FRANCIS HYDRO TURBINE-GENERATOR MODEL FOR MODEL PREDICTIVE SPEED CONTROL

Gláucio Sansevero
GE Optimization Services - Campinas - SP
glaucio.sansevero@ge.com

Celso Pascoli Bottura
DMCSI - FEEC - UNICAMP
bottura@dmcsi.fee.unicamp.br

Abstract. *In this paper a few classical models for Francis type hydraulic turbine-generator are revisited. From its analysis, and focusing on the development of model predictive control algorithms, a non-liner model is constructed. In this model, the main dynamical characteristics of this type of machine, like water column viscosity and compressibility, are incorporated. Phenomenons like water hammer and hydraulic turbine inverse response are reproduced in digital computer simulations. A locally linearized state space model is also presented and simulations comparing the two models are performed.*

Keywords: *model, turbine, hydraulic, Francis, non-linear*

1. Introduction

Since the introduction of digital computers in the field of turbo-machinery control, not many changes have occurred in the way hydraulic turbines are controlled. Despite the advances in digital computer technology, the algorithms used on turbine speed control are essentially PID type algorithms. Recently, in attempt to make a better use of the larger processing capacity available today, model based control algorithms have been proposed[6][10][11][13]. A very important stage in the development of model based controllers is the construction of a system model, that is in this case part of the controller itself. The ability of this model to closely reproduce the modeled system behaviour is crucial. However, as real time simulations using the model are performed for each new control input calculation, a compromise must exist between model accuracy and computational complexity. Models including system non-linearities may be more accurate in terms of reproducing real systems behaviour, but the extra computational complexity involved with the non-linearities may prevent it from being used in real time simulations.

2. System Modeling

The block diagram shown in Fig. 1 displays the relationships between the several dynamic elements present on a typical hydro turbine-generator arrangement[9]. It will be the ground reference for all the developments that follow. The proposed model will cover the Turbine, Conduit and Rotor/Load Dynamics. Models for the gate positioner will not be part of this paper. A simple model for this subsystem will be used in the simulation.

The gate positioner is a servo-mechanism controlled by the turbine governor that positions the turbine wicket gates. The gates position determines the water flow through the turbine. Water flowing inside a closed conduit determines head variations at the turbine admission. The net mechanical power on the turbine axis is determine by flow and head across the turbine. This mechanical power, combined with the electrical power determined by the electrical load connected to the generator act on the spinning mass of the turbine-generator set, determining its angular speed. The angular speed on its turn, determines the electrical frequency of the generator, that might influence the electrical load itself.

2.1 Turbine and Conduit

To facilitate understanding, we summarize the definitions of the constants and variables to be used in this section in Tab. 1. The dynamical behaviour of a hydraulic turbine power is practically an instantaneous function of head and flow across the turbine when compared to the dynamics of water flowing inside the conduit[1]. Turbine inefficiency can be accounted by subtracting the flow necessary to compensate for efficiency losses from the actual flow across the turbine. It can also be demonstrated that a speed deviation damping effect is present[12]. These relations can be expressed by the equation

$$P_m = A_t h(q - q_{nL}) - D_{tur} G \Delta\omega_m \tag{1}$$

A_t is a proportionality used to convert the per unit gate opening into per unit turbine power on the volt-ampere base of the generator. It takes into account the turbine gain and is defined as

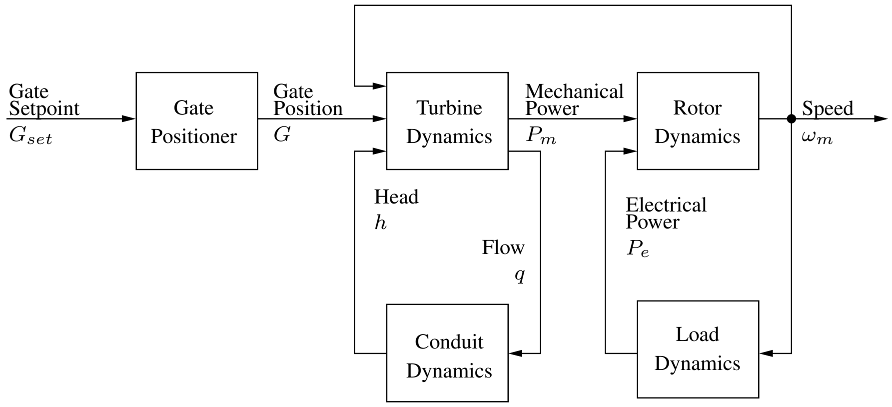

Figure 1. Block Diagram for Hydro Turbine and Generator Dynamical Relationship

Parameter	*Definition*
ρ	Density of water
K	Bulk modulus of water
D	Internal penstock diameter, m
f	Wall thickness of penstock, m
E	Young's modulus of pipe wall material
α	$\rho g(\frac{1}{K} + \frac{D}{fE})$
g	Acceleration of gravity, m/s^2
L	Penstock length, m
A	Penstock cross section area, m^2
a	Wave velocity $\sqrt{\frac{g}{\alpha}}$, m/s
G	Per unit instantaneous gate opening
q	Per unit instantaneous turbine flow
h	Per unit instantaneous head at turbine admission
q_{nL}	Per unit no-load flow
P_m	Per unit instantaneous turbine mechanical power output
ω_m	Per unit instantaneous angular speed
$\Delta\omega_m$	Per unit deviation from rated angular speed

Table 1. Turbine Model Parameters

$$A_t = \frac{\text{Turbine MW Rating}}{(\text{Generator MVA Rating})h_r(q_r - q_{nL})} \tag{2}$$

Where h_r is the per unit head at rated flow and q_r is the per unit flow at rated power. D_{tur} is called the turbine nominal dumping factor, with typical values between 0.5 and 2[12]. The relation between flow, head and gate position is determined by the turbine valve characteristic[9]:

$$q = G\sqrt{h} \tag{3}$$

We now need to analyse the dynamical behaviour of the conduit. It can be shown[8] that if fluid compressibility and conduit wall elasticity is considered, the flow through a closed conduit with length L and constant cross section area A produces a pressure perturbation at the turbine admission that can be approximated by

$$\frac{h_p(s)}{q(s)} = \frac{-Z_0[1 - exp(-2T_e s)]}{[1 + exp(-2T_e s)]} \tag{4}$$

where Z_0, the surge impedance of the penstock is defined by

$$Z_0 = \frac{q_R}{Ah_R}\frac{1}{\sqrt{g\alpha}} \tag{5}$$

and T_e, the wave travel time is

$$T_e = \frac{L}{a} \tag{6}$$

In Eq. 5, q_R and h_R are the the base values for flow and head used in the per unit definitions. The water time constant T_W will be defined as

$$T_W = (\frac{L}{A})\frac{q_R}{h_R g} \tag{7}$$

From Eq. (5) and Eq. (6) and from the definition of wave velocity a presented in Tab. 1, it is easy to see that

$$T_W = Z_0 T_e \tag{8}$$

The viscosity of the fluid can be take into account by considering a head loss that is proportional to the square of the instantaneous flow[4][7], as expressed by

$$h_l = f_p q^2 \tag{9}$$

The static water head determined by the water column, the head perturbation determined by flow inside the conduit and the pressure loss due to fluid viscosity combined determine the instantaneous head at the turbine admission[7]:

$$h = 1 - h_l + q\frac{-Z_0[1 - exp(-2T_e s)]}{[1 + exp(-2T_e s)]} \tag{10}$$

The relations expressed in Eq. (1), Eq. (3) and Eq. (10) are summarized in Fig. 2.

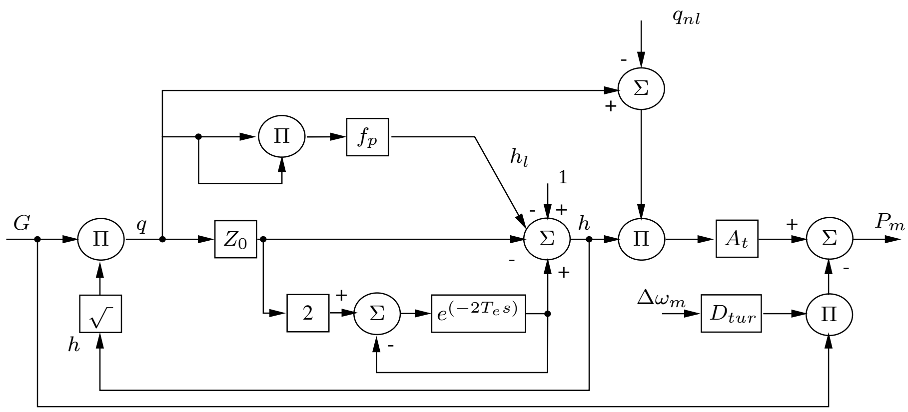

Figure 2. Non-Linear Model of Turbine - Elastic Water Column

Our next goal is to obtain a linear state-space model of the system described by Fig. 2. Neglecting friction losses, linearization and small signal analysis of this system lead us to the block diagram illustrated in Fig. 3. The linearization of equation (4) introduces an approximation for the elasticity of the water column in the system equations.

Making the input vector $\boldsymbol{u} = \begin{bmatrix} G \\ \omega \end{bmatrix}$, state vector $\boldsymbol{x} = q$ and the output vector $\boldsymbol{y} = P_m$, through a straightforward process the system matrices can be shown to be:

$$\mathbf{A} = -\frac{2}{G_0 T_W} \qquad \mathbf{B} = \begin{bmatrix} \frac{2}{G_0 T_W} & 0 \end{bmatrix}$$

$$\mathbf{C} = A_t \begin{bmatrix} \frac{G_0 + 2(q_0 - q_{nl})}{G_0} \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} \frac{-2A_t(q_0 - q_{nl})}{G_0} & -D_{tur}G_0 \end{bmatrix}$$

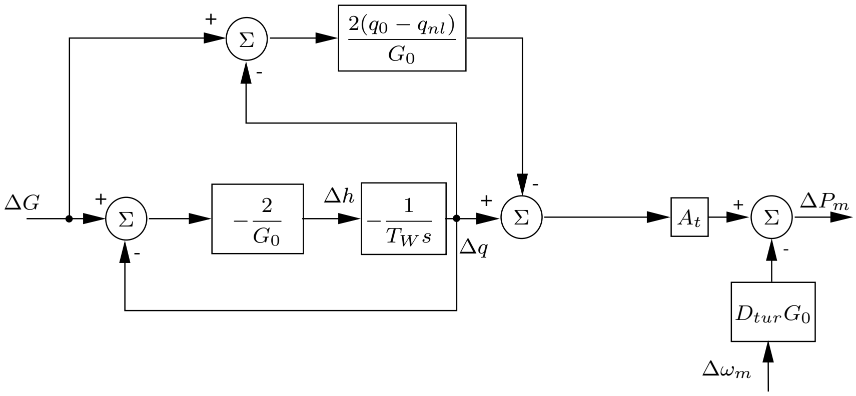

Figure 3. Linear Model of Turbine - Non-elastic Water Column

2.2 Rotor and Load Dynamics

We will now discuss the dynamical elements associated to the rotor and load dynamics. In this paper we will discuss the case of a generator feeding an isolated load. Models for this arrangement are easily found in the literature[3][5][2]. We will use the model illustrated in Fig. 4. The parameters in this model are listed in Table. 2. The same analysis can be performed for other electrical system configuration, like for example a generator paralleled to an infinite bus.

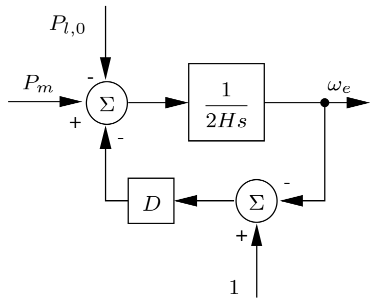

Figure 4. Rotor and Load Model

Parameter	*Definition*
P_m	Per unit instantaneous turbine mechanical power output
P_L	Per unit instantaneous electrical load
H	Rotor inertia constant
D	Electrical load frequency variation constant
ω_m	Per unit rotor angular speed

Table 2. Rotor/Load Model Parameters

With the input vector $\boldsymbol{u} = \begin{bmatrix} P_m \\ P_L \end{bmatrix}$, state vector $\boldsymbol{x} = \omega$ and the output vector $\boldsymbol{y} = \omega$, the system matrices are:

$$\mathbf{A} = -\frac{D}{2H} \qquad \mathbf{B} = \begin{bmatrix} \frac{1}{2H} & -\frac{1}{2H} \end{bmatrix}$$

$$\mathbf{C} = 1 \qquad \mathbf{D} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

3. Simulations and Results

In order to evaluate the models proposed in the previous sections, we will now present a few simulations. We will use the sample system listed in the appendix of reference[9]. The surge tank described will be ignored. We repeat in

Tab. 3 the parameters for this sample system. The gate positioner will be considered as having unitary response with rate of movement and absolute position limits as described in Tab. 3. The electrical load connected to the generator will be considered as purely resistive, i.e., $D = 0$. The governor acting on the gate positioner will be a PID controller tuned accordingly to the guidelines in reference [9].

Parameter	*Value*
Rated Generator MVA	100 MVA
Rated Turbine Power	90.94 MW
Rate Turbine Flow	71.43 m^3/s
Rated Turbine Head	138.9 m
Gate Position at Rated Condition	Rotor speed
No-Load Flow	4.3 m^3/s
Maximum Gate Opening Rate	0.1 pu/s
Maximum Gate Closing Rate	-0.1 pu/s
Maximum Gate Limit	1 pu
Minimum Gate Limit	0 pu
Lake Head	307 m
Tail Head	166.4 m
Penstock Length	465 m
Penstock Cross Section	15.2 m^2
Penstock Wave Velocity	1100 m/s
Penstock Head Loss Coeff.	0.0003042 $m/(m^3/s)^2$
Turbine Damping	0.5 pu/pu
Rotor Inertia Constant	4 s

Table 3. Sample System Parameters

The first experiment performed is a load rejection of 0.2 pu with system initially operating with 50% gate opening. The results are presented in Fig.5.

Clearly, for this level of disturbance, the linearized model approximates very well the behaviour of the non-linear model that incorporates fluid compressibility and viscosity and conduit walls elasticity. The only remarkable difference between the two simulations is the overpressure at the turbine input. The overpressure calculated in the non-linear simulation is 0.2 p.u. higher than the same overpressure obtained with the linear model. This however is expected, as we have seen that the pressure waves inside the conduit are caused by fluid compressibility and conduit walls elasticity. When the model is linearized, an error is introduced on the calculations of pressure variations due to these non-linear phenomena.

Nevertheless, as can be seen in the mechanical power output plot, both linear and non-linear models are successful in reproducing an interesting characteristic of a hydraulic turbine, its inverse power response. Figure 6 shows the results of the same experiment of Fig. 5 with a different time scale. It can be seen that when the wicket gates starts to close, mechanical power output of turbine actually rises. At first, this would seem inconsistent, as a smaller opening of the wicket gates mean less flow trough turbine and consequently less power. However, as stated in Eq. (4), flow variations produce head variations with opposite signal. While the flow varies with the rate determined by the wicket gates positioner, the head varies with rates determined by the conduit physical parameters. As can bee seen in Fig. 6, head raises faster than the movement of the wicket gates. The sudden increase in head is responsible for the inverse initial response of the turbine. As the head initial transitory response fades out with time, the power output starts to decrease, taking the turbine to a direct response region of its dynamical behaviour.

In order to evaluate the fidelity of the linear model for large disturbances, a new experiment has been performed. Starting with gates fully open, a total load rejection on the unit is simulated. Results are presented in Fig. 7

It is clear from the results that the linear model fails to represent the dynamics of the system when large excursion of the systems variables are considered. While mechanical power response is reasonably well approximated by the linear model, head and speed responses differ a lot from one model to the other. Specially interesting is the head response curve. After a full load rejection and with the gate position totally closed, poorly attenuated traveling pressure waves develop inside the conduit. This is a very well known phenomenon when dealing with fluid flow inside a closed pipe and is commonly called water hammer. The linear model is successful in representing the low frequency response of this pressure wave inside the conduit. It fails however to correctly describe the high frequency oscillation created by the sudden restriction to flow. As the gate position is near the full closed position, mechanical output of the turbine is close to zero, and the high-frequency effects do not affect turbine speed control. However, as the pressure oscillations can have serious effects on the penstock structure itself, the high-frequency effects cannot be ignored when analyzing system

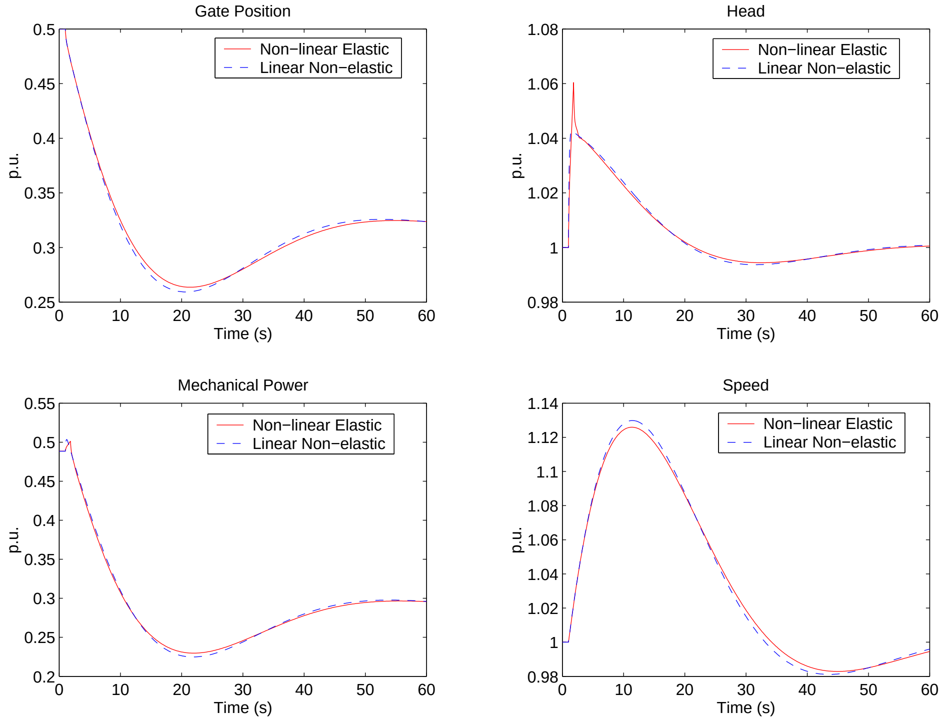

Figure 5. Simulation - 0.2 p.u. Load Rejection

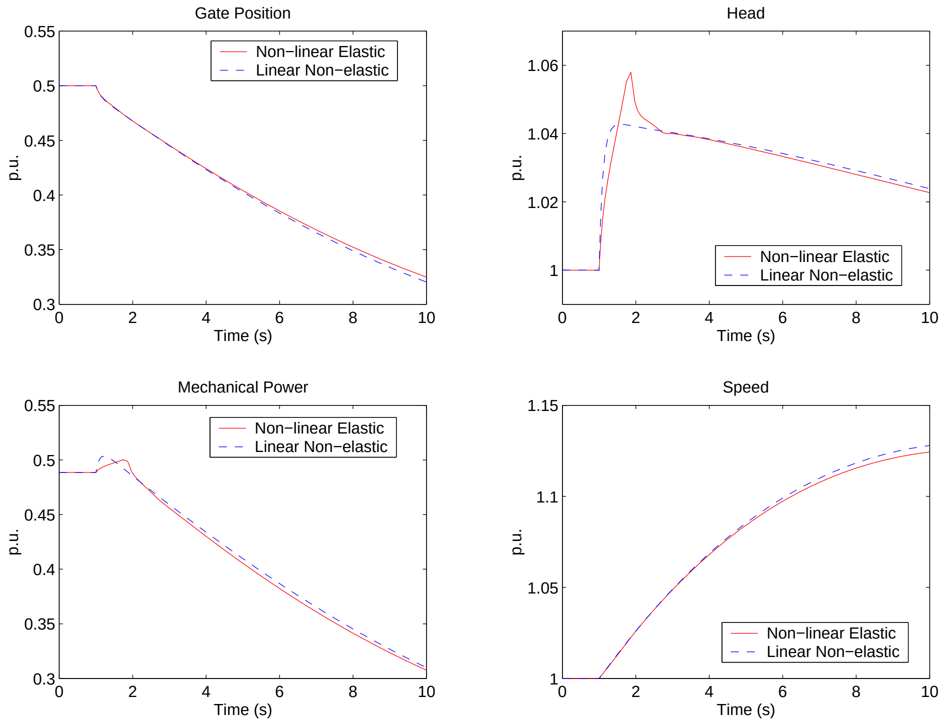

Figure 6. Simulation - 0.2 p.u. Load Rejection Detail

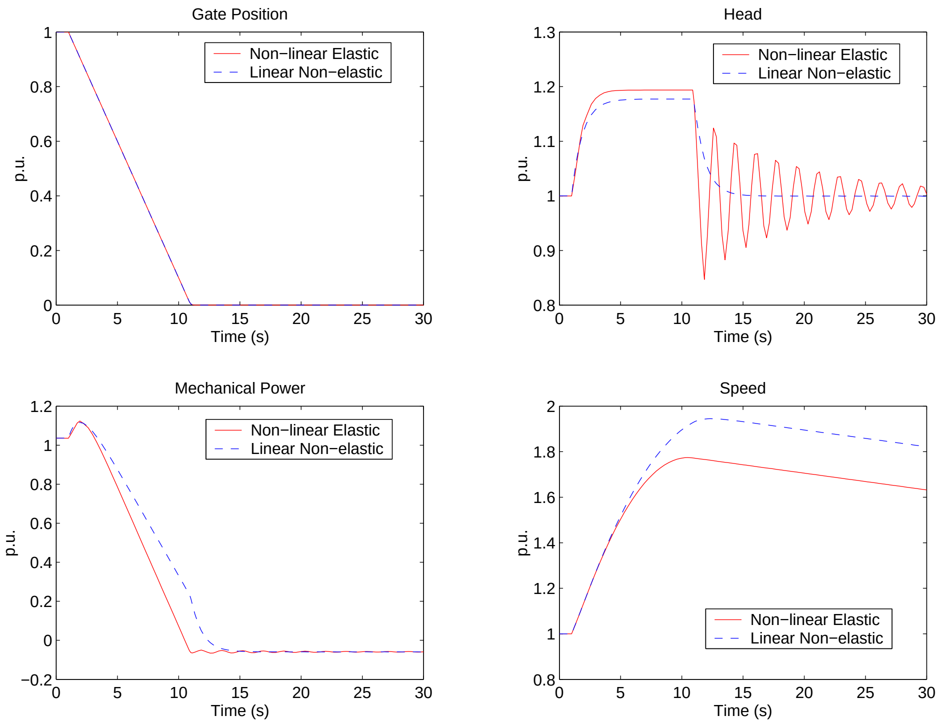

Figure 7. Simulation - Full Load Rejection

behaviour over a wide operational range.

4. Conclusion

From the physical equations describing a Francis hydro turbine-generator set, both a non-linear and a linear model have been constructed. From the simulations presented, it is clear that there are differences between the results obtained with each model. While both models lead to practically the same results when small disturbances are introduced to the system, the linear model is not adequate for calculations where large disturbances are present, or when the wicket gate position is close to zero. In this case, the differences are significative and important behaviour of system variables is simply missed.

This however does not mean linear models are not adequate for use in model predictive control. Better results for calculations involving large disturbances can be obtained using a successive linearization rather than a one-time linearization approach. Also, some implementations of a model predictive control algorithm require more than one model of the system[13]. While the more precise and computational complex non-linear model can be used as an observation model of the system, used to reconstruct system states from the observations, the simpler linear model can be used in the heavy calculations involved in determining the next control output.

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