matrix calculated. Solving the domain frequency problem again, new velocities and amplitudes are obtained and so on. This is repeated until a predetermined precision is achieved.

2.9 Boundary Layer Correction

The final source of non-linearity that must be removed is the soil-riser contact. According to MARTINS (2000), it can be proved that in typical cases there is no impact force among the riser and the soil, and an articulation can be placed on the TDP to linearize the problem. To consider the movement of the TDP due to the flexional rigidity effect, a boundary-layer correction is performed. This means that the global solution given by the ideal cable model is matched to a local solution which takes flexional rigidity into account. The results given by the latter tends asymptotically to the cable model as the distance from the TDP increases. This approach allows expressions for the curvature and bending moment to be derived, thus reproducing the dynamic behavior of the portion of the riser near to the TDP, including the flexional rigidity effect.

3. Data and Metodology

All the simulations were performed using a riser with three segments as in Figure 1:

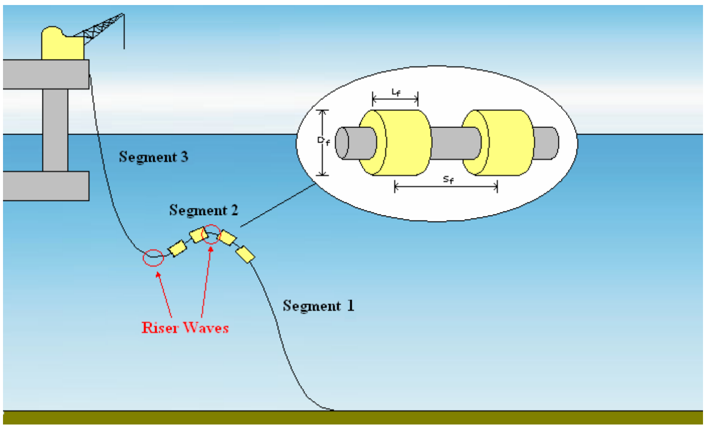

Figure 1. Segments of the simulated riser

The three segments share the same properties, except for the floater on the second segment. These properties are given below:

Inner Diameter	0.4064	m
Outer Diameter	0.4572	m
Young Modulus	207800	MPa

ρ	7.85	ton/m^3
ρ_{oil}	0.917	ton/m^3
Normal Drag Coefficient	1	
Tangential Drag Coefficient	0.1	
Added Mass Coefficient	1	

The Floater on the second segment has the following properties:

ρ_{float}	0.501	ton/m^3
Floater Diameter (Df)	1.37	m
Floater Length (Lf)	1.3	m
Floater Pitch (Sf)	2.0	m

The lengths of the three segments vary from configuration to configuration, being chosen as the variables for the parametric analysis. From now on they will be referred to as L1, L2 and L3, respectively. Total cable length will be referred to as Lt and can be calculated as the sum of the lengths of the three segments. The ranges are given in Table 1.

Table 1. Variable Ranges

Variable	Range (m)
L1	500 – 2133
L2	200 - 1200
L3	666 - 1800

The sea depth is 1255m, and the riser is suspended 1247.9 m over the seabed, with a horizontal projection of 2339.899 m. Local gravity acceleration is 9.81 m/s^2 and the water density is assumed to be 1.03 ton/m^3.

All configurations were tested in 192 different conditions, which can be divided in three groups of 64. The first group was created using a combination of decenary currents and centenary waves, in the second were grouped centenary currents and decenary waves, and in the last group centenary currents and centenary waves were combined.

The 64 cases of each group were created combining 8 currents which directions vary 45 degrees from each other (North, Northwest, etc…) with 8 waves inline with the current surface velocity directions in all possible pairs. The offset of the platform was fixed at 3.5 % of the water depth, in the same direction of the current of the corresponding case. For all cases the same RAO was used. Tables with the currents' and waves' parameters follow:

Table 2. Parameters from the waves

Direction (degrees clockwise from north)	Height (m)		Period (s)	
	Decenary	Centenary	Decenary	Centenary
0	6.1	7.0	14.00	14.70
45	6.9	7.8	14.62	15.35
90	4.0	4.6	8.14	8.19
135	4.0	4.6	8.14	8.19
180	5.2	6.3	9.28	10.00
225	4.7	5.4	8.99	9.40
270	4.2	4.7	9.05	9.21
315	5.5	6.7	10.76	11.35

Table 3. Parameters from the currents

Direction (degrees clockwise from north)	Surface Speed (m/s)	
	Decenary	Centenary
0	1.15	1.32
45	1.22	1.34
90	1.15	1.25
135	1.60	1.75
180	1.80	1.96
225	1.70	1.89
270	1.22	1.36
315	1.18	1.30

These 3 groups of cases were created with the intent of comparing the usual simulated conditions (groups 1 and 2) with the worst-case scenarios (group 3) to determine to which degree the results are different, and if this difference is big enough to justify the simulation of twice the number of cases.

4. Results

In this section the obtained results are presented. Two dimensionless variables have been used to simplify result comparison. The first is L1/L3 which represents the floater position. The smaller L1/L3 is, the closer to the TDP the floater is. The other is L2/Lt, which represents the proportion of the cable covered with floaters. The presented results are the envelops of the 128 cases formed by combining decenary waves with centenary currents and centenary waves with decenary currents.

In the first set of results (Figure 2), the total cable length was kept constant at 3000 m, while L1, L2 and L3 varied. Other lengths were tested as well and similar behavior observed. The figure shows the dependency of the obtained results on both the position of the floater (L1/L3) and its size (L2/Lt).

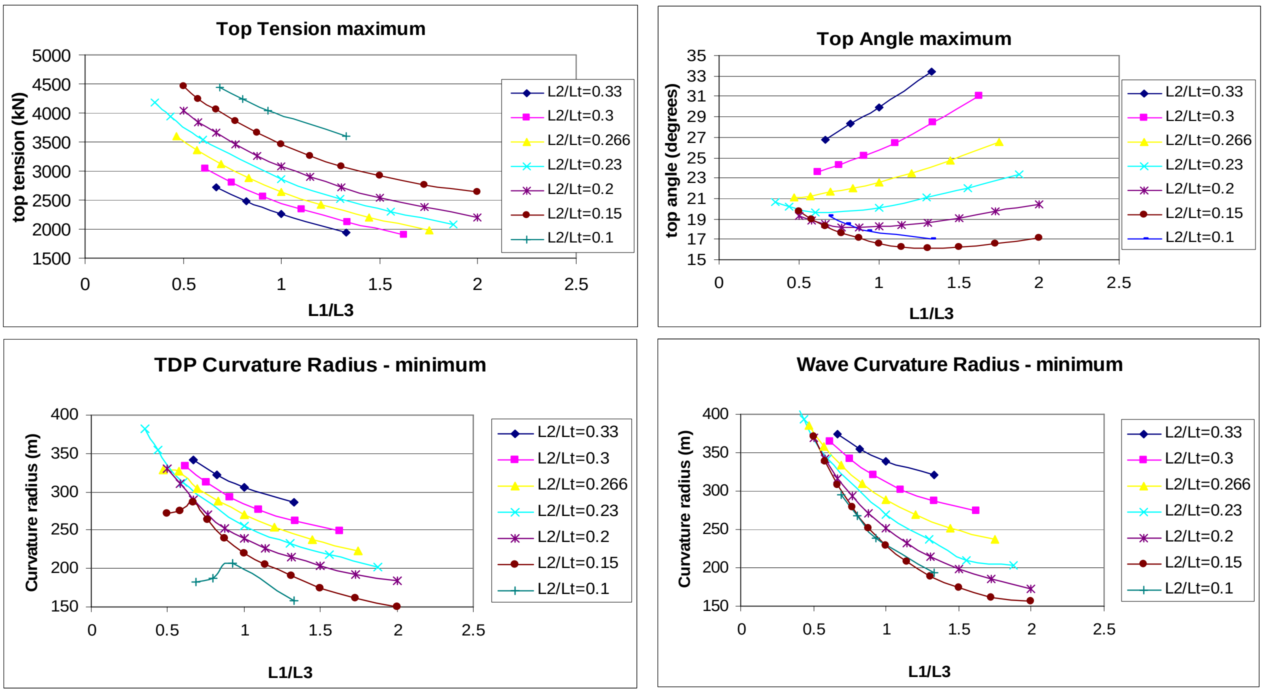

Figure 2. Results for constant cable length

Figure 2 shows that an increase in floater size reduces top tension and increases the minimum curvature radius at TDP, but increases angles at the top. The floater also cannot be too short because short floaters cause small curvature radius at the waves (for the definition of a wave, see figure 1). Placing the floater closer to the TDP increases both top tension and minimum curvature radius at the TDP and the waves, so a compromise solution must be achieved.

In the second set of results (Figure 3), the total length varied as well as L1, L2 and L3. However, the relation L2/Lt was kept constant at 0.2. Other values for L2/Lt have been tested and similar behavior detected, but this particular value has been chosen as a representative case due to space considerations.

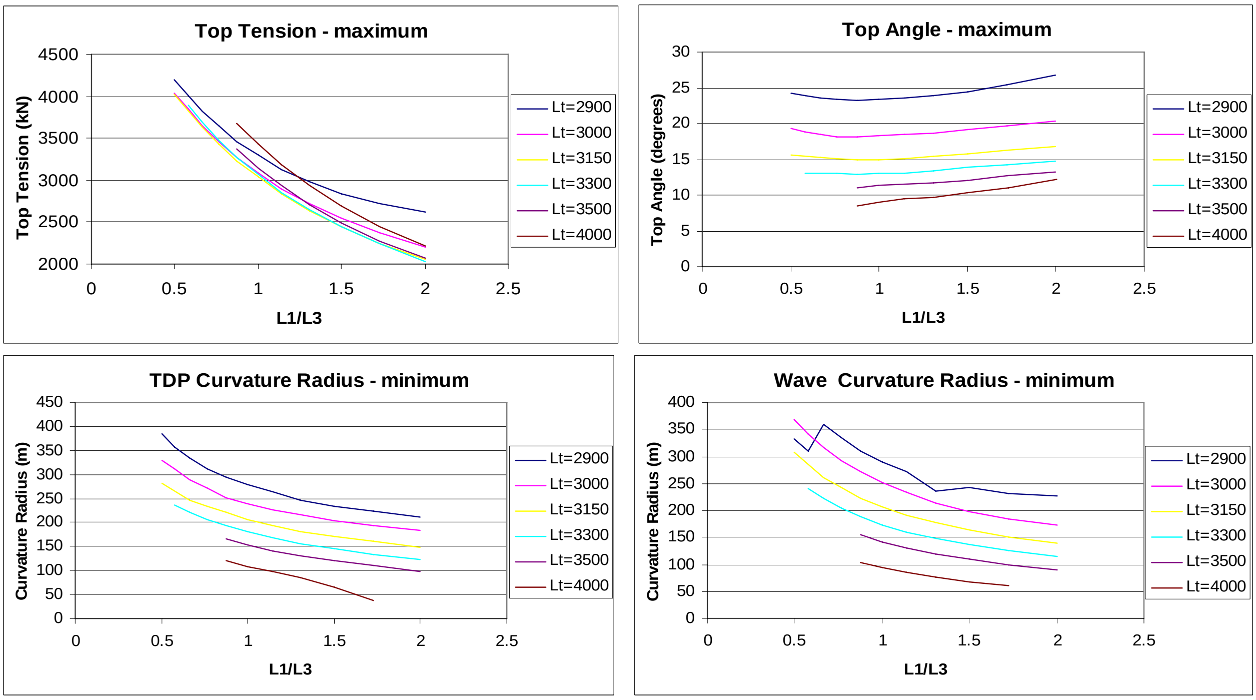

Figure 3. Results for variable cable length (L2/Lt =0.2)

Figure 3 shows that cable size has little influence on top tension, but a smaller cable brings bigger top angles and TDP and wave curvature radii. The distance from the floater to the TDP influenced the results in the same way it did in the previous test, so the same conclusions can be drawn.

Figure 4 presents the results of a series of configurations tested in 192 dynamic cases. 64 of these cases were created combining centenary currents and decenary waves. Other 64 cases were created combining decenary currents and centenary waves. The final 64 cases were created combining centenary currents and centenary waves. The results were plotted against total riser length and show little difference on the envelops of the 128 first cases when compared to the ones of the final 64. This suggests that an adequate dynamic analysis could be made using only combinations of centenary waves and currents, simulating only half the cases and with little difference on the results.

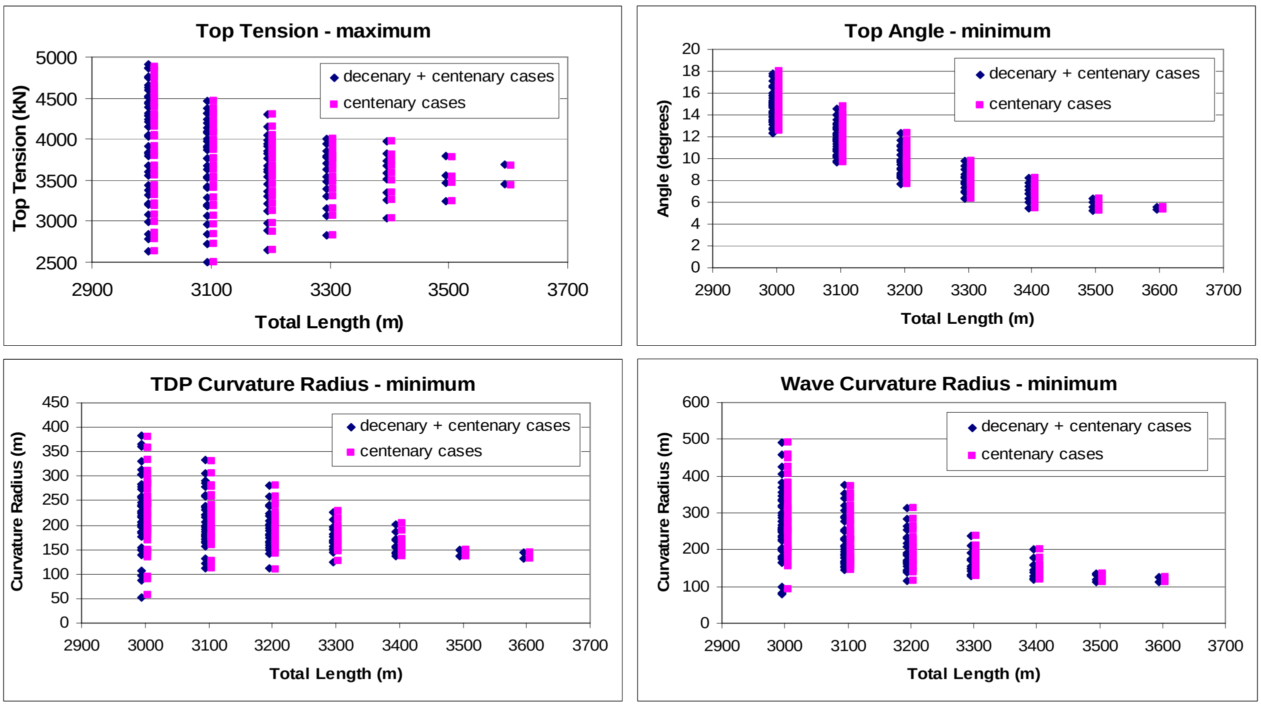

Figure 4. Results for load case comparison

5. Conclusions

A large parametric analysis was performed, where approximately 2000 configurations where simulated in 192 different load cases, in a total sum of 384.000 dynamic simulations. The computers used for these simulations were 2 Pentium IV 2.4 GHz, with 1 GB RAM, and performed a (rough) mean of 1200 simulations each hour, thus giving a total of 320 computational hours. This is quite fast if compared to most commercially available tools.

This analysis' output was used to study the effect of the parameters on top tension, top angle and TDP and wave curvature radius. The effects of the change of each parameter on each result were discussed, and the following conclusions can be drawn, based on the presented graphics:

- A smaller cable has bigger TDP and wave curvature radii, but has also bigger angles at the top.
- The bigger the floater, the smaller top tension and the bigger curvature radii at the TDP and wave it has. Top tension decreases with floater size, so a bigger floater is better for all results.
- The closer to the TDP the floater is, the bigger the TDP and wave curvature radii. However, top tension also increases, so care must be taken. Top angle shows little variation with this parameter.

Finally, a riser was simulated subject to two different groups of load cases. The first group combined centenary currents and decenary waves and centenary waves and decenary currents, while the second group combined centenary currents and centenary waves. Even though the second group only required half the simulations the first group did, the envelop of results for both groups were very close. This suggests that the dynamic analysis of a riser in the initial design phases can be performed much faster, with little difference in the obtained results. As only a single riser has been studied, this conclusion must be taken carefully, being this a topic for further research.

6. Acknowledgements

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7. References

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