A TURBULENCE MODEL FOR THE PREDICTION OF THE HEAT TRANSFER REDUCTION OF VISCOELASTIC FLUID FLOWS

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Abstract. *A recently developed viscoelastic k-epsilon turbulence model is used to analyze the reduction of the heat transfer coefficient of viscoelastic fluid flow. The new turbulence model is able to link fluid rheology and turbulence and constitutes a reliable approach to the prediction of turbulent flow of drag reducing fluids. Here, the momentum equations are numerically solved for a fully developed pipe flow and the flow field solution is used to solve the thermal energy equation, in order to obtain the heat transfer coefficient along the developing boundary-layer. The numerical calculations of momentum compare favorably with experimental data, whereas the heat transfer simulations do show a significant heat transfer reduction in relation to that of Newtonian fluid, indicating that the new formulation is able to predict adequately both drag and heat transfer reductions in pipe flows.*

Keywords*: heat transfer reduction, viscoelastic, pipe flow.*

1. Introduction

Drag reduction is a fascinating subject that occurs in turbulent flow of some dilute polymer solutions and surfactant solutions, but is still poorly understood. One of the theories for explaining drag reduction in non-Newtonian fluids relates the suppression of turbulent fluctuations with some elastic properties (Lumley, 1977; Virk, 1975), especially the normal stresses.

More recently, experimental and DNS investigations have confirmed the role of fluid elasticity in drag reduction (Massah and Hanratty, 1997 and Warholic *et al.*, 1999), especially the effects of polymer extensibility and enhanced elongational viscosity (Sureshkumar *et al.*, 1997 and Dimitropoulos *et al.*, 1998) but so far no single physical model has clearly emerged for explaining drag reduction. Still, as a possible cause for drag reduction an enhanced elongational viscosity has been quoted more often at least since the nineteen seventies (Lumley, 1977). Other tentative theories, such as stress or viscosity anisotropy, have been less successful and the corresponding results confusing. For instance, the numerical investigations of Den Toonder *et al.* (1997) has predicted higher drag reductions for anisotropic purely viscous models than for anisotropic viscoelastic fluids, in contradiction with experimental findings.

In parallel with the reduction in turbulent momentum transport those fluids also exhibit a reduction in convective heat transfer. Although both phenomena (drag and heat transfer reduction) are important in many branches of chemical and mechanical engineering industries, the heat transfer has been far less investigated than the corresponding fluid dynamic problem (Matthys, 1991). This can be partially explained, considering that heat transfer experiments are in general more complex to perform accurately, but also because developments in predicting turbulent drag reduction with polymer solutions are required prior to attempting to predict heat transfer with the same fluids. The non-existence of a widely accepted turbulence model for drag reducing fluids has been the greatest obstacle to theoretical and numerical investigations on their heat transfer characteristics.

The major problem in the description of turbulent flow characteristics of viscoelastic fluids, is the correct inclusion of rheological parameters of the fluid in order to make it as general as possible. In the late nineteen seventies, some authors (Mizushina, 1973, Durst and Rastogi, 1977 and Hassid and Poreh 1975, 1977, 1978) used experimental results of pipe drag reduction to adjusts the constants, wall functions and damping functions in standard and low Reynolds number turbulence models and then were able to predict the characteristics of the same flows. However they could not successfully predict the behavior of the same fluids in other flows or in pipe flows of different diameter and, as far as we are aware, there was no further progress in deriving appropriate single-point closures. Since then most of the research in the field, based on DNS simulations of elastic fluids, has been directed at understanding the molecular configurations and corresponding fluid properties (De Angelis *et al.*, 2002).

Without knowing the correct physics behind drag reduction and the correct rheological constitutive equation for a given drag reducing fluid, which at present nobody really knows for sure, it will not be possible to arrive at the correct turbulence models. However, it is the duty of the engineer to improve the current situation since there is clearly the need to further advances in single-point closures for drag reducing fluids. These must include some, if not all, of the assumed relevant fluid rheological properties, and the best candidate is a strain-thickening elongational viscosity.

Recently, Pinho (2003) and Cruz and Pinho (2003) proposed a new turbulence model for drag reducing fluids that was developed from a Generalised Newtonian Fluid, and is based on the classical, low Reynolds number k-ε model. The GNF constitutive model was modified to mimic some of the elastic fluid properties that are held responsible for drag reduction, namely enhanced elongational viscosity and a viscoelastic damping function was introduced. So far, the results have been quite successful and, consequently, in the present work this turbulence model is extended to deal with the heat transfer of a drag reducing fluid and its capabilities, in terms of heat transfer performance, are assessed. As part of the solution, the thermal energy equation for turbulent flow will be solved for a thermally developing pipe flow, but under the conditions of fully-developed hydrodynamic flow, i.e., corresponding to high Prandtl number condition.

2. The Constitutive Equation

As a first step, it is necessary to consider a constitutive equation for the viscosity. An algebraic form for the viscosity function can be a Bird-Carreau type of equation containing a shear-rate dependent term that is multiplied by a strain-rate dependent term. While the former gives the appropriate variation of the shear-viscosity, the latter mimics some of the strain-thickening effects that are held responsible for drag reduction.

$$\mu = \mu_0 \left[1 + \left(\lambda_s \dot{\gamma}\right)^2\right]^{\frac{n-1}{2}} \left[1 + \left(\lambda_e \dot{\varepsilon}\right)^2\right]^{\frac{p-1}{2}} \tag{1}$$

However, for simplicity in the derivation of the turbulence model, a power law based equation was preferred (Pinho, 2002) and, consequently, the same form is adopted here in Eq. 2.

$$\mu = \mu_v(\dot{\gamma})\mu_e(\dot{\varepsilon}) = K_v \left[(\dot{\gamma})^2\right]^{\frac{n-1}{2}} K_e \left[(\dot{\varepsilon})^2\right]^{\frac{p-1}{2}} \tag{2}$$

In Eq. (2) the viscometric function $\mu_v(\dot{\gamma})$ is obtained by fitting the viscometric viscosity η_v with a power law whereas the non-dimensional elastic function $\mu_e = 1/3\,\eta_e(\dot{\varepsilon})/\eta_v(\dot{\gamma})$ at $\dot{\gamma} = \sqrt{3}\dot{\varepsilon}$ in order to respect the limiting physical behavior.

For this Generalized Newtonian fluid it is now necessary to derive the corresponding conservation equations, bearing in mind that there are turbulent fluctuations in the viscosity, because of its dependence on the flow kinematics.

3. The Turbulence Model

The details of the derivation of the transport equations are presented in Pinho (2003) and here only the main features of the model are presented. The mean flow conservation equation for the GNF fluid, with the two-dimensional boundary layer simplifications, is:

$$\frac{\partial \rho U^2}{\partial x} + \frac{1}{r}\frac{\partial \rho r U V}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\left(\overline{\mu}\frac{\partial U}{\partial r} - \rho\,\overline{uv}\right)\right) \tag{3}$$

It is important to note here that a mean value of the molecular viscosity must be used, which is defined in Pinho (2003), since the viscosity functions no longer linear.

Since this average molecular viscosity was derived with high Reynolds number arguments, it must be corrected in the vicinity of a wall to where f_v is a molecular viscosity damping function. In this model, this damping function is made equal to f_μ.

$$\overline{\mu} = f_v \overline{\mu}_h + (1 - f_v) K_v \left[(\dot{\gamma})^2\right]^{\frac{n-1}{2}} \tag{4}$$

To determine the shear stress the Boussinesq approximation is invoked by which

$$-\rho\overline{uv} = \rho \nu_t \frac{\partial U}{\partial x} \tag{5}$$

The eddy diffusivity ν_t is given by the Prandtl - Kolmogorov equation, which is modified for low Reynolds number effects with the damping function f_μ:

$$\nu_t = C_\mu f_\mu \frac{k^2}{\widetilde{\varepsilon}} \tag{6}$$

In Eq. (6) k stands for the turbulence kinetic energy and $\widetilde{\varepsilon}$ is the modified rate of dissipation of turbulent kinetic energy which is used here as in most near wall, low Reynolds number k-ε models (Patel *et al.*, 1985). It is related to the true rate of dissipation of turbulent kinetic energy ε by $\varepsilon = \widetilde{\varepsilon} + D$, where D takes a specific form for each turbulence model.

The turbulence kinetic energy equation is not deeply affected by the new definition of the viscosity and it can be written as follows, with the boundary layer simplifications:

$$\frac{\partial Uk}{\partial x} + \frac{1}{r}\frac{\partial rVk}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\left(\frac{\overline{\mu}}{\rho} + \frac{\nu_t}{\sigma_k} \right)\frac{\partial k}{\partial r} \right) + P - \varepsilon \tag{7}$$

For the modified rate of dissipation of turbulence kinetic energy the transport equation is:

$$\frac{\partial U\widetilde{\varepsilon}}{\partial x} + \frac{1}{r}\frac{\partial rV\widetilde{\varepsilon}}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\left(\overline{\nu} + \frac{\nu_t}{\sigma_\varepsilon} \right)\frac{\partial \widetilde{\varepsilon}}{\partial r} \right) + f_1 C_{\varepsilon 1}\frac{\widetilde{\varepsilon}}{k}P - f_2 C_{\varepsilon 2}\frac{\widetilde{\varepsilon}^2}{k} + E + \frac{\nu_t}{\sigma_E \overline{\nu}}\frac{\partial \widetilde{\varepsilon}}{\partial r}\frac{\partial \overline{\nu}}{\partial r} \tag{8}$$

In Eqs. (8) and (9) we adopted a modified version of the model proposed by Nagano and Hishida (1987), with the kinematic viscosity ν substituted by the average kinematic viscosity $\overline{\nu}$ and with a different damping function f_μ. The various viscous extra terms and damping functions take the following form:

$$D = 2\overline{\nu}\left(\frac{\partial \sqrt{k}}{\partial y} \right)^2 \tag{9}$$

$$E = \overline{\nu}\nu_t\left(1 - f_\mu\right)\left(\frac{\partial^2 U}{\partial y^2} \right)^2 \tag{10}$$

$$f_1 = 1 \text{ and } f_2 = 1 - 0.3e^{-R_T^2} \tag{11}$$

$$R_T = k^2 / \overline{\nu}\widetilde{\varepsilon} \tag{12}$$

The coefficients were taken form Nagano and Hishida's model that are basically those of the standard model and are listed in Tab. 1.

Table 1. Values of the parameters assigned to Nagano and Hishida's low Reynolds k-ε model.

C_μ	σ_k	σ_ε	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$
0.09	1.0	1.3	1.45	1.9

The damping function f_μ has to be modified in order to consider the non-Newtonian behavior of the fluids. The complete deduction of the damping function f_μ used here can be found in Cruz and Pinho (2003) and is given by:

$$f_v = f_\mu = \left(1 - \left(1 + \left| \frac{1-n}{1+n} \right| y^+ \right)^{-\left| \frac{1+n}{1-n} \right| \frac{1}{26}} \right) x \left(1 - \left(1 + \left| \frac{p-1}{3-p} \right| y^+ C^{\frac{1-p}{2-p}} \right)^{-\left| \frac{3-p}{p-1} \right| \frac{1}{26}} \right), \tag{13}$$

where

$$y^+ = y\frac{\sqrt{\tau_w/\rho}}{\vec{\nu}} \tag{14}$$

Constant C is equal to 45 and was obtained in Cruz and Pinho (2003) by comparing predictions with experimental data of Escudier *et al.* (1999) for an aqueous solution of 0.125% PAA.

The thermal energy equation adopted here for the turbulent flow simulations is the same used by Durst and Rastogi (1977), which is written as:

$$\frac{\partial UT}{\partial x} = \frac{\partial}{\partial r}\left(\frac{\nu_t}{P_{rt}}\frac{\partial T}{\partial r}\right) + \alpha\left(\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\right) \tag{15}$$

According to the literature (Loulou *et al.*, 1992 and Shin, 1996) the variation of thermal conductivity with shear rate is weak and so, here the thermal properties of the fluid are assumed constant.

4. The Numerical Procedure and Results

The set of Eqs. (3) to (13) were solved numerically using a finite volume formulation to obtain the fully-developed pipe flow solution. Since the flow problem is fully decoupled from the thermal problem, the momentum solution was then used as input to solve the thermal energy balance for the development of the thermal boundary layer, a situation that is physically consistent with a high Prandtl number flow. Here, however, the emphasis is not on investigating the characteristics of a high Prandtl number flow, but on assessing the capabilities of the current turbulence model to predict heat transfer reduction.

Under these conditions the second term on the left-hand-side of Eq. (12) vanishes, and a finite difference formulation was used to compose the following ordinary differential equation:

$$-UT_I = -UT_{I+1} + \Delta x\left(\frac{d}{d\,r}\left(\frac{\nu_t}{P_{rt}}\frac{dT_{I+1}}{d\,r}\right) + \alpha\left(\frac{1}{r}\frac{dT_{I+1}}{dr} + \frac{d^2T_{I+1}}{d\,r^2}\right)\right), \tag{16}$$

which was numerically solved for each axial step x using a finite difference code. The mesh was refined in the near wall region where the temperatures gradients are high. The convergence criterion adopted was 10^{-9}.

Figure 1 compares predictions of the Fanning friction factor with the corresponding experimental measurements of Escudier *et al.* (1999) for an aqueous solution of 0.125% polyacrylamide (PAA). As mentioned above, parameter C was made equal to 45 based on these comparisons, but the value worked equally well when predicting the behavior of other aqueous polymer solutions based on xanthan gum (XG), carboxymethilcellulose sodium salt (CMC) and blend of both.

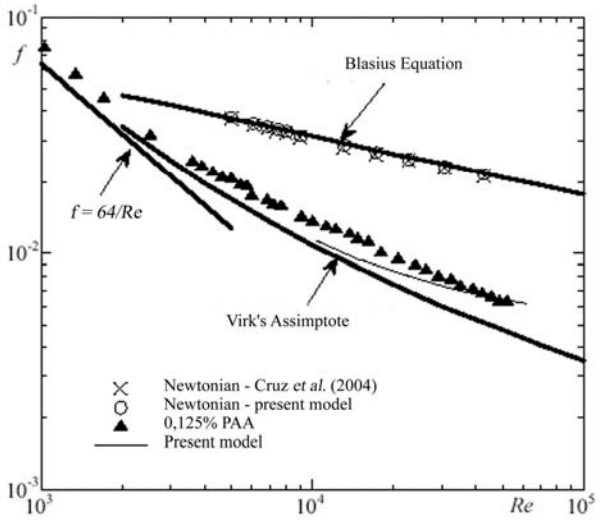

Figure 1. Variation of the friction factor with Reynolds number.

The values of n, p, K_e and K_v, listed in Tab. 2, were obtained from least-square fitting to the shear and elongational viscosity data of 0.125% PAA, presented in the paper of Escudier *et al.* (1999).

Table 2. Parameters of the viscosity model (Eq. 2) for an aqueous solution of 0.125% PAA.

n	p	K_e	K_v
0.425	1.4796	8.25	0.2491

Results of the solution of the thermal energy equation are presented below, for a wall Reynolds number of 31,000 (the Reynolds number is based on the wall viscosity). The thermal diffusivity (α), axial step (Δx), wall temperature (T_{wall}) and inlet temperature (T_{inlet}) used are listed in Tab. 3. First, the thermal behavior of a Newtonian fluid flowing at the same Reynolds number was calculated and is used here as a reference for the comparisons, since there are no experimental data for the heat transfer characteristics of this fluid. Considering that the experimental data are scarce, the thermal diffusion coefficient was estimated considering a constant Prandtl number for all fluids.

Table 3. Thermodynamic and thermal properties of the fluid and numerical parameter

α	Δx	Radius	T_{wall}	T_{inlet}
$1.51 \cdot 10^{-7}$ m^2/s	0.01 m	0.0502 m	313 K	283 K

In Fig. 2, the nondimentional mean temperature derivative (*dTm/dr*) at the wall is shown. It is clear that the de decaying of the wall heat flux for the water is far more intense than for any of the other fluids considered here, indicating a more effective wall heat transfer. This fact demonstrates that the present theory can predict a strong heat transfer reduction, caused by viscoelastic effects.

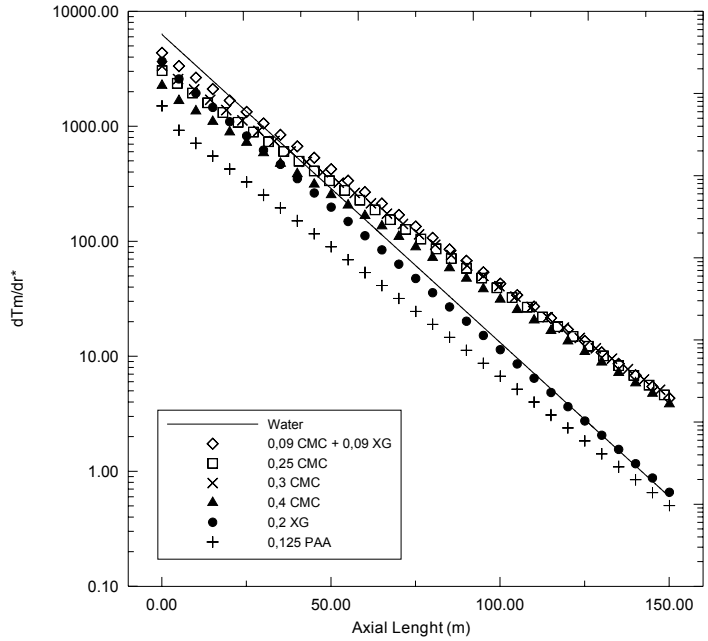

Figure 2. Mean temperature derivative versus the axial length.

Figure 3 show axial variation of the nondimentional temperature profile versus the nondimentional radius for the six non-Newtonian fluids considered in this work.It should be noted that, as for the fluid dynamic case, most of the temperature gradient is concentrated at the wall region, indicating that the correct modeling of the wall region is fundamental for the accurate prediction of heat transfer phenomenon.

An important issue concerning heat transfer of viscoelastic fluids is the true value of the turbulent Prandtl number. It is known (Matthys, 1991) that the amounts of maximum drag and heat transfer reductions are not identical as they should if the Reynolds analogy held. By comparing some predictions with experimental data, Matthys (1991) has suggested that the turbulent Prandtl number for viscoelastic fluids is much higher than for Newtonian fluids. It is important to note, however, that such high values of the turbulent Prandtl number were obtained form modified Newtonian eddy viscosity models, which may not be entirely applicable to viscoelastic turbulent flows, as pointed out by Matthys (1991). Nevertheless, such findings are in agreement with experimental results (Kostic, 1994) that show the heat transfer reduction to be higher than the drag reduction, at least in the asymptotic condition of maximum reductions. Therefore, for turbulent flow of viscoelastic fluids the Reynolds analogy does not work.

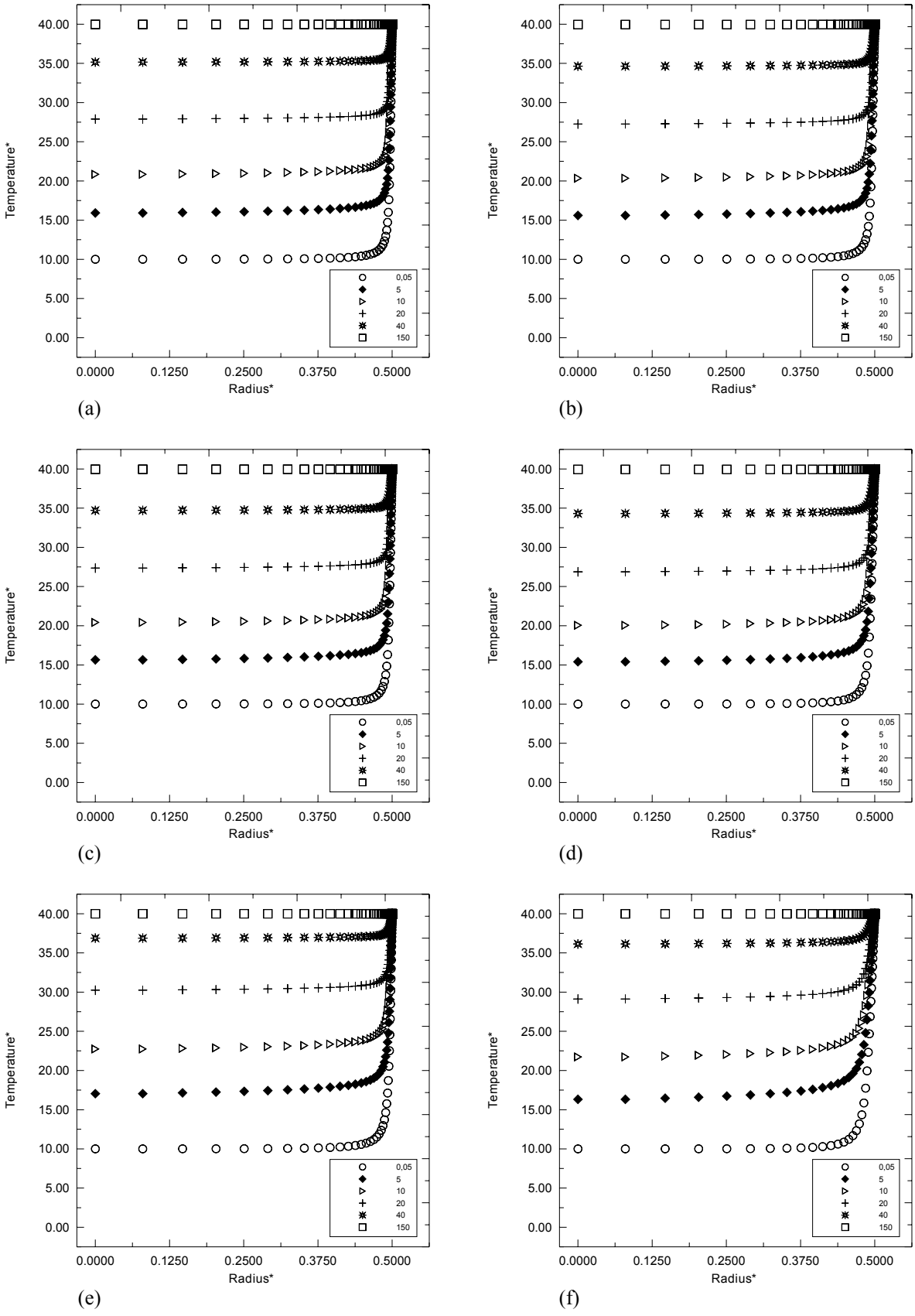

Figure 3. Axial variation of the nondimentional temperature profile versus the nondimentional radius for the six non-Newtonian fluids: (a) 0.09 CMC + 0.09 XG blend; (b) 0.25 CMC; (c) 0.3 CMC; (d) 0.4 CMC; (e) 0.2 XG; (f) 0.125 PAA

To assess the effect of the turbulent Prandtl number (Pr_t) in the current turbulence model, some simulations were carried out of 0.7 to 1.1, these results are listed in Tab. 4 where, U is axial mean velocity, f is the friction factor and $f_{\%}$ is the respective drag reduction. It is clear that the amount of heat transfer reduction is always higher than the drag reduction. It is worth to note that the influence of the turbulent Prandtl is not as important as expected, just a minor variation of the Nusselt is observed as the Pr_t is changed. Since the experimental data are scarce, any assumption concerning the influence of the turbulent Prandtl number on the wall heat transfer should be made with care but, the present results indicate that it has little importance, and that the viscoelastic influence on the turbulent viscosity ν_t, play a major role on the heat transfer reduction. This is not unexpected since the same effects that causes the drag reduction, are the responsible for the heat transfer reduction.

The Nusselt number calculated by the empirical correlation at Eq. (17) (Incropera, 1991) was introduced in Tab. 4 for comparison, using the water molecular Prandtl number (Pr_{water}) at 298 K obtained at the mean between the inlet and the outlet temperature. Although the present formulation furnishes higher values of heat transfer reduction, at least most fluids analyzed here, it is clear that Eq. (17) can also predict strong values of the heat transfer reduction. It is worth to mention that the Eq. (17) was deduced considering the Reynolds analogy hypothesis, and as discussed by Matthys (1991), this analogy, at least as proposed for Newtonian fluid, does not hold for viscoelastic fluids.

$$Nu_D = \frac{(f_{Darcy}/8)(Re_D - 1000)\,Pr}{1 + 12.7(f_{Darcy}/8)^{1/2}(Pr^{2/3} - 1)} \tag{17}$$

Table 4. The influence of the turbulent Prandtl number variation on the Nusselt number and the drag reduction for each fluid

Fluid	$Pr_t = 0.7$		$Pr_t = 0.9$		$Pr_t = 1.1$		$Pr_{water} = 6.146$		U	f_{Darcy}	$f_{\%}$
	$\overline{Nu}_D$	$Nu_{\%}$	$\overline{Nu}_D$	$Nu_{\%}$	$\overline{Nu}_D$	$Nu_{\%}$	Nu_D (1)	$Nu_{\%}$			
Water $\overline{Nu}_D$	226,40	---	211.91	---	200.64	---	203.66	---	1.55	0.023	---
Blend	155.76	31.20	142.86	32.58	133.05	33.69	162.05	20.43	2.21	0.017	27.88
0.2 XG	132.22	41.60	118.70	43.99	108.68	45.83	155.11	23.84	1.73	0.015	32.52
0.3 CMC	117.25	48.21	108.30	48.89	101.47	49.43	148.66	27.00	4.85	0.015	36.28
0.25 CMC	106.39	53.01	98.34	53.59	92.20	54.05	136.65	32.90	4.32	0.013	43.05
0.4 CMC	76.71	66.12	71.00	66.50	66.70	66.76	108.94	46.51	4.60	0.009	58.64
0.125 PAA	47.18	79.16	43.03	79.69	40.07	80.03	84.98	58.27	2.06	0.007	70.02

(1): Nusselt number calculated using the empirical correlation Eq. (17)

5. Conclusion

In the present work a recently proposed turbulent-viscoelastic fluid formulation was used to analyze the heat transfer reduction in viscoleastic pipe flow. The results have shown that the present formulation is capable to predict a strong reduction of the heat transfer coefficient in parallel to a reduction in drag. It was also shown that an increase in the turbulent Prandtl number can increase the heat transfer reduction but has little effect on it, as suggested by experimental data, but further work is necessary to improve the Reynolds flux closure model and to quantify the true value of the turbulent Prandtl number in order to improve the prediction of the heat transfer reductions.

In any case, these results indicate that the present formulation has the potential to be used on the solution of engineering problems because it only requires information on the fluid rheology, thus overcoming the difficulties of previous formulations, which needed some intrinsic flow properties as input.

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