Portfolio Optimization of Oil Production Projects Using Mathematical Programming and Utility Theory

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Abstract. The correct valuation of E&P projects and composition of the portfolio is a fundamental step towards investment's profit, especially if the corporation has a number of potential projects whose outlays overcome its capital availability. This paper presents a study of portfolio optimization of a set of 25 petroleum production projects located in the offshore sedimentary Brazilian basins. The methodology is based on mathematical programming (using genetic algorithm), where the objective function is the maximization of NPV, together with: i) corporation's strategic and operational goals (production profile, reserve addition or losses, etc); ii) capital constraints (investments, operational costs, etc); iii) requirements of revenue. In addition, risk is incorporated into this model by taking account the standard deviation of NPV after a Monte Carlo simulation of main economic, technical and financial variables of each project's cash flow. Although, this optimization model based on return-risk is an important tool, it does not take into account explicitly the risk attitudes of the corporate decision makers. In order to deal with these limitations, a model using the utility theory for risk attitudes of decision makers is included in the portfolio analysis. The results indicate that the application of preference theory models is an important tool for choosing the optimal portfolio, especially in case of a large number of viable portfolios with different return and risk values.

Keywords: Risk, Monte Carlo simulation, utility function, portfolio optimization and portfolio selection.

1. Introduction

Worldwide oil reserves account to 1.27 trillion bbl and thousands of projects are in the way to extract this oil (O&G Journal, 2004). Since the middle the 1980's, the majority of the petroleum E&P international companies recognized that the average size of new world discoveries was diminishing, together with the increase of risk, in part due to oil price volatility. As a result, major companies begun to analyze with more rigor some of their prospects and concluded that less than 1% really proved to have a potential profitability (Rose, 2002).

On the other hand, over the last two decades, the technological advances in the petroleum E&P have been expressive (Bohi,1999)[1]. One of the main results of these recent technological advances is the increase of the number of potential projects to compose E&P portfolios, starting to compete with each other in the company limited E&P budgets. Thus, the integrated tasks of project valuation, selection and portfolio construction and optimization have gained increasing importance in the petroleum business.

Traditionally, portfolio optimization has been based on two distinct (but, in some cases, equivalent) criteria, as discussed by Luenberger (1998) and Campbell et al. (2001). The first one uses a higher discount rate (hurdle rate) in order to select only those projects presenting better cash flows. The second criteria rank projects according to NPV up to the point that the entire budget is consumed. For valuation and decision-making in the petroleum industry, these two proposals have failed by ignoring important risk implications. A portfolio with high NPV and high risk may be rejected in favor of other with lower NPV, but also much less risky. In advance, it is understood that some determinants of optimal return/risk profile depend on the magnitude of budget and utility function of decision-maker. Details will be given over the next sections.

[1] Bohi (1999) argues that discovery costs had been reduced in more than 30%, while new techniques of seismic interpretation had increased the petroleum success rates in the exploration and the development phases.

Nowadays, in the petroleum industry, especially in case of majors companies, there are a large number of projects concurring with scarce resources, in a way of compelling the decision makers to place the capital in projects with minor risks levels and greater returns. These new considerations, give rise to concerns of corporation towards implementing a more realistic portfolio optimization. In this context, according to Denney (2003), portfolio optimization is a whole process that allows decision makers to create strategies - like maximization and/or minimization of the objective function based on NPV, IRR, etc -, respecting corporative goals (production, growth, etc) and constraints (capital, time, labor, etc). In petroleum projects, these corporative goals and constraints can be technical, geological, financial, and economic.

The objective of this paper is the study of portfolio optimization of oil production projects and it is organized in three sections. Section 2 presents the proposed methodology to valuation, selection and portfolio optimization. Section 3 shows a case study considering a portfolio of 25 oil production projects. Section 4 discusses the main implications and conclusions.

2. Methodology for portfolio optimization of oil projects

Sharpe (1970) defines portfolio optimization as a mixture of art and science, since it involves three steps: a) asset valuation; b) asset selection; c) investment allocation in most appropriated manner among those chosen assets. In specific case of capital budgeting, Denney (2003) considers that portfolio optimization is divided into four phases: asset valuation, choice of objective functions, insertion of corporation goals and restrictions and, finally, analysis of viable portfolios.

- ***Project evaluation*** – In this phase the analyst has to evaluate each project, using discounted cash flow, contingent claim approach or market proxies. The main indicator is the expected value and standard deviation of the random variable NPV.
- ***Definition of the objective function*** – This incorporates what is most important for corporation, such financial results (NPV, IRR, growth rate, etc), operational performance (production, reserve addition, remaining reserve, etc), and technological advances, among others. In this paper, the objective function is maximization of NPV.
- ***Corporative restrictions*** – They refer to those financial and technical limitations of the corporation, such as yearly capital spending, production to meet contractual obligations, etc.
- ***Analysis of viable portfolios*** – After defining the objective function and construction of goals and restrictions, the generation of viable portfolios is a pure mathematic programming problem. Main input data are expected value, such as the variance of the chosen measure of value, usually NPV. In addition, there must be taken into account the interdependencies among projects.

In this context, the methodology proposed for optimization of portfolio of projects is based on a combination of distinct tools ranging from pure mathematical up to pure economic ones. In addition, the risk treatment is based on an extended mean-variance portfolio, complemented by indifference return/risk curves according to the utility functions of the decision-maker.

2.1. Risk accounting I – Extended Mean-Variance Portfolio Theory

Rational investors will choose those projects with higher NPV, since they prefer more to less richness. However, usually projects with higher NPV present also higher risk. Then, the risk-averse investor faces a continuous tradeoff of how to select projects so that expected NPV and risk of portfolio are appropriate to his expectations and tolerances. In this context, Markowitz (1959) developed the mean-variance analysis, an useful tool linking return and risk of portfolios in a way suitable for financial decision-making. Sharpe (1970) shows that the expected NPV and risk of a portfolio composed of N projects are:

$$E[NPV_p] = \sum_{i=1}^{n} NPV_i . x_i \, , \tag{1}$$

$$\sigma[NPV_p] = \sqrt{\sum_{i=1}^{n} x_i^2 . \sigma_i^2 + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq 1}}^{n} x_i . x_j . \sigma_{\mathrm{i}} . \sigma_{j.} . \rho_{i,j}} \tag{2}$$

where $E[NPV_p]$ and $\sigma[NPV_p]$ are the expected NPV and risk of portfolio, NPV_i is the NPV of each project, $\sigma_{I,j}$ is the project's risk and x_i refers to the fraction of the corporation's budgetary investment allocated in each paired project, and $\rho_{i,j}$ is the correlation coefficient between each pairs of projects. Note that equations (1) and (2) allow the estimation of two first statistical moments of the random variable NPV of portfolio.

By choosing projects merely according to equations (1) and (2), it is implicit assumed that the budget is infinite, so that a more realistic model has to be bounded by the following constraint:

$$\sum_{i=1}^{n} x_i = 1 \text{ or I} \tag{3}$$

Equation (3) says that the total available outlay is I, a finite amount, which is allocated to finance and produce N projects. Note that in equation (3), the analyst may use percentage or absolute values, so that the total corporation's investment (I) corresponds to 100%.

The use of Markowitz model in capital budgeting involve limitations about how to estimate risk (variance, lower probability, kurtosis, etc), especially in case that NPV has a probabilistic distribution other than Gaussian. However, in practice, it is easier to estimate the variance of projects, so that it has been used in the optimization of portfolios in the oil and gas industry (Walls, 2004). In addition, it is worth to mention that standard deviation is a measure of uncertainty (dispersion around mean value). Differently, risk refers only to that portion of uncertainty in the downside part of statistical distribution. As a result, standard deviation measure does not differentiate between "downside" and "upside" uncertainty, what is an important reference in capital budgeting.

The advance of mean-variance model consists in considering the impact of risk of each project in the portfolio. From equation (2), the portfolio risk depends on two terms: i) contributions of projects in additive way, which is called systematic risk[2], since it depends solely on project's characteristics; and ii) contribution due to interrelations among projects, which is called non-systematic or diversifiable risk, since it can be reduced by choosing non-positively correlated assets. In practice, we can say that the unsystematic risks depends solely on the business or industry, whereas the systematic risk depends on the behavior of the whole economy - inflation, purchasing power, interest rate, unemployment, etc.

In the process of portfolio optimization, the analyst has two main degrees of freedom. First is the choice of potential projects through estimation of NPV and its risk. The second consists in the choice of how much of budget is allocated to each project. These two degrees of freedom are sufficient to generate an infinite number of viable portfolios. The traditional optimization method is quadratic optimization, an especial case of mathematical programming, which is described in Markowitz (1959), Sharpe (1978) and Luenberger (1998). Meanwhile, if the number of projects is large and corporate goals and constraints are added to the model, this technique becomes complex. In the petroleum industry, the main constraints and goals are:

$$\sum_{i=1}^{n} F(t)_i . x_i \geq F(t)^* \tag{4}$$

$$\sum_{i=1}^{n} Q(t)_i . x_i \geq Q(t)^* \tag{5}$$

$$\sum_{i=1}^{n} OPEX\,(t)_i . x_i \leq OPEX\,(t)^* \tag{6}$$

Equation (4) says that the corporation wishes that the yearly minimum portfolio cash flow is F(t)*. Equation (5) says that the corporation has production targets for next years, so that the oil production of these projects must be at least equal to Q(t)*. Equation (6) says that the corporation has concerns about cash out and will choose projects so that the yearly OPEX is bounded by OPEX (t)*.

Methods to solve this problem are complex. Lessard (2003) argue that a medium size petroleum company with a modest set of 30 potential projects will have a billion of possible portfolios combinations, what confirms the limitations of quadratic programming in solving problems of corporate finance. Alternatively, in this paper, the optimization technique is done using genetic algorithms (GA), which was pioneered described in Holland (1975) in his work on adaptative systems and artificial systems. According to Goldberg (1989), GA is a numerical procedure able to carry out optimization of nonlinear and discontinuous problems and different kind of restrictions, whose result aims at being close to the optimal solution. Due to its robustness, Lazo (2000) has applied GA in optimization problems of portfolio of common stocks, which has some similarities with portfolio of oil and gas projects.

This extended mean-variance model for portfolio optimization is an advance compared to the traditional project ranking methods or optimization based solely on expected NPV, but it just shows viable portfolios under the general assumption of existence of a return/risk tradeoff. In order to find the optimal portfolio, additional premises and tools about the preferences of decision-makers are necessary, and the next section goes along this route.

2.3. Risk accounting II – Complementing results of mean-variance through indifference risk/return curves

Classically, the selection and ranking of risky projects is based solely on expected monetary value (EMV)[3] until the budget is consumed. This practice has been criticized, among others, in Walls and Dyer (1996) who complains about: i) the proper way of accounting the project value in the time, ii) risk inconsistencies by caring only about project risk and ignoring the attitudes of decision-maker and iii) the project valuations with different durations.

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[2] The mean-variance theory is the basis of CAPM, since this pricing model states that only systematic risk must be rewarded. See Sharpe (1978), Merton (1973), Ross, Westerfield and Jaffe (1995) for more details on CAPM.

[3] Indeed, EMV is equal to NPV, since the later is found from expected value of price, production, cost, etc.

Alternatively, utility functions developed by Von Neumann and Morgenstern (1953) was applying by Newendorp (1975) and Cozzolino (1977) in a proposed model of preference theory (PT) for exploration and production oil projects to describe investor's preference under risk in a more realistic way. In theory, the functional expression of the preference theory can assumed different forms[4], but, in practice, it is defined empirically. For example, the exponential utility function is widely used in the decision-making of petroleum exploration (Hammond III, 1974; Cozollino, 1977; Walls, 1995 and Nepomuceno and Suslick, 2000). Meanwhile, in certain cases the companies need to define a priori their best utility functions.

The utility function more used in the petroleum industry is an exponential function:

$$U(x) = -e^{cx} \quad , \tag{7}$$

where U(x) is the utility function; c is the risk aversion coefficient and x is the uncertain monetary value or project NPV.

Equation (7) says that U(x) is a dimensionless quantity suitable for making comparisons and not for valuations of assets, as stated by Lima (2004). In addition, PT does not indicate the level of risk aversion that a company would have to use in its decision-making, since this depends on a number of factors such as budget size, psychological profile of decision-maker, history of corporation, goals of short, mid and long-term, etc. In order to use the PT, Walls and Dyer (1996) investigated the risk-aversion coefficient of petroleum exploration industry. They analyzed 50 petroleum companies in USA market in the period of 1981 the 1990 and found that its value is the inverse of risk tolerance (T), which is close to 25% of the yearly budget of firms. Then, we have: c = ¼ T.

For a risk-averse investor, an increase in risk must be followed by increase in return. Then, it is possible to get what is called indifference curves to study not only a single, rather a set of projects. Nepomuceno (1997) applied this methodology for decision-making in the petroleum exploration business. The general equation of indifference curves for a risk-averse individual is:

$$\mu = -TR^{*}\ln(e^{c^{*}EqC})^{*}(p_1 {}^{*} e^{(-c^{*}\sigma_1{}^{*}NPV_1)}) + (p_{2*} e^{(-c^{*}\sigma_2{}^{*}NPV_2)}) + \ldots + p_n(e^{-c^{*}\sigma_n{}^{*}NPV_n)})) \tag{8}$$

where μ is the minimum NPV of portfolio, T is the corporation risk tolerance, c is risk-aversion coefficient, CE is certainty-equivalent; σ_i is portfolio risk, and NPV_i is net present value of portfolio.

From Equation (8), note that the indifference curves is the geometric place where the decision-maker has the same level of satisfaction by choosing different portfolios with distinct levels of risk and return. In addition, as risk increases, return also increases and the rate of increase need not be the same for these two curves. An especial case is when there is no risk ($\sigma = 0$) and these curves begin with a return whose values are equal to the CE of each decision maker.

Since decision-makers are rational, those portfolios located below the indifference curve will not be selected. Although their NPV are higher than decision maker' CE, their risks are too high. These models seems to be an improvement over the traditional results of mean-variance model since optimal portfolio will be found by considering risk characteristics of both projects (expected NPV and its standard-deviation) and the corporation (via utility functions).

3 – Results and discussion of portfolio optimization of 25 potential oil production projects

This model is applied to solve a problem of Gama Petroleum, a hypothetical oil and gas company. This corporation has a set of 25 oil production projects, but cannot develop all these projects simultaneously and has some requirements in terms of production, cash flows and limitations of yearly cash out flow. Most of these projects come from a database of Laboratory of Geoeconomic Analyses of Mineral Resources (LAGE) of the Center of Petroleum Studies from State University of Campinas (UNICAMP). They are evaluated considering oil production, OPEX, price, and the Brazilian fiscal regime. In Table 1, the main characteristics of these projects are presented such as production, revenue, OPEX and NPV.

The oil volume characteristics (deep waters and a low ºAPI -heavy oil) is estimated from seismic data interpretation and, later, confirmed for pioneering wells and formation tests. The reserves have different sizes, ranging from 270 million barrels up to 1.4 billion barrels and decline rate is around 10% for all the projects. Each project has an operational life of 12 to 20 years, what it is typical for the petroleum E&P industry in this region.

The optimal choice of projects to compose the portfolio is done taking into account the following corporate strategy of return:

$$\text{Maximize } \sum_{i=1}^{N} NPV_i \tag{9}$$

In addition, there are two concerns related to CAPEX and OPEX:

[4] The only prerequisite is that utility functions must be increasing, so that individuals prefer more wealth to less.

$$\sum_{i=1}^{N} OPEX_i \leq 0{,}7 * OPEX * \tag{10}$$

$$\sum_{i=1}^{N} CAPEX_i \leq 0{,}7 * CAPEX * \tag{11}$$

Equation (9) says that the corporation intends to maximize the generation of wealth through getting the highest NPV that is possible. Equations (10) implies in a limitation so that the selected projects must not consume more than 70% of the total cash that flows out (OPEX). Equation (11) says that selected projects may consume no more than 70% of the corporation's CAPEX.

After running 10,000 iterations, results are displayed in Table 2. Note that, there is no significant correlation between risk and certainty-equivalent of all projects. These results are found by employing a deterministic base, what is unrealistic for petroleum industry. On the other hand, a more real-world analysis must take the risk of NPV into account. This process is generated by attributing probabilistic distribution to model uncertain variables such production, spot price, and OPEX. Normal and lognormal statistical distributions had been attributed to the four mentioned variables (stochastic variables).

Now, we can consider return and financial restrictions in order to generate portfolios. By pursuing this strategy, around 20 viable[5] portfolios are generated and the twenty ones with higher NPV are selected and included into the risk analysis, the next step of the portfolio optimization, what is based on mean-variance theory. Table 4 shows these 20 selected portfolios. Note that portfolios number 15, 16 and 17 are the main component of these portfolios.

The values of NPV and risk of each project are used as input into the mean-variance theory according to equations (1), (2) and (3). In case of pure financial assets, optimization of portfolios is obtained by considering that investor will choose the optimal allocation level of resource in each asset, whereas in the case of projects it is assumed that this level (working interest) is previously chosen.

In order to use equation (2), the correlation among projects is need. In this paper, for the sake of simplicity, it is assumed that correlation between each pair of projects (ρ) is 0.7. The main reason is that all projects are dependent on the same oil price, from which the NPV is directly contingent.

The next step consists in selecting the optimal portfolio among those viable ones. The corporation has a budget of USS120 million, whereas to develop all projects an investment capital of USS 129.59 million is required. By using equations (9), (10) and (11) and an optimization procedure based upon genetic algorithm, a set of generated viable portfolios is shown in Table 3. Note that the risk of each portfolio is not absolute values, but in percentages since they were standardized in relation to the largest risk's portfolio.

Next step deals with the indifference curve analysis. In order to make clear the methodology, two decision-makers with different CE are analyzed - decision-maker A has a CE of 70.16 MM and decision-maker B has a CE of 163.72 MM.

Figure 1 displays the results of viable portfolios of this set of 25 oil production projects together with indifferences curves.

[5] Viable portfolios are combinations that respect the constraints, which not always they have simultaneously the highest return.

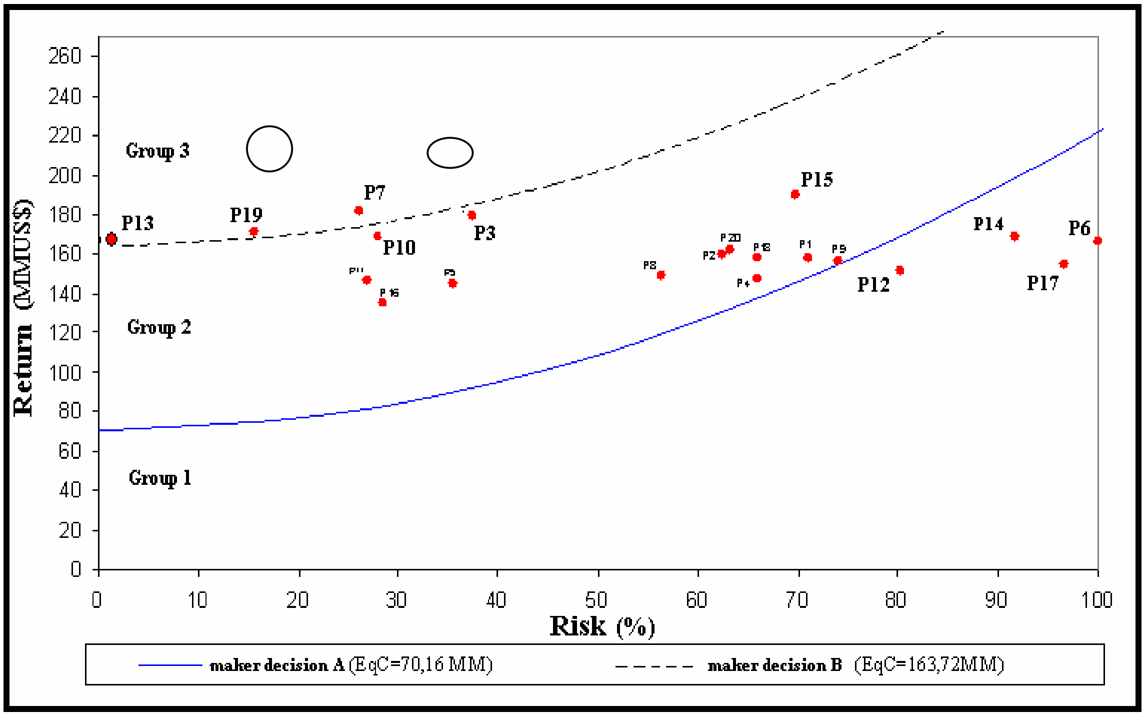

Figure 1: Tradeoff risk/return coupled with indifference curves for a portfolio of 25 oil projects.

For rational decision-makers, a portfolio with less return than the certainty equivalent should never be selected, but only those ones that presents higher return. Therefore, the choice will depend on decision-maker's risk-attitude. For this case study, there are three distinct groups of portfolios with different risk and return levels.

Group 1: This group contains viable portfolios, but rejected by both decision-makers A and B, since they are located bellow their indifference curve. Note that if the CE is reduced, portfolios P12, P14, P17 and P6 may be selected. Meanwhile, for the actual CE their risk is too high for their offsetting return. In addition, of all viable portfolios, only four are immediately rejected.

The message is clear. The decision-maker A prefers a CE of USS 70.16 MM instead of the possibility of gaining more by choosing one of these portfolios with greater NPV, but also more risky. Consequently, in this proposed model, the portfolio selection depends not only on returns/risk ratio, but also on the decision-maker's profile upfront risk.

Group 2: This group has portfolios possible for decision-maker A, but insufficient for decision-maker B. Note that the risk of its portfolios ranges from 26.87% 74.02%, what covers about 65% of the company total number of viable portfolios. Any of these portfolios may be selected by decision-maker A, but he will look for those ones with higher return and risk compatible with the indifference function. For example, portfolios P2 and P20 have different return and risk, but promote the same level of satisfaction for the decision maker, since they are on the same indifference curve. The same occurs with portfolios P3 and P10, which are over an indifference curve.

Then, the indifference curve of decision serves to tell the analyst that portfolios above it are suitable to be selected, whereas the other ones must be rejected.

Group 3: In this group are located those portfolios selected by maker decision B, since their have NPV higher than its CE. Portfolios P13, P19 and P7 have higher levels of returns and low risks and represent 15% of the company's total number of portfolios. Note that if the CE is US$ 163 MM, the satisfaction generated by portfolios P13, P19 and P7 are the same.

In this case, the choice of portfolios is strongly depend on utility function, risk tolerance and CE. The main remark of these results is the portfolio with the biggest NPV is not always the optimal for the corporation, since additional considerations are needed, especially project risk level and corporation utility functions. In addition, if project risk is too low and decision-maker is risk-averse, then the results of the traditional and this proposed model will be convergent.

It is worth to mention that Figure 1 only shows different relationships among risk and returns, together with indifference curves. It is not intended to point out which portfolio is the best, but only to be used as a tool for choosing different combinations of risk and return.

4. Conclusions

The specific observations presented in this paper cannot be generalized to any portfolio optimization and selection problem, but the underline process to include the corporate risk-aversion can be a potential path for future algorithm for portfolio selection. The inclusion of risk preferences can significantly affect the portfolio selection. Furthermore by examining new alternatives for inclusion of risk preferences, important insights can be gained, allowing for a more effective portfolio management. This procedure is very useful for highly correlated portfolios and certain restrictions that posses an indirect impact in the portfolio valuation.

The optimal portfolio according to the uncertain approach does not coincide with the best one indicated from the deterministic approach, what is not a surprise, since the uncertain approach takes into account project risk and risk-attitude of decision-maker through utility functions.

The results of case study comprehending 25 oil projects indicate that the optimal portfolio selected according to the two approaches are different. The stochastic process presents significant differences among portfolio selection. Another aspect is the inclusion of risk preference attitudes in the selection process.

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Table 1 – Main cash flow economic and technical characteristics of all projects.

Projects	Total oil production (MMbbl)	Revenue (MMUS$)	CAPEX (MMUS$)	OPEX (MMUS$)	Net present value (MUS$)
Proj 1	695.210	13,904.21	1,669.30	1,873.43	46,278.96
Proj 2	999.400	19,988.00	1,466.86	1,437.80	346,050.80
Proj 3	748.127	14,962.40	1,157.37	1,825.75	28,562.40
Proj 4	214.750	4,295.00	1,087.52	445.73	140,172.27
Proj 5	643.200	12,864.00	580.00	1,551.84	339,119.55
Proj 6	499.706	9,994.00	1,020.93	1,141.91	48,354.53
Proj 7	389.305	7,786.00	1,022.00	1,254.46	130,332.73
Proj 8	305.300	6,106.00	1,263.00	849.00	22,314.76
Proj 9	410.790	8,215.80	660.94	1,174.80	535,741.33
Proj 10	703.391	14,067.80	1,790.83	1,688.00	243,595.91
Proj 11	193.272	3,865.50	992.42	401.15	126,638.26
Proj 12	268.444	5,368.72	1,083.44	445.73	233,245.61
Proj 13	335.833	6,716.60	1,226.00	849.00	85,887.38
Proj 14	350.375	7,007.40	914.58	1,129.01	155,555.96
Proj 15	287.555	5,751.06	454.96	822.36	459,233.76
Proj 16	449.731	8,994.60	910.62	1,027.71	93,419.94
Proj 17	514.560	10,291.20	518.67	1,396.65	254,179.93
Proj 18	625.690	12,513.78	1,560.45	1,686.08	84,482.13
Proj 19	773.730	15,474.58	1,980.91	1,856.80	187,566.08
Proj 20	897.740	17,954.88	1,557.82	2,288.42	19,360.56
Proj 21	832.834	16,656.57	1,524.64	1,437.80	186,017.33
Proj 22	1,199.20	23,985.60	1,686.77	1,725.36	41,867.82
Proj 23	1,399.16	27,983.20	2,002.00	2,012.92	474,417.09
Proj 24	214.750	4,295.00	1,050.35	490.30	145,824.82
Proj 25	410.790	8,215.80	855.10	903.69	497,372.86

Table 2 - Working interest (WI), portfolio composition, risk and return of all projects

Projects	WI (%)	Composition (%)	Risky capital (MMUS$)	Return (MUS$)	Risk (MUS$)
Proj 1	100	5.40	-7.00	43.39	83.87
Proj 2	33	6.89	-8.93	314.75	164.30
Proj 3	33	11.57	-15.00	24.26	113.24
Proj 4	33	12.35	-16.00	121.38	44.60
Proj 5	100	5.17	-6.70	297.18	90.69
Proj 6	100	2.16	-2.80	45.05	120.88
Proj 7	50	1.50	-1.95	121.67	101.37
Proj 8	50	4.24	-5.50	20.88	58.30
Proj 9	50	5.65	-7.32	468.68	95.21
Proj 10	40	1.66	-2.15	214.35	149.30
Proj 11	50	1.88	-2.44	108.90	40.22
Proj 12	33	3.90	-5.05	207.23	56.35
Proj 13	25	3.09	-4.01	78.97	65.29
Proj 14	33	3.77	-4.88	136.96	91.15
Proj 15	33	4.33	-5.61	405.59	68.50
Proj 16	32	1.68	-2.18	81.62	103.88
Proj 17	33	3.12	-4.04	218.47	72.56
Proj 18	40	2.79	-3.62	79.45	78.18
Proj 19	40	1.67	-2.17	165.51	163.15
Proj 20	50	4.68	-6.07	19.55	139.33
Proj 21	25	1.92	-2.49	168.36	134.27
Proj 22	12	2.71	-3.51	371.02	195.50
Proj 23	100	2.66	-3.45	421.41	227.92
Proj 24	25	2.62	-3.39	146.97	44.73
Proj 25	33	2.57	-3.33	436.81	94.09
Total		100	-129.59	4,718.40	2,596.88

Table 3: Portfolio risks and return

Portfolios	Actual participation		100 % of participation	
	Return (MMUS$)	Risk (%)	Return (MMUS$)	Risk (%)
1	157.76	71.03	218.12	42.97
2	159.84	62.36	219.58	39.38
3	178.78	38.36	254.31	89.49
4	147.15	65.84	202.26	46.75
5	144.85	35.50	210.60	50.60
6	165.87	100.00	223.38	50.51
7	181.56	26.12	257.50	87.41
8	149.19	56.38	203.24	43.14
9	155.87	74.02	222.56	4.79
10	168.19	28.09	237.18	96.82
11	146.48	26.87	211.78	48.40
12	151.42	80.22	212.31	50.38
13	161.39	1.00	243.88	95.95
14	168.25	91.70	225.07	48.07
15	189.98	70.75	262.00	100.00
16	134.84	28.52	195.00	55.52
17	155.00	97.65	207.25	54.72
18	157.79	65.97	224.08	1.00
19	170.98	15.56	239.83	94.99
20	162.37	63.19	223.43	46.39

Table 4 – Different optimum portfolios generated by genetic algorithm (without considering risk).

Portfolios	**Number of projects of portfolio**	**NPV (MMUS$)**	**Feasibility (%)**
Port 1	17	3,841,32	100
Port 2	17	3,817.73	100
Port 3	16	3,815.00	100
Port 4	17	3,804.71	100
Port 5	17	3,800.57	100
Port 6	16	3,791.91	100
Port 7	16	3,791.41	100
Port 8	17	3,781.12	100
Port 9	17	3,778.48	100
Port 10	16	3,778.39	100
Port 11	17	3,776.98	100
Port 12	17	3,775.30	100
Port 13	16	3,774.25	100
Port 14	16	3,768.32	100
Port 15	15	3,765.59	100
Port 16	17	3,763.95	100
Port 17	16	3,755.29	100
Port 18	17	3,754.89	100
Port 19	16	3,754.80	100
Port 20	16	3,754.14	100