The log-law method as a means for evaluation of skin-friction on flows over abrupt changes in surface roughness

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Abstract. *This work discusses the applicability of the log-law method for the evaluation of skin-friction on flows that develop over surfaces with an abrupt change in surface roughness. In particular, the work deals with flows that move from one rough surface to another smooth surface. When that happens, the strong memory effects present in the inner flow region make the log-profile depart from a straight line, resulting in a curve bended to the left. The main objective of the present work is to argue that the Perry and Joubert (JFM, vol. 17, pp. 193–211, 1963) fitting procedure can still be used to evaluate the skin-friction and to determine the length of the flow transition region. Validation of the proposed method will be provided by a comparison of predicted skin-friction with values measured directly through a Preston tube.*

Keywords: *Roughness, skin-friction, Preston tube, hot-wire anemometry.*

1. Introduction

In a recent contribution, Brasil et al. (2004) have discussed the problem of turbulent flows over surfaces with changes in roughness. The emphasis then was on studying the statistical properties of the flow in the transition region that results downstream of the point in change in surface roughness, from rough to smooth. As argued in that paper, following the change in surface roughness, the flow is very slow in adjusting to the new surface condition. Under this situation, the rough-wall upstream flow dominates the rate of diffusion of the disturbances for a considerable length, dictating conditions that are far from self-preserving (Antonia and Luxton (1971), Antonia and Luxton (1972), Antonia et al. (1977)).

The objective of the work was then to identify and to quantify the statistical parameters that best characterized the flow in a transitional state. The conclusion was that the flow could be was characterized by two parameters: the error in origin and the third order momentum, also known was skewness. The paper showed further that the variance and the kurtosis do not vary much with the change in roughness. The work further assessed the longitudinal mean velocity, longitudinal velocity fluctuations, the Taylor micro scales of time and length, the Kolmogorov micro-scales of time and length, the dissipation rate and the one dimensional spectrum.

Additional investigations on the statistical properties of a turbulent flow passing from a rough surface to a smooth surface are, for example, the investigations of Ligrani and Moffat (1985), Perry et al. (1987), Bandyopadhyay (1987), Bandyopadhyay and Watson (1988), Krogstad and Antonia (1994). On the other hand, Avelino and Silva Freire (2002) and Loureiro and Silva Freire (2004) showed that much remains to be understood about turbulent flows over rough surfaces even in what concerns some of their mean properties. This is typically the case of the roughness displacement function and of the error in origin.

In particular, in Brasil et al. (2004) the skin-friction in the transition region was evaluated from an extension of the method of Perry and Joubert (1963) to the smooth surface. Basically, it was assumed that in the transition region, where the flow is still accommodating to equilibrium condition, the graphical method of Perry et al. (1969) developed for rough surfaces of types 'K' and 'd' could still be used to smooth surfaces. The implication was that in the transitional region the flow developed over a 'virtual' error in origin which could be evaluated with the standard practice of arbitrarily adding values of ε to the wall distance measured from the top of the roughness elements until a well discriminated straight line can be identified in the log-law region. Then, and provided von Karman's constant is known, the slope of the straight line can be used to find the friction velocity.

At the time, however, the hypothesis advanced by Brasil et al. (2004) was not properly corroborated. A great difficulty was, of course, to obtain independent skin-friction measurements in the non-equilibrium, transitional region. This is a major undertaking since many of the methods that are available in literature rely, in one way or another, on the existence of the classical law of the wall. The purpose of this work is to propose an alternative method to be used on the characterization of the transition region by inspection of the readings of a Preston tube as compared to the skin-friction data furnished by the method of Perry et al. (1969). This will, somehow, yield two independent, but correlated methods, to find the length of the transition region. The first method is based on an assessment of the behavior of the error in origin – an absolute method. The second method is based on a comparison between the readings of a Preston tube and the values of skin-friction evaluated by the method of Perry et al. (1969).

Thus, the present work aims specifically at assessing the effects of the rough-to-smooth step change in surface on the skin-friction, through the error in origin and on the roughness function. To the best of the present author's knowledge, this is the first time where predictions of the skin-friction values in the transitional region are evaluated by the Perry et al. (1969) method. Furthermore, this is the first time that this procedure is compared with data obtained by Preston tubes so as to determine the length of the transition region.

2. Theory

2.1 The law of the wall for flows over rough surfaces

The arguments that lead to the establishment of the error in origin as a fundamental parameter for the understanding of flows over rough surfaces will be laid down here.

For flows over rough surfaces, Einstein and El-Samni (1949) were the first to realize that if the experimental mean velocity data for a rough surface were displaced some distance below the top of the roughness elements, then they would shape to a well defined straight line. Subsequently, Moore (1951) confirmed that a universal expression can be written for the wall region provided the origin for measuring the velocity profile is set some distance below the crest of the roughness elements. This displacement in origin observed by the above authors is normally referred to in literature as the error in origin, ε.

A rigorous method for the determination of the displaced origin was developed by Perry and Joubert (1963) and by Perry et al. (1969).

In general, for any kind of rough surface, it is possible to write

$$\frac{u}{u_\tau} = \frac{1}{\varkappa} \ln \Big[\frac{(y_T + \varepsilon)u_\tau}{\nu}\Big] + A - \frac{\Delta u}{u_\tau} \tag{1}$$

where,

$$\frac{\Delta u}{u_\tau} = \frac{1}{\varkappa} \ln \Big[\frac{\varepsilon u_\tau}{\nu}\Big] + C_i \tag{2}$$

and $\varkappa$ = 0.4, A = 5.0, and $C_i, i = K, D$; is a parameter characteristic of the roughness (see, for example, Perry and Joubert (1963)). The value of y_T indicate the measured distance from the top of the roughness element.

Equations 1 and 2, although of universal character, have the inconvenience of needing two unknown parameters for their definition, the skin-friction velocity, u_τ, and the error in origin, ε. A chief concern of many works on the subject is, hence, to characterize these two parameters.

The reader must notice that once ε is determined the skin-friction velocity follows directly from the inclination defined by Eq. 1. Is this, in fact, a classical way of determining u_τ for flows over rough surfaces. The difficulty is that ε is a quantity which is, in principle, unknown.

In fact, the fundamental concepts and ideas on the problem of a fluid flowing over a rough surface were first established by Nikuradse (1933) who investigated the flow in sand-roughened pipes. Even at that early age, Nikuradse was capable to establish that, at high Reynolds number, the near wall flow becomes independent of viscosity, being a function of the roughness scale, the pipe diameter and Reynolds number. He also found that, for the defect layer, the universal laws apply to the bulk of the flow irrespective of the conditions at the wall. The roughness effects are, therefore, restricted to a thin wall layer.

Thus, considering that Coles's wake hypothesis (Coles (1956)) applies to the outer region of the flow, the law of the wall can be re-written as

$$\frac{u}{u_\tau} = \frac{1}{\varkappa} \ln \Big[\frac{(y_T + \varepsilon)u_\tau}{\nu}\Big] + A - \frac{\Delta u}{u_\tau} + \frac{\Pi}{\varkappa} W\big(\frac{y}{\delta}\big) \tag{3}$$

where W is a universal function of y/δ and Π is a parameter dependent on the upstream shear stress and pressure distribution.

Equation 3 provides a representation of the velocity field over the whole of the turbulent and defect regions of the boundary layer.

Substitution of (y, u)=(δ, U_∞) into equation 3 furnishes

$$\frac{U_\infty}{u_\tau} = \frac{1}{\varkappa} \ln \Big[\frac{\delta + \varepsilon}{\varepsilon}\Big] + A - C_i + \frac{2\Pi}{\varkappa} \tag{4}$$

This is a simple algebraic equation that furnishes values of C_f $(= 2\, u_\tau^2/U_\infty^2)$ for known values of U_∞, δ and ε.

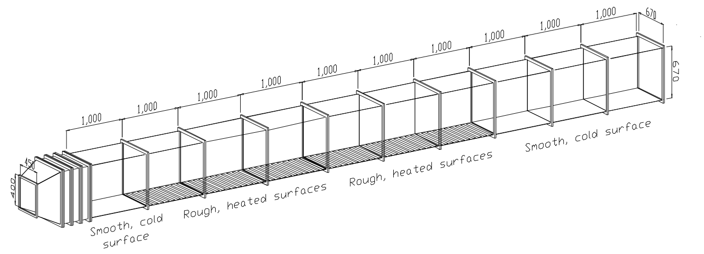

Figure 1. General view of wind tunnel.

For flows over rough surfaces, we have seen that the characteristic length scale for the near wall region must be the displacement in origin. Indeed, in this situation, the viscosity becomes irrelevant for the determination of the inner wall scale because the stress is transmitted by pressure forces in the wakes formed by the crests of the roughness elements. It is also clear that, if the roughness elements penetrate well into the fully turbulent region, then the displaced origin for both the velocity and the temperature profiles will always be located in the overlap fully turbulent region.

2.2 Preston tubes

The principle of operation for a Preston tube obeys the equation for a Pitot tube. Thus, the operational conditions for a Preston tube depends on the existence of a region of flow similarity adjacent to the wall. Under this assumption and further considering that the Preston tube lies entirely in the viscous region, the following relation can be advanced for a turbulent flow

$$\frac{\overline{\Delta P}}{\tau_w} = \frac{1}{2}\Big(\frac{1}{2}K_t\Big)^2 \frac{d_t^2 \overline{\tau_w}}{\rho \nu^2}\Big[1 + \frac{\overline{\tau_w'^2}}{\overline{\tau_w}^2}\Big] \tag{5}$$

where K_t = 1.30, d_t is the outside diameter of the tube, $\overline{\Delta P}$ is the mean difference between the static pressure and the total pressure measure by Preston tube, $\overline{\tau_w}$ is the time-averaged wall stress, and $\overline{\tau_w'^2}$ is the mean square of the fluctuations in the wall stress. Measurements performed in many laboratories indicate that $\overline{\tau_w'^2}/\overline{\tau_w}^2 \cong 0.15$.

Preston tubes are calibrated in fully developed pipes, where there is a direct relation between pressure gradients and wall shear stress. Head and Rechenberg (1962) and Patel (1965) published calibration curves for different-diameter Preston tubes. These curves, as mentioned before, are dependent on the accuracy of the law of the wall. For flows with small pressure gradients, the law of the wall is usually observed to hold. For flows under large pressure gradients where, for example, separation regions are present, the Preston tube should not be used.

3. EXPERIMENTAL PROCEDURE

The experiments were carried out in the high-turbulence wind tunnel located in the Laboratory of Turbulence Mechanics of PEM/COPPE/UFRJ. The tunnel is an open circuit tunnel with a test section of dimensions 670 mm x 670 mm x 10.000 mm (Fig. 1). The test section is divided into three sections of equal length which can be fitted with surfaces having different types of roughness. The first section, which is normally kept at ambient temperature, consists of a smooth glass wall. The second and third parts of the test section are equipped with independent surface types. A general view of the wind tunnel is slhown in Fig. 1. For further details of the wind tunnel see Cataldi et al. (2001) and Cataldi et al (2002).

At first, data was taken to characterize the behavior of the Preston tube to the smooth surface case. A comparison between the values for the friction velocity (u_τ) obtained through the Preston tubes and to Clausert's method (1954) was made. This procedure enabled the validation of the data obtained through the Preston tube.

Next, the flow was subjected to a step change in roughness, from smooth to rough, after travelling one meter over the glass wall. Then, a second change in surface, from rough to smooth, occurred after 6 meters. In the final 2 meters the fluid flowed again over the glass smooth surface. One type of rough surface was considered where the roughness elements consisted of equally spaced transversal rectangular slats. The dimensions of the roughness elements and the coordinate system are shown in Fig. 2 were K denotes the height, S the length, W the gap, and λ the pitch. In constructing the surface, extreme care was taken to keep the first roughness element always depressed below the smooth surface, its crest kept aligned with the smooth glass wall surface.

The measurements were performed for values of the free-stream velocity of 3.12 m/s; the free stream-level of turbulence was about 2%. The stream-wise pressure gradient was closely set to zero by adjusting the roof of the tunnel

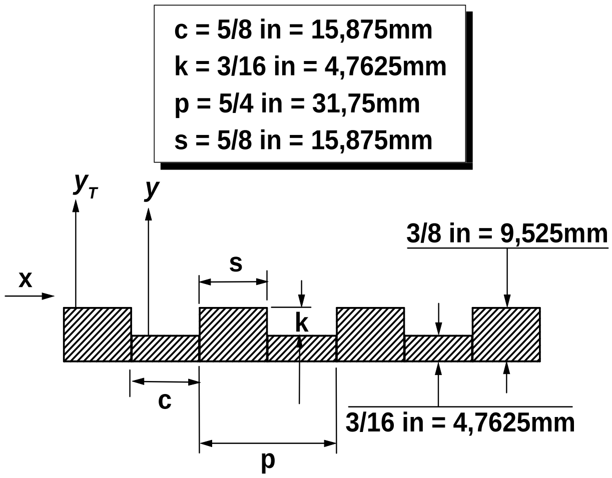

Figure 2. Geometry of roughness elements.

according to the readings of eight equally spaced pressure taps.

Mean velocity profiles and turbulence intensity levels were obtained using a DANTEC hot-wire system series 56N. The boundary layer probe was of the type 55P15. A Pitot tube, an electronic manometer, and a computer controlled traverse gear were also used. In getting the data, 10,000 samples were considered. The profiles were constructed from about 100 points. The mean temperature profiles were obtained through a chromel-constantan micro-thermocouple mounted on the same traverse gear system used for the hot-wire probe. An uncertainty analysis of the data was performed according to the procedure described in Kline (1985). The uncertainty associated with the velocity and temperature measurements were: U = 0.064 m/s precision, 0 bias (P=0.95); T = 0.214 ^oC precision, 0 bias (P=0.99).

To obtain accurate measurements (Bruun (1985)), the mean and fluctuating components of the output signal of the anemometer were treated separately. Two output channels of the anemometer were used. The mean velocity profiles were calculated directly from the untreated signal of channel one. The signal given by channel two was 1 Hz high-pass filtered leaving, therefore, only the fluctuating velocity. The later signal was then amplified with a gain controlled between 1 and 500 and shifted by an offset so as to adjust the amplitude of the signal to the range of the A/N converter.

The experimental setup is illustrated in Fig. 3.

4. Results

4.1 Comparison between skin friction data to the smooth case

A comparison was made between skin friction data obtained through the Preston tube and trough Clausert's method over the smooth surface. The velocity distribution across the boundary layer for several external flow velocities was measured through the hot-wire anemometry; these profiles were subsequently used to calculate the friction velocity through Clauser's method, which consists of a manipulation of the classic law of the wall to

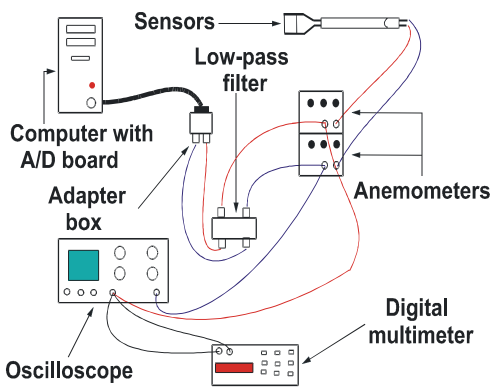

Figure 3. Experimental setup.

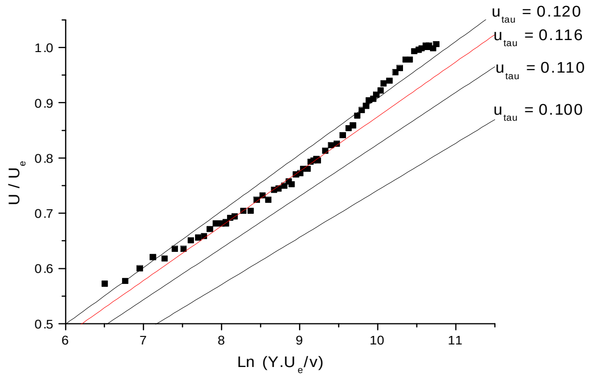

Figure 4. Calculating U_τ through Clauser's method.

$$\frac{\overline{u}}{\overline{u_e}} = \left[\frac{u_\tau}{\overline{u_e}\cdot\varkappa} ln\left(\frac{\overline{u_e}y}{\nu}\right)\right] + \underbrace{\left[\frac{u_\tau}{\overline{u_e}\cdot\varkappa} ln\left(\frac{u_\tau}{\overline{u_e}}\right) + \frac{A\cdot u_\tau}{\overline{u_e}}\right]}_{A_e \rightarrow Linear\ coefficient} \tag{6}$$

By plotting the local mean velocity data in a $\frac{\overline{u}}{\overline{u_e}} \times ln\left(\frac{\overline{u_e}y}{\nu}\right)$plot, the value of u_τ is selected so as that the resulting straight line coincides with the tangent to the turbulent region of the profile. This procedure is seen in Fig. 4. Measurements taken with the Preston tube were then used to calculate the same quantity. The results obtained through both methods were then compared to each other to validate the Preston tube data. General agreement was within 5%.

$U_\infty(m/s)$	$u_{\tau HWA}(m/s)$	$u_{\tau Preston}(m/s)$	$Error(\%)$
3.30	0.131	0.135	3.1
2.86	0.116	0.122	5.2
2.44	0.108	0.103	4.6

Table 1. Experimental friction velocity results over a smooth surface.

4.2 Velocity profile results in the transition region

The measured velocity profiles at stations 5 mm and 25 mm, referred to the point in change in surface roughness, are shown in Fig. 5 in logarithmic form. This Figure shows clearly that the logarithmic regions of the flow have suffered a bending to the left. In fact, and as we shall see, the method to be used here to find the error in origin, ε, is based on a procedure to restore the turbulent region portion of the velocity profile to a logarithmic profile.

The global parameters of the velocity boundary layers are shown in Fig. 6, where δ denotes the boundary layer thickness, δ^* the displacement thickness and θ the momentum thickness. Of particular note are the results for the Clauser factor, G, defined by

$$G = \frac{1}{\Delta}\int_0^\infty \left(\frac{U_e - \overline{u}}{u_\tau}\right)^2 dy. \tag{7}$$

where

$$\Delta = \int_0^\infty \frac{U_e - \overline{u}}{u_\tau} dy = \delta^* \sqrt{2/C_f} \tag{8}$$

The Clauser parameter indicates the state of equilibrium of the boundary layer. For the values found here, between 6.0 and 7.0, the boundary layer is in a self-preserving state. Please, note that the evaluation of G depends on the knowledge of $C_f/2$ which, in principle, is not known at the moment. The determination of $C_f/2$ is explained next.

4.3 The error in origin and the skin-friction

The error in origin, ε, was estimated through the procedures by Perry and Joubert (1963) and by Perry et al. (1969). These procedures are very rigorous so that the data resulting from them must be seen as very reliable. The procedures of Thompson (1978) and of Bandyopadhyay (1987) are very simplified, so the values of ε obtained through them can only be seen as a first approximation.

In the Perry and Joubert (1963) and Perry et al. (1969) methods, arbitrary values of ε are added to the wall distance measured from the top of the roughness elements, and a straight line is fitted to the log-law region. The value of ε that furnishes the best discriminated logarithmic region is then considered to be the correct value for the error in origin. The alternative method of Perry et al. (1986) is more sophisticated, resorting to a cross plot of ε vs $2\Pi/\varkappa$, where Π stands for Coles's wake profile.

Thus, to determine the error in origin, the velocity profiles were plotted in semi-log graphs in dimensional coordinates. Next, the normal distance from the wall was incremented by 0.1 mm and a straight line fit was applied to the resulting points. The best fit was chosen by searching for the maximum coefficient of determination, R-squared. Other statistical parameters were also observed, the residual sum of squares and the residual mean square. Normally, a coefficient of determination superior to 0.99 was obtained.

Having found ε, we can now use the gradient of the log-law to determine u_τ. Another method to obtain u_τ is the momentum-integral equation. This latter method, however, is very sensitive to any three-dimensionality of the flow and the determination of the derivatives of the various mean flow parameters is a highly inaccurate process.

The difficulty with both cited methods is that they depend on the evaluation of derivatives. For flows subjected to step changes in surface roughness, the momentum-integral method further suffers from the ill definition of the boundary layer origin. The process of finding adequate parameters for a good curve fitting is, therefore, highly aggravated.

The graphical method described above is shown in Fig. 7.

The results for the prediction of ε in the flow transitional region are presented in Fig. 8. Considering the high degree of difficulty involved in finding these results, and the coherent agreement between the predictions based on the expected flow behavior, we might think that the results of ε and consequently of C_f will be likely to be very representative of the flow.

Figure 8 illustrates how ε relaxes from its high upstream value, representative of the roughness, to a much lower values representative of the smooth surface flow. Thus, it is in this region that the flow retains its memory effects. This fact had been clearly identified by Brasil et al. (2004) through measurements of the flow skewness. The evolution of ε on the smooth surface characterizes, thus, the transition region form the fully rough regime to a fully smooth regime. The length of the transition region according to Fig. 8 is somewhere slightly over 50 mm.

The values of u_τ obtained through the velocity gradient method (Perry and Joubert (1963)) an by the Preston tube are shown in Fig. 9.

The results found for the skin-friction through Preston tube are clearly different from the results found by the velocity gradient method. This fact occurs because the Preston tube is unable to furnish measurements on a rough surface, or yet, in regions where the canonical law of the wall does not hold. For the rough surface, the failure is due to the reduction of the dynamic pressure near the wall because of the reduced speeds caused by the protuberances which are interpreted by the Preston tube as a lower friction. As a fact, Preston tube's calibration curve is only valid for the smooth surface case.

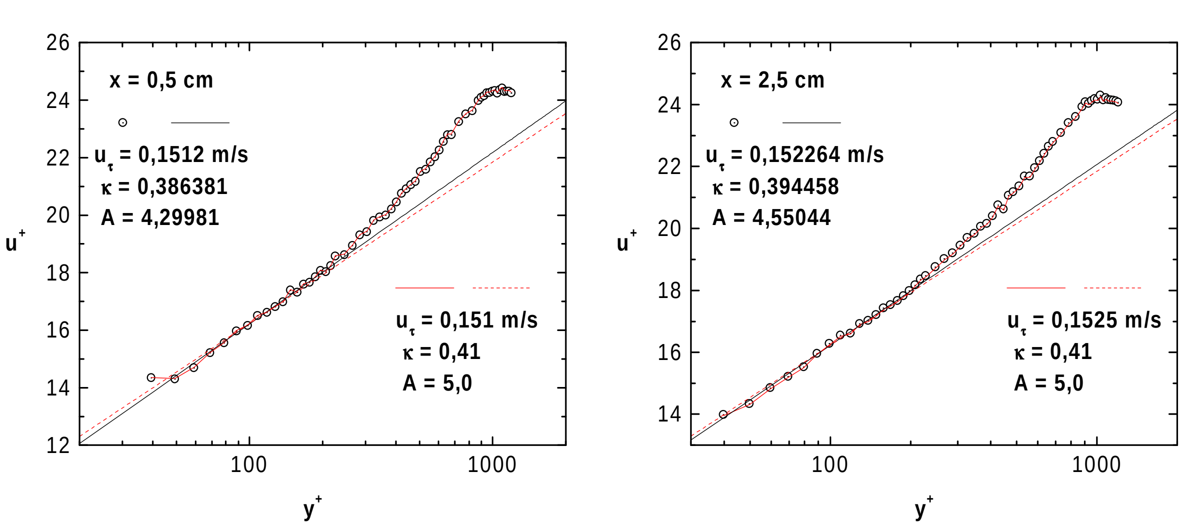

Figure 5. Velocity profiles in wall coordinates at 5 mm and 25 mm from the end of the roughness.

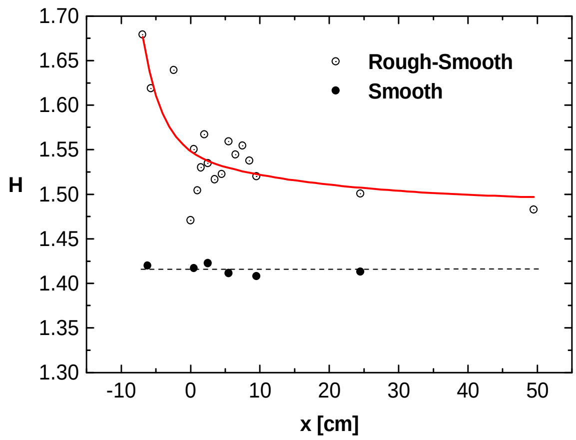

Figure 6. Global parameters of the boundary layer.

Therefore, the intercept of the curves defined by the Preston tube readings and the velocity gradient method data can be interpreted as the length of the transition region. This intercept is located somewhere between 50 and 60 mm.

We may than look at the curve defined by the Preston tube as a reference line for the specification of the transition region. Picking the point where ε goes to zero as the reference point where the transition region is defined poses the difficulty of establishing a criterion which is absolute in itself. Using the curve defined by the Preston tube has the advantage of defining an independent level to which u_τ must tend as the transition region is over.

5. FINAL REMARKS

The calculated values of u_τ over a smooth surface were obtained through Clauser's method and through a Preston tube. In the first method, a line tangent to the velocity points in the turbulent region gives the correct value of u_τ. The second method uses the universal calibration curves of Head and Rechenberg (1962) and of Patel (1965). The results for skin friction in the smooth surface case obtained through the Preston tube have shown a good agreement with those obtained through Clauser's method, staying typically within a 3% error band.

In the rough case, the values of u_τ were obtained both through Perry and Joubert's method and through Preston tubes. The first method consist in systematically adding an arbitrary value to the distance from the top of the roughness elements; a least square procedure was built to furnish the best discriminated straight line fit. The second method uses the universal calibration curves of Head and Rechenberg (1962) and of Patel (1965). The results obtained through the Preston tubes are used as reference values to which the return to equilibrium flow must adhere. The Preston tubes are calibrated for use within equilibrium flows and over a smooth surface so that they must be used in the transition region as an independent standard level for the skin friction.

Determining the wall shear stress has always been a difficult problem that has plagued many authors. In transitional, non-equilibrium regimes, we remind the reader that this problem is aggravated manifold. In this paper we have proposed to use the Perry and Joubert method (derivative method) to find u_τ. In addition, the length of the transition region has

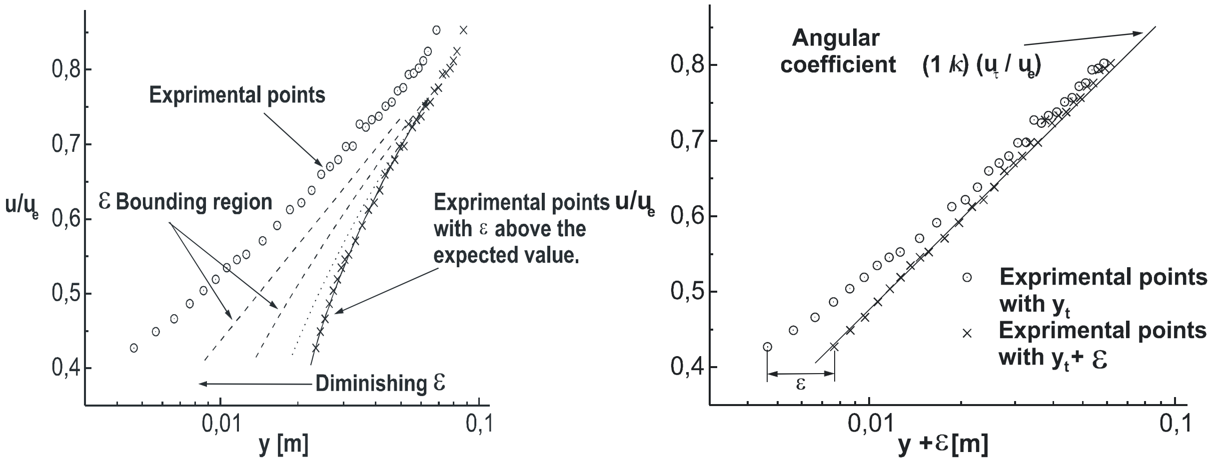

Figure 7. Determination of the error in origin and of the skin-friction velocity.

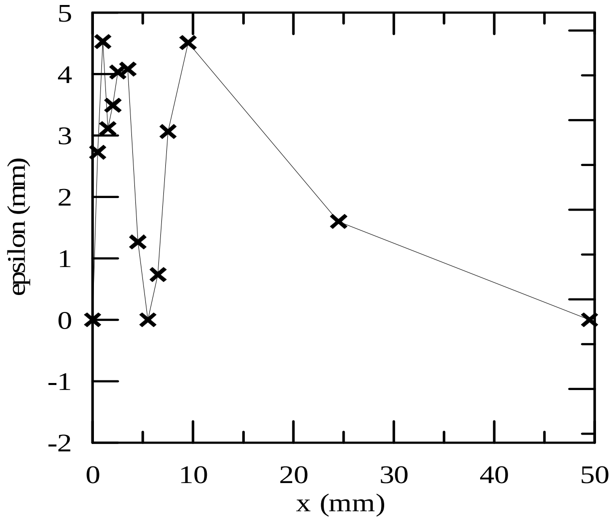

Figure 8. Measured values of the error in origin.

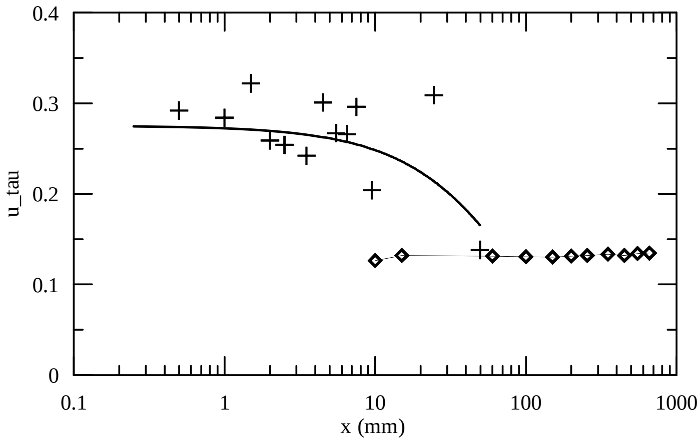

Figure 9. Measured values of the skin-friction velocity though velocity gradient and though Preston's method.

been evaluated through two related but independent methods: the return to zero value of the error in origin – an absolute method, and, the return to undisturbed values of the skin-friction – a relative method.

Since the main objective of this work has been to assess the usefulness of the derivative method to evaluate the skin-friction and the length of the transition region, we feel that this task has been successfully accomplished. Measurements of turbulent quantities and a further processing of the available data had already been published by Brasil et al. (2004).

Acknowledgements. APSF is grateful to the Brazilian National Research Council (CNPq) for the award of a research fellowship (Grant No 304919/2003-9). The work was financially supported by CNPq through Grant No 472215/2003-5 and by the Rio de Janeiro Research Foundation (FAPERJ) through Grants E-26/171.198/2003 and E-26/152.368/2002. ASM benefited from a Research Scholarship from the Brazilian Ministry of Science and Technology through CNPQ.

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