A NUMERICAL STUDY OF NATURAL CONVECTION IN AN INTERNAL FINNED ANULUS WITH CONDUCTION IN THE HORIZONTAL CYLINDERS AND FINS

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Abstract. *It is studied the natural convection of fluid confined between two concentric cylinders with conduction in their walls and in the fins placed on the internal cylinder. The finite element method was used to solve the conservation equations in terms of stream function ψ, dimensionless temperature θ, and vorticity ω. The flow is considered to be two dimensional, laminar, incompressible, and unsteady. The fluid is air with Prandtl number equal to 0.7. The stream function, dimensionless temperature and vorticity distributions are obtained in the whole domain. The Nusselt number is computed on the internal surface of the outer cylinder as well as on the fin and inner cylinder surfaces by ranging the diffusivity ratio (0.1, 1.0, 10.0) and the Grashof number ($2x10^4 \leq Gr \leq 10^6$).*

Keywords: *natural convection, finite element method, finned cylinders*

1. Introduction

Natural convection in horizontal cylindrical annuli has been studied numerically and experimentally due to its importance in engineering. Some of its most important applications are: electric cable isolation, solar collectors, airplane isolation, nuclear reactors, and energy thermal storage.

A thorough literature review was carried out in 1976 by Kuhen and Goldstein (1976, 1978). A numerical and experimental on laminar natural convection inside horizontal concentric cylinders was performed by Kuhen and Goldstein (1976). In the experimental study, the laminar-turbulent transitional state took place when the Rayleigh number was 10^6.

An excellent review on laminar and turbulent natural convection in horizontal annuli was done by Miki et al. (1993). They studied the turbulent natural convection in horizontal annuli using large eddy simulation. The numerical solution using the finite difference method was obtained for Rayleigh number up to 1.18×10^9. It was verified that the average Nusselt number had a reasonable agreement with the experimental results from literature.

Lai. (1993) conducted a study on laminar natural convection in concentric cylinders having a porous medium and fins placed on their inner surfaces. A numerical analysis was carried out considering the Rayleigh number ranging from 50 to 500. The effect of the heat transfer reduction due to the presence of fins with low conductivity was verified. The geometry with the best performance was the one with partial fins placed on the inner cylinder.

Chai and Patankar (1993) studied the heat and mass transfer by laminar natural convection in annuli with six radial fins placed on the inner cylinder. Different configurations were considered. This kind of study was also performed by Rahnama et all (1999).

The study of Farinas. et al. (1997) investigated numerically the effect of inner fins on the velocity and temperature fields in horizontal annuli considering laminar natural convection air flow with Rayleigh numbers ranging from 10^3 to 10^6. It was also verified the effect of the shape of the fin tops and three fin lengths.

More recently, Rahnama and Fahadi (2003) numerically studied the turbulent natural convection between two concentric horizontal tubes with radial fins placed on the inner tube. The number of fins and the Rayleigh numbers varied from 2 to 12 and from 10^6 a 10^9, respectively. It was observed that the local Nusselt numbers increased as the Rayleigh numbers became higher. Keeping the Rayleigh number constant, the heat transfer rate decreased in all cases studied when comparing to the cases without fins.

It is observed through a literature review that more works on the study of laminar natural convection in horizontal concentric tubes with internal fins on the inner tube are needed being that almost all of them consider thin fins without the effect of thermal diffusivity of the tube and fin walls.

The present work performs a study on laminar natural convection in horizontal concentric tubes with six fins placed inside the inner tube. All walls have considerable thicknesses. Figure 1 shows the geometry studied and Fig. 2, the finite element mesh. One should note that due to symmetry in relation to the vertical y-direction, just half domain is considered in the numerical analysis using the finite element method.

2. Dimensionless parameters

In order to have a generalization of the analysis, the following dimensionless parameters are defined:

$$\tau = \frac{\nu t}{H^2}\ ,\ X = \frac{x}{H}\ ,\ Y = \frac{y}{H}\ ,\ U = \frac{uH}{\nu}\ ,\ V = \frac{vH}{\nu}\ ,\ P = \frac{pH^2}{\rho \nu^2}\ ,\ \theta = \frac{T - T_0}{T_h - T_0}\ ,\ D = \frac{\alpha_s}{\alpha_f}\ , \tag{1}$$

where ν is the kinematics viscosity [m^2/s], $H = 2 \cdot R5$ is the outer diameter [m], x and y are the coordinates [m], ρ is the density [kg/m^3], T_0 is a reference temperature [K], u and v are the velocities [m/s], t is the time [s], p is the pressure [Pa], T is the temperature, T_h is the temperature on the hot surface [K], α_s and α_f are the solid and liquid thermal diffusivities [m^2/s], respectively.

3. Governing equations

The natural convection is based on the conservation of mass, momentum, and energy. The following hypotheses are considered: i) unsteady regime; ii) laminar and two-dimensional flow; iii) incompressible flow; iv) viscous dissipation function neglected; v) the physical fluid properties $\left(\rho, \mu, c_p, K\right)$ are constant, except the density in the buoyancy terms; vi) no internal heat generation.

By applying the considerations described before and the dimensionless parameters, the conservation equations in terms of streamfunction (ψ), temperature (θ) and vorticity (ω), are:

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = -\omega\ , \tag{2}$$

$$\frac{D}{Pr}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) + \left(\frac{\partial \psi}{\partial X}\frac{\partial \theta}{\partial Y} - \frac{\partial \psi}{\partial Y}\frac{\partial \theta}{\partial X}\right) = \frac{\partial \theta}{\partial \tau}\ , \tag{3}$$

$$\frac{\partial^2 \omega}{\partial X^2} + \frac{\partial^2 \omega}{\partial Y^2} + \left(\frac{\partial \psi}{\partial X}\frac{\partial \omega}{\partial Y} - \frac{\partial \psi}{\partial Y}\frac{\partial \omega}{\partial X}\right) + \frac{Gr}{2}\frac{\partial \theta}{\partial X} = \frac{\partial \omega}{\partial \tau}\ , \tag{4}$$

where the streamfunction (ψ), temperature (θ) and vorticity (ω) are defined as follows:

$$\frac{\partial \psi}{\partial Y} = U\ ,\ -\frac{\partial \psi}{\partial X} = V\ \text{ and }\ \omega = \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y}\ , \tag{5}$$

The dimensionless numbers Prandtl (Pr) and Grashof (Gr) are given by:

$$Pr = \frac{\nu}{\alpha_f}\ ,\ Gr = \frac{g\beta\left(T_h - T_c\right)H^3}{\nu^2}\ , \tag{6}$$

where g is the gravity acceleration (m/s^2) and β is the volumetric expansion coefficient (K^{-1}).

In equation 3, the following is considered:

$$\psi = 0\,,\; D = \alpha_s / \alpha_f \quad (\text{on } \Omega_s), \tag{7a}$$

$$D = 1 \quad (\text{on } \Omega_f). \tag{7b}$$

The average Nusselt numbers on the cold (Nu_c) and hot (Nu_h) walls are respectively given by:

$$Nu_c = \frac{1}{S}\int_S \sqrt{Nu_x^2 + Nu_y^2}\, dS \quad \text{and} \quad Nu_h = \frac{1}{S}\int_S \sqrt{Nu_x^2 + Nu_y^2}\, dS\,, \tag{8}$$

where the local Nusselt numbers are computed using the following expressions:

$$Nu_x = \frac{1}{2}\left(\frac{\partial \theta}{\partial X}\right)\bigg|_S \quad \text{and} \quad Nu_y = \frac{1}{2}\left(\frac{\partial \theta}{\partial Y}\right)\bigg|_S\,. \tag{9}$$

The geometrical parameters shown in figure 1 have the following values:

$$H = 2 \cdot R5 = 1\,,\; R1 = 0.150\,,\; R2 = 0.225\,,\; R4 = 0.425\,,\; R5 = 0.500\,,$$

$$d = 0.075\,,\; L = 0.075\,. \tag{10}$$

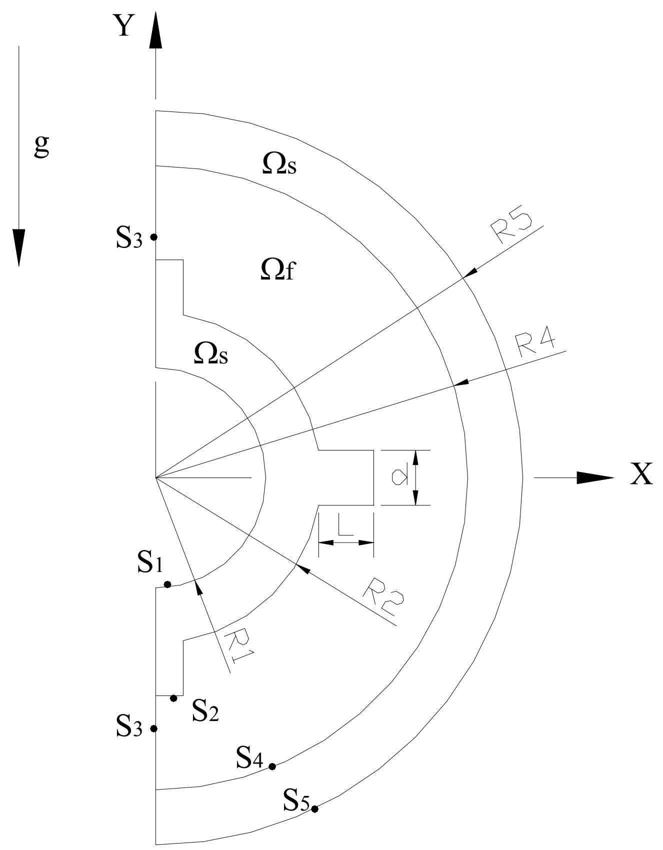


Figure 1: Geometry.

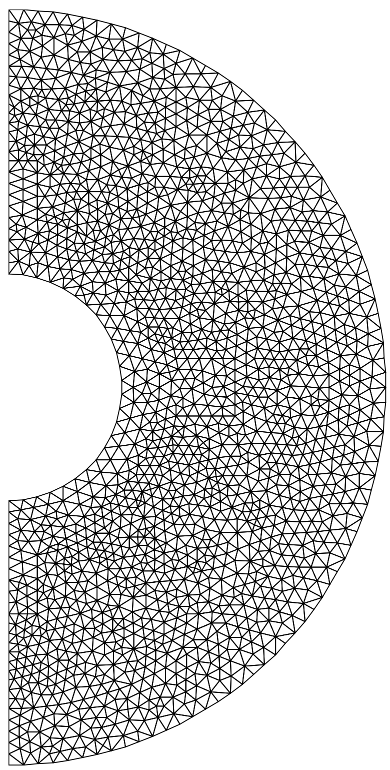


Figure 2: Mesh

4. Initial and boundary conditions

The initial and boundary conditions are given by:

$\tau = 0 : \quad \psi = \theta = \omega = 0$ (on the whole domain), (11)

$\theta = 1$, $\psi = 0$, $\omega = 0$ (on the external surface of the finned cylinder S_1), (12)

$\psi = 0$, $\omega = \omega_W$ (on the external surface of the finned cylinder S_1), (13)

$\psi = 0$, $\omega = 0$, $\dfrac{\partial \theta}{\partial X} = 0$ (isolation on the imaginary symmetry line S_3), (14)

$\psi = 0$, $\omega = \omega_W$ (on the internal surface of the outer cylinder S_4), (15)

$\theta = -1$, $\psi = 0$, $\omega = 0$ $\omega = \omega_W$ (on the external surface of the outer cylinder S_4), (16)

where ω_W is the fluid vorticity on the boundary, calculated as in (Brito, 1999).

5. Validation

The validation of the computational code was carried out by comparing the numerical results of the Nusselt numbers Nu_h with the results found in literature for the flow in an enclosure with an internal body. Table 1 shows this comparison. The symbols found in the table mean: + mesh 42x42, * mesh 21x21, ** mesh 42x42 and *** mesh 22x22. From Tab. 1, it can be noted that a good agreement.

Table 1: Comparison for a square geometry with Pr = 0,733.

Ref.	Figueredo et al.		Present work	Wong e Raithby		Present work
Gr	Nu^{*}_{h1}	Nu^{**}_{h1}	$\dfrac{Nu_h - Nu_{h1}}{Nu_h} \cdot 100[\%]$	Nu^{***}_{h2}	Nu^{**}_{h}	$\dfrac{Nu_h - Nu_{h2}}{Nu_h} \cdot 100[\%]$
34.110	2.884	3.023	4.60	2.972^{+}	3.023	1.69
60.000	3.468	3.588	3.34	----	----	----
100.000	4.160	4.190	0.72	----	----	----
136.430	4.686	4.602	-1.83	4.51	4.602	2.00
341.070	----	----	----	5.92	6.033	1.87

6. Results and discussion

In order to choose the mesh to be used in this work, a mesh refinement was performed to analyze the average Nusselt numbers on cold and hot surfaces for some Grashof numbers. The diffusivity ratio was set equal to 0.1. Figures 3 and 4 show the results.

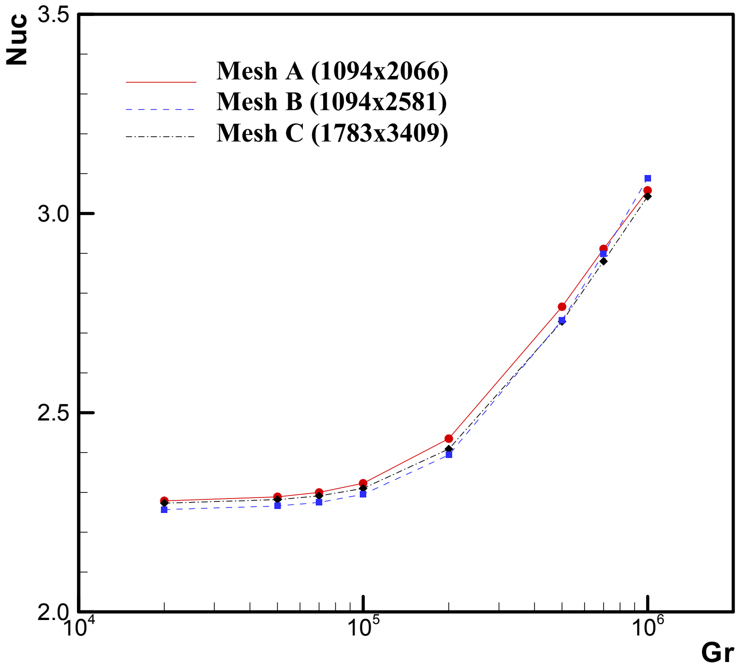


Figure 3: Average Nusselt numbers on the cold surface versus Gr for D = 1.

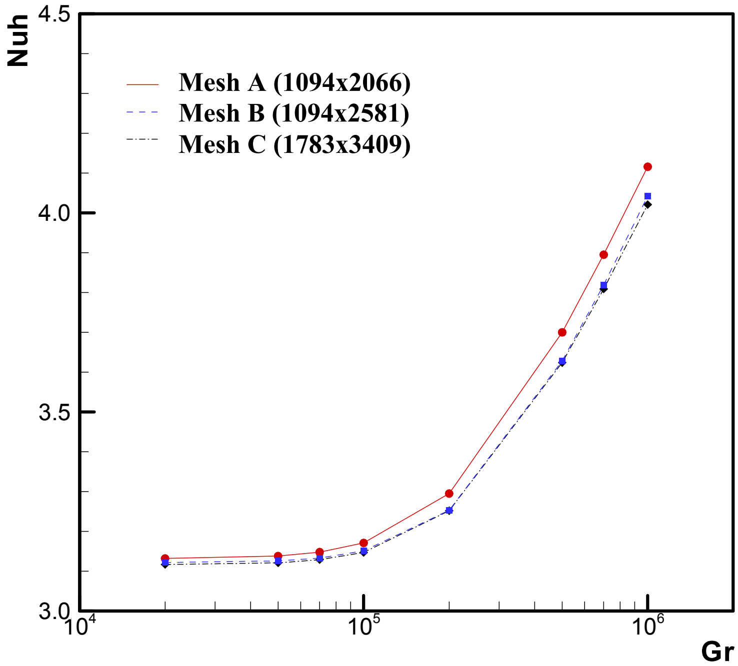


Figure 4: Average Nusselt numbers on the hot surface versus Gr for D = 1.

Analyzing the previous figures, mainly Fig. 4, one can observe that the Nusselt numbers for meshes B and C are very close, hence showing convergence. Therefore, mesh C with 3409 elements and 1783 nodes is chosen to run the cases of the present work. Figure 5 pictures the effect of the number of elements NE in function of the computational time per iteration. Just one case is studied here considering Pr = 0.7; Gr = 2×10^5; $\Delta\tau$ = 0.01 and D = 10. As expected, one can note that the increase of the number of the elements makes the computing time be higher. This is one of the biggest problems encountered in the numerical analyzes in an attempt to look for more accurate results, and thus, making it necessary more powerful machines. Figure 6 presents the average Nusselt numbers on the internal surface of the cold cylinder (Nu_c) versus the Grashof number (Gr) for different diffusivities D. In a similar way, figure 7 shows the average Nusselt numbers on the internal surface and on the fins of the heated cylinder (Nu_h). As Grashof number increases, the average Nusselt numbers, for both cases, also increases. This behavior is more significant as D is higher. Clearly, this is explained by the presence of a more conductive solid material benefiting the heat transfer. However, when D = 10, the Nusselt number increase in relation to the Grashof number, that is, the difference between Nu_c and Nu_h , is almost negligible.

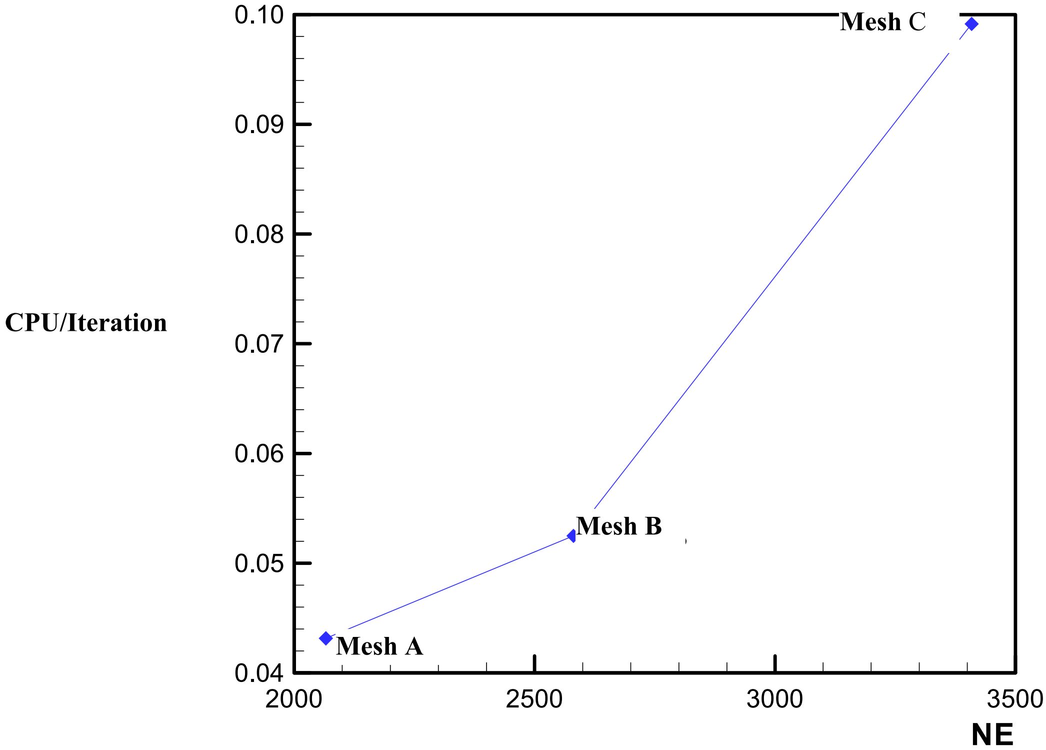
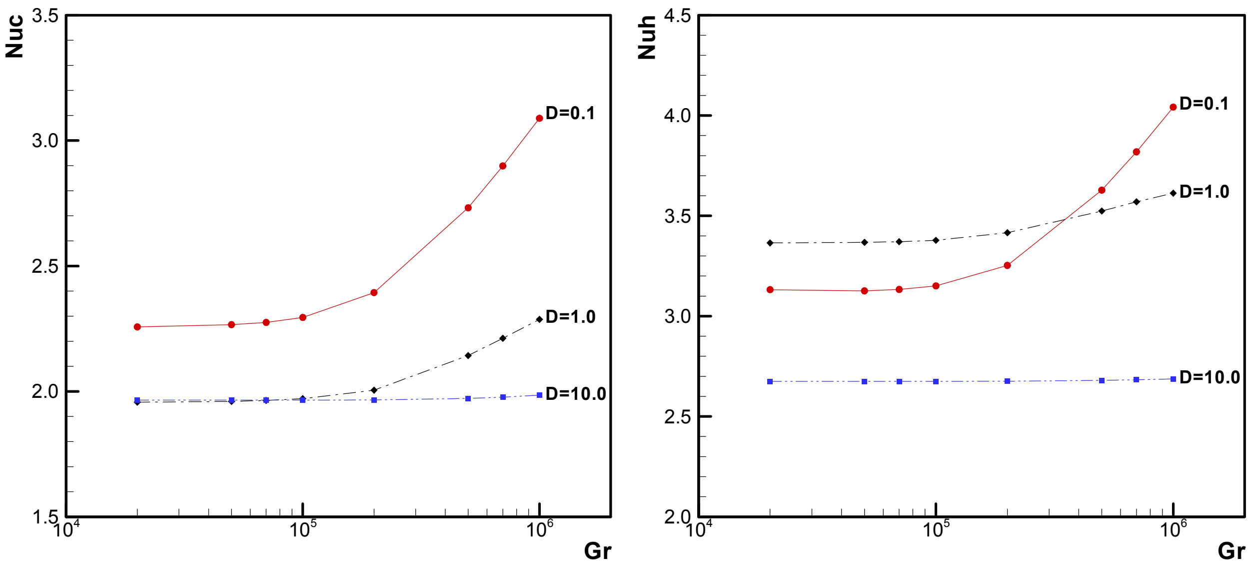


Figure 5: CPU[s]/iteration versus number of elements NE for Pr = 0.70; Gr = $2x10^5$; D = 10 e $\Delta\tau$ = 0.01.



Figures 6 and 7: Average Nusselt number on the cold and hot(finned) surfaces, Nu_c and Nu_h, respectively, versus Gr for different diffusivity ratios D.

Figure 8 shows the streamfunction (ψ) and the temperature (θ) fields in the left and right sides of each case, respectively. It is noted that for Gr $=5x10^4$, only when D = 0.1, two distinct recirculation cells appear due to the presence of the horizontal fins which partially block the flow. The hot finned cylinder makes the fluid go upwards till it reaches the cold surface where it is cooled and starts traveling downwards. As Grashof number increases, the velocities also increase, and when it reaches $5x10^5$, the two recirculation cells mix giving rise to a peripheral cell involving two smaller ones. When D = 1 and D =10, it can be noted that the recirculations only appear for high Grashof numbers. In the cases D = 10, even when Gr = 10^6, the recirculations are very weak. This features the conductive regime. This can also be verified in the temperature field for D = 10, where a thermal stratification occurs.

When D = 0.1 and 1, in Figs. 7 and 8, as Grashof number increases, there is a stronger variation of the temperature gradient near the internal surfaces for both cylinders, thus increasing the average Nusselt numbers Nu_c and Nu_h. On the contrary, when D = 10, the temperature field does not change.

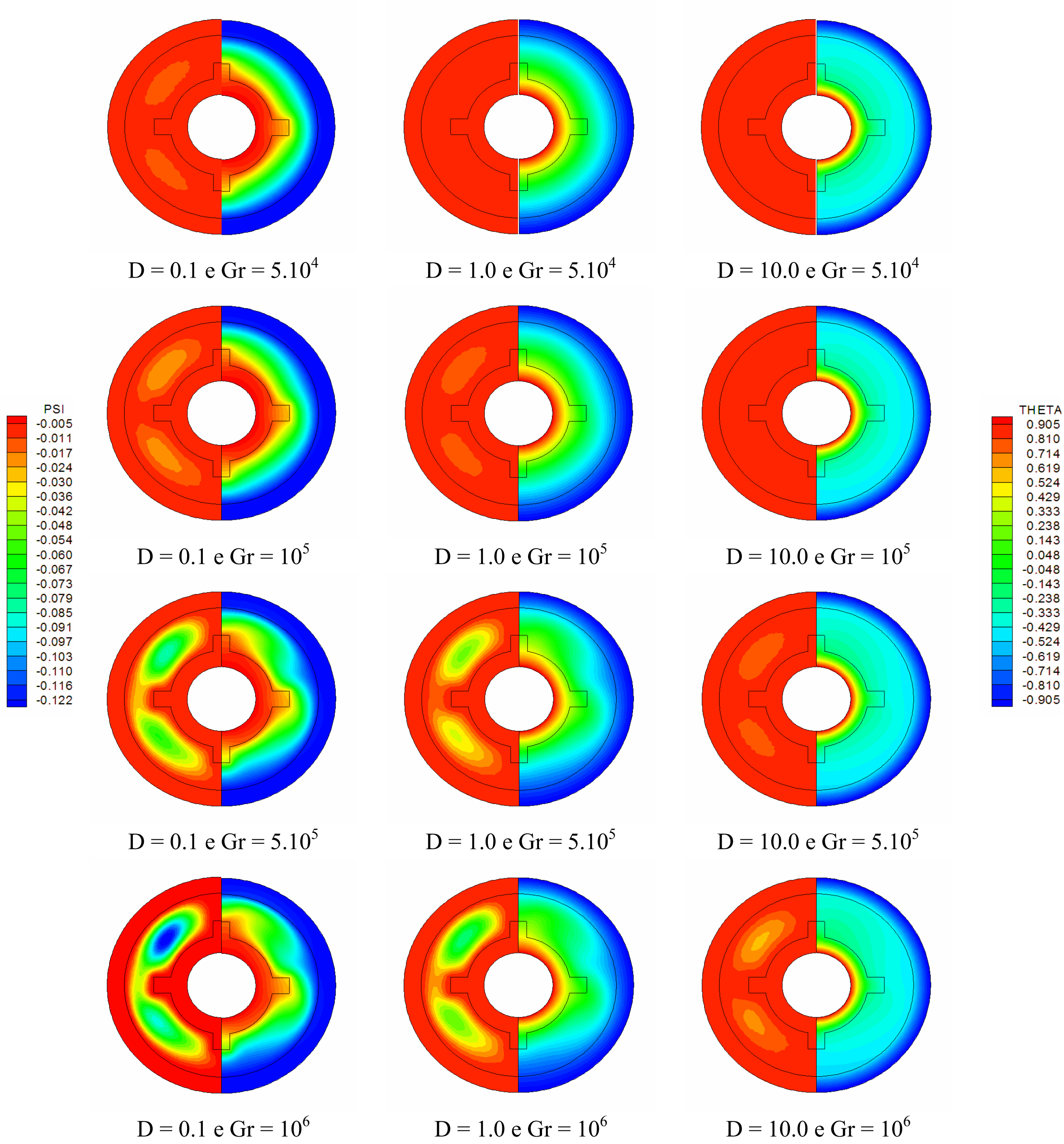


Figure 8: Streamfunction and temperature fields for Gr = $5x10^4$, 10^5, $5x10^5$ and 10^6.

7. Conclusions

Laminar natural convection study in finned annuli is carried out in this study. The finite element method is used to approximate the conservation equations. The fluid between the cylinders is air with Prandtl number Pr = 0.7. Four conductive thick fins are placed on the inner cylinder surface.

It is analyzed the effect of the diffusivity ratios between the solid and fluid materials D = 0.1, 1.0, 10.0, and Grashof numbers ranging from $2x10^4$ to 10^6.

It is verified that for low diffusivity ratios, D = 0.1 and 1.0, the velocities of fluid particles are higher due to the convective effects, making the Nusselt number get higher as Grashof number increases from Gr = 10^5 over.

As the diffusivity ratio increases D ≥ 10.0, the convective effects weaken considerably, and therefore, the Nusselt number behavior practically does not change by only varying the the Grashof number. The Grashof number ranging does not alter the heat transfer significantly.

8. Acknowledgements

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