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# THERMODYNAMIC CONSISTENT VERSION OF THE GRIFFITH FRACTURE CRITERION

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**Abstract.** *A thermodynamic consistent version of the Griffith`s fracture criterion is presented. Although Griffith has not constructed his criterion based explicitly on thermodynamic reasons, he compared two states, one defined just before (infinitesimally in time) the crack initiation and other immediately after the crack advance. In fact, he used implicitly the First Thermodynamic Principle. Otherwise this work is a new exercise on the fundamental concepts of a thermodynamic approach, that also includes the Second Thermodynamic Principle in the interpretation of the fracture phenomenon. The Griffith`s criterion thus reconstructed reveals its original imprecision.*

**Keywords**: fracture thermodynamics, fracture, Griffith`s criterion

## 1. Introduction

This paper aims to present a thermodynamic consistent version of the Griffith's criterion. In fact this is an initial exercise on the fundamental concepts of a new thermodynamic approach.

First of all, the thermodynamic consistent interpretation is required because fracture is considered an irreversible phenomenon, as emphasised here. Although this aspect has not been included in the Griffith's theory, it contains a relevant contribution to start a deeper study in fracture. In fact, the so-called energetic interpretation, present in the pioneer papers by Griffith (1920 and 1924), is the basis for the present study.

Griffith (1920) extended the concept of *total potential energy* of elasticity to the interpretation of the fracture process, introducing a new quantity, the *surface energy*, associated with the crack advance. In fact, this is a skilful resource used by Griffith but the crack growing energy dissipation is not considered by him. Obviously the linear elasticity theory, basis of the Griffith's theory, does not take into account this dissipation. In fact, the Griffith's surface energy is only the reversible part of the energy associated to the linear elastic behaviour, but there is another part of the fracture energy that is associated to the irreversible break of material links. This part is equivalent to the heat released when the irreversible crack advances in the solid. So, the rigorous thermodynamic treatment of the phenomenon seems to be the only solution for this problem, requiring therefore the inclusion of the Second Thermodynamic Principle. Thus, this approach is an attempt to complete the pioneer Griffith's idea to use the Thermodynamic (First Principle or Law of Conservation of Energy) for the interpretation of the fracture phenomenon.

For the theoretical deformation modelling in the Mechanics of Material context, fracture is the more *sever* phenomenon, in the sense that indicates a configuration limit of the solid, associated with an equilibrium bifurcation. This means that *plasticity* and *damage*, where the integrity is preserved, are less *sever* dissipative *p*rocesses than fracture, because the sequence of configurations that characterise the deformation process, preserves the global identity of the solid. For any of these cases, however, heat dissipation occurs and the thermodynamic consistency is necessary in the theoretical modelling. Thus, if the study of fracture were also based in the same approach, this indicates the possibility of unification to model the entire deformation process in Mechanics of Materials.

In the concept of *energy release rate*, suggested by Griffith (1920,1924) and after formally proposed by Irwin (1957), the Principle of Minimum Potential Total Energy is also implicit. This conclusion is due to Westergaard (1939), which was the base of Irwin's work (1957). In fact, the *stress intensity factors*, conceived by Irwin, are singular term coefficients of series of functions used to represent the components of stress in elastic linear plane problems. Thus, using that Minimum Potencial Total Principle in the Irwin's theory deserves the same criticism given to the Griffith's theory, i.e., the heat dissipation for the crack advances cannot be considered, because it is obviously impossible in the linear elasticity theory.

Similarly, the idea of *J integral* due to Rice (1968), other historic and important contribution to Fracture Mechanics, also based on the *energy release rate* of Griffith and Irwin, accord to Taroco (1996), suffers of the same described limitation, i. e., the lack of the thermodynamic consistency.

This paper aims the reconstruction of the Griffith's criterion as an introductory text of the fracture mechanic thermodynamic consistent interpretation, without any intention of getting hide of the historical limitation of his original criterion.

## 2. The thermodynamically consistent version of the Griffith criterion

Although Griffith has not constructed his criterion based explicitly on thermodynamic reasons, he compared two states, one defined just before (infinitesimally in time) the crack initiation and other immediately after the crack advance. The reconstruction of the Griffith's criterion is based on the assumption of the hypothesis *quasi-static* process. With this assumption (Dym and Shames, 1973) the thermodynamic balance local expression of the First Thermodynamic Principle is given by:

$$d\varepsilon = d'q + d'w, \tag{1}$$

where $\varepsilon$ is the *internal energy* (state function) *per unity volume*; $q$ is the *heat exchanged with the exterior* (*positive* when the solid *receives* heat), *per unity volume*; and $w$ is the *work per unity volume* made i*n the solid* (*positive* when the external actions make works in the solid).

The terms $d'q$ and $d'w$, in Eq. (1), are not exact differentials, because heat and work are not state functions, i.e., they are path dependent during the local process of the thermal changes and the deformation. However, the sum of $d'q$ and $d'w$, as in Eq.(1), is an exact differential, i. e., a state function.

Let us consider an initial void in the solid, an elliptical flaw for example, that is assumed a pre-crack. An irreversible growth can be developed only when the material links break, producing an advance of the crack lateral surfaces. This crack surface advance is as well irreversible. According with this hypothesis, heat is released in the process and its magnitude is associated with the irreversible growth of that lateral surface. This approach is clearly different of the one established by Griffith, in which the *surface energy* is not the dissipated in the crack surfaces. For the Griffith' process, given in the context of Linear Elasticity, the surface energy, uniformly distributed over the crack surface advance, is part of the solid *total potential energy*. Thus, following the treatment proposed by Griffith, the cracking phenomenon is a reversible process.

So the new aspect of the crack process analysis thermodynamically consistent here presented is that there are *two* surface energy densities associated to the advance of the crack: the first is the parameter $\gamma$ due to Griffith, related to the process reversible part and associated to $w$, and the other parameter, a new density, $\gamma_{diss}$, related to the heat released during the crack growth.

There are two types of dissipative processes, one associated with volume, plasticity for example, and other associated with internal surface formation. Herein, the quantities referred to the volume process dissipation will be denoted by *. Thus, if larger axis of the elliptical crack surface is augmented by $da$, then the *incremental work* $d'w$ is equal to $d'w^* + 4\gamma da$, where the positive sign, in the second part, means work realised in the system by the external actions.

On the other hand, the supplied (or lost) and the produced heat inside the solid and over the surface varies of a quantity $d'q^* - 4\gamma_{diss}$, where the negative sign mean a heat decrease dissipated during the process. Assuming that the internal surfaces grow irreversibly inside the solid, as a cracking process consequence, the integral of Eq. (1) in the volume between two considered states, one obtains the internal energy global balance:

$$\int_V d\mathcal{E} dV = \int_V d'w^* dV + \int_V d'q^* dV + 4\gamma da - 4\gamma_{diss} da . \tag{2}$$

To build up a theory that considers the irreversibility of the cracking process the entropy balance is required as an instrument of analysis. The entropy, $S$, is a quantity that links the heat variation and the equilibrium absolute temperature $T$.

Assuming that that no irreversible growth occurs, then $d'q^*$ will be equal $TdS$. Otherwise, if a crack grows, the corresponding energy dissipation has to be subtracted from $d'q^*$. Thus, the energy balance is given by:

$$\int_{dV} d'q^* dV - 4\gamma_{diss} da = \int_V (TdS) dV . \tag{3}$$

Let us now consider a state function $\psi = \varepsilon - TS$, the Helmholtz Free Energy. As the process is irreversible, $\psi$ gives the greatest amounts of *energy* capable of realising *work*, (see Lewis & Randall, 1961). Differentiating $\psi$ leads to:

$$d\psi = d\varepsilon - TdS - SdT. \tag{4}$$

Replacing Eqs.(2) and (3) into Eq. (4) and integrating it over the entire volume gives:

$$\int_V (d\psi) dV = \int_V (d'w^*) dV + 4\gamma da - \int_V (SdT) dV . \tag{5}$$

The first terms in the right hand side of the Eq.(5) is the strain energy variation between the two states, immediately before and after the irreversible crack growth. The measurement of this variation (integrated over the entire body) is here represented by $dE_d$, that is *the infinitesimal difference of the strain energy, between the two considered states.* Assuming that the fracture is a isothermal process, thus $\mathrm{d}T=0$, and the thermodynamic balance is represented in Eq. (5), the following equation can be written:

$$\int_{dV} d\psi \mathrm{d}V = \mathrm{d}E_d + 4\gamma \mathrm{d}a \ , \tag{6}$$

where the left hand side corresponds to the free energy variation of the entire solid. Finally, reordering the Eq. (6), leads to:

$$-dE_d = 4\gamma da + \left( - \int_{dV} d\psi dV \right) . \tag{7}$$

The crack phenomenon can now be interpreted using the Second Principle of Thermodynamic. Accordingly the concept of Helmholtz free energy: *if a phenomenon initiation spontaneously occurs, what characterises its irreversibility*, *then a free energy decrease will necessarily appear and this decrease is the least one possible (see* Lewis &Randall, 1961). *This means that when some crack advance is assumed, the quantity between parentheses in the right hand of Eq. (7) is positive. Thus:*

$$-dE_d > 4\gamma da. \tag{8}$$

This inequality is formally similar to the one derived by Griffith (1920). This result is the synthesis of the thermodynamic consistent interpretation of fracture developed here. It is clear that in the way used to reach the inequality (8) the magnitude of the neglect quantity in Eq. (7), last integral in the right hand side, was not take into account. Thus, this demonstrated that there is an imprecision in the traditional Griffith's criterion resulting from the Helmholtz free energy neglecting.

Although Eq.(8) apparently preserves the structure of the Griffit's theory, it shows an important difference when compared with the Griffith's criterion expression: the left hand side in Eq. (8) is not the absolute value of the *total potential energy* variation *(*$\mathrm{d}U$*) in the Griffith sense*, but it is the symmetric value of the strain energy variation ($\mathrm{d}E_\mathrm{d}$), evaluated between the two states defined immediately before and immediately after the crack advance. Thus, according with the left hand side term in Eq. (8), if an irreversible crack advance occurs, the strain energy (nor the total potential energy as used by Griffith) measured after the crack growth is smaller that its value measured before the crack advance.

From Eq. (8) and motivated by the Irwin's concept of *energy release rate*, $G$ (which is obtained by differentiating potential total energy with respect to a geometric fracture parameter), one can analogously defines the derivative of strain energy with respect to the same geometric parameter, i.e., $G_\mathrm{t}=dE_\mathrm{d}/da$, which measures the strain energy sensitivity with respect to the geometric parameter of fracture $a$.

The thermodynamically consistent criterion given by Eq. (8) is the essential part of this contribution. As this contribution could not be derived without the Griffith's theory, the new criterion suggested here is named here *Griffit's thermodynamically consistent fracture criterion.*

## 3. Discussion and conclusions

As the thermodynamic consistent parameter, $G_\mathrm{t}$, for fracture analysis has been proposed, some questions naturally arise about the traditional Fracture Mechanics concepts, such as the stress intensity factors and $J$ integral. By definition, these quantities are related to $G$, the *energy release rate*, arising as a consequence of the extension of the Principle of Minimum Total Potential Energy to Fracture Mechanics, in the Griffith's sense. Using this principle apparently becomes improper, because it does not consider the heat dissipation in the thermodynamic balance.

Although much works based on the classical Fracture Mechanics principles have already been carried out, the thermodynamically consistent interpretation shows that the stress intensity factors and the $J$ integral are not enough for the complete characterisation of the fracture phenomenon.

Naturally, to elaborate a general thermodynamically consistent Fracture Mechanics theory, some obstacles of mathematical nature have to taken over. In this way, the contributions of Griffith, Irwin, Rice among others are very much useful. Accordingly Rocha (1999), the methodology for $J$ integral evaluating, associated with the sensitivity analysis, due to Taroco (1996), is an important tool to be used for new approach.

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## 5. Responsibility notice

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