OPTIMIZED DESIGN OF AXIAL-FLOW HYDRAULIC TURBINES

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Abstract. *This work reports the development of a computational methodology for the optimized design of axial-flow hydraulic turbines (propeller turbines), with non-adjustable guide vanes. The methodology has been developed with a quasi-two dimensional flow model, employing empirical correlations for cascade losses and flow deviations. The study is based on the conservation principles for mass and angular momentum and on velocity triangle relations. The methodology has been implemented by means of a computational code written in MATLAB™. The code searches for a basic geometry that maximizes the machine efficiency, given the design flow rate, rotational speed and bounds for the geometric parameters and also for the available head. Application examples of the methodology are presented and discussed for the case of a real turbine model, previously tested in a laboratory rig. According to the proposed hydrodynamic modeling, the code furnishes design solutions close to the initial guessed design but with better efficiencies. Recommendations for the practical use of the program and methodology improvements are also made.*

Keywords: *propeller turbines, loss and deviation modeling, optimization techniques, optimized design.*

1. Introduction

Optimized design methodologies for hydraulic turbines are scarce in the technical literature. Often the designer or analyst has few available options besides those based on complex CFD codes (Peng et al., 2002; Lipej, 2004).

With this motivation in mind, the present work reports the development and test of a computational methodology for optimization of the basic geometry of propeller turbines, given the design flow rate, rotational speed and bounds for the design variables and also for the resulting available head. The optimization process starts from an initial guessed design. The details of this methodology are given in Albuquerque (2004), where a program for analysis (Biruel Jr. and Manzanares Filho, 1999) has been converted into a program for optimized design (POTHA code, in MATLAB™).

The methodology does not account for details of the blade shapes, since this kind of information is not always available or is of difficult assessment. Thus, sophisticated computational methodologies were early rejected. Instead, a quasi two-dimensional flow model was adopted by using velocity triangles in some suitable flow sections of the turbine and applying simplified correlations for loss and flow deviations.

Test examples of the POTHA code are presented in the case of a real propeller turbine (Souza, 1989). For this case, all the necessary information about the geometry and measured performance characteristics is available for comparison.

2. Flow calculation through an axial-flow hydraulic turbine

In this section one presents the necessary mathematical formulation for the numerical simulation of a stationary, incompressible, quasi two-dimensional flow throughout a propeller turbine with non-adjustable vanes. The following basic assumptions are made: 1) incompressible flow; 2) the absolute flow is stationary in the stator (guide vanes); 3) the relative flow is stationary in the rotor (rotating blades); 4) the flow occurs in stream surfaces of revolution, concentric to the machine shaft; 5) the flow in the main flow sections (before and after the stator and before and after the rotor) is axisymmetric, i. e., is characterized by mean variables in each circumference concentric to the machine shaft; 6) the meridian velocities do not change in the direction normal to the stream surfaces; 7) the stream surfaces are cylindrical in the rotor (pure axial rotor).

The general calculation methodology can briefly be described as follows: given the turbine geometry, the flow rate and the rotational speed, one first constructs the velocity triangles at the main flow sections with basis on (i) the principle of mass conservation, (ii) empirical correlations for the flow deviation in turbine cascades; (iii) relations directly obtained from the velocity triangles; (iv) considerations about the maximum deflection capacity of cascades. Thence the power absorbed by the blades is calculated by applying the principle of angular momentum (Euler equation). The hydraulic losses are classified and calculated by means of empirical correlations, as well as the tip clearance and lateral friction losses. In this way, it becomes possible to determine the available head and the efficiency.

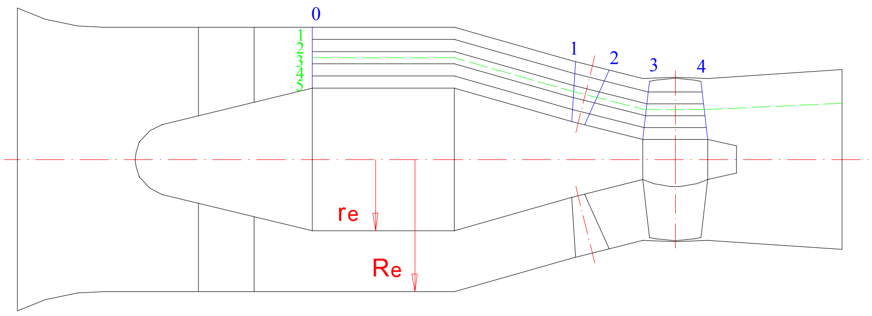

Figure 1. Turbine divided in N =5 elementary turbines
(the mean stream line of the 3th elementary turbine is in green)

In the POTHA code, the optimizer changes the design geometric variables (starting from an initial estimate) and searches for a configuration that maximizes the turbine efficiency, subject to imposed constraints. The basic calculation procedure described above is employed to compute the objective function — represented by the negative of efficiency, for minimization — and also the available head which is constrained to rest below a given superior bound.

2.1. Hydraulic efficiency and available head

The flow domain throughout the turbine is divided into N hypothetical "elementary turbines", defined by N+1 revolution flow surfaces concentric to the machine shaft (Fig. 1). For simplicity, all elementary turbines have a same radial height Δr constant. The i-th elementary turbine is defined by the surfaces i and i+1, i=1,2,...N, with i=1 and i=N+1 corresponding to the shroud and hub surfaces, respectively.

The characteristic properties of each elementary turbine i are evaluated on the mean stream surface between i and i+1. These properties are: a) geometric dimensions of the cascades that intercept the surface (lengths and angles); b) velocities and flow angles on the surface.

The main flow sections receive the following indices: 0 → section representing the inlet of the turbine; 1 → section immediately before the stator; 2 → section immediately after the stator; 3 → section immediately before the rotor; 4 → section immediately after the rotor and before the draft tube.

Once one has determined the power absorbed by the blades, P_{blade}, and the hydraulic loss power throughout the turbine, P_{hyloss}, the hydraulic efficiency, η_h, and the available head, H, are calculated as follows:

$$\eta_h = \frac{P_{blade}}{P_{blade} + P_{hyloss}} \quad ; \quad H = \frac{P_{blade} + P_{hyloss}}{\rho g Q} \tag{1}$$

where ρ is the water density, g is the local acceleration of gravity and Q is the (volumetric) flow rate.

2.2. Blade power

The Euler equation for turbomachines is applied for calculating the specific work done over the blades, Y_{blade}:

$$Y_{blade} = u_3 c_{u3} - u_4 c_{u4} \tag{2}$$

where u is the blade velocity and c_u is the circumferential component of the absolute flow velocity.

The total power absorbed by the rotor blades, P_{blade} (blade power), is calculated by summing up the contributions of all elementary turbines:

$$P_{blade} = \sum Y_{blade} \Delta\dot{m} \quad ; \quad \Delta\dot{m} = \rho c_m 2\pi r \Delta r \tag{3}$$

In the Eq. (3), $\Delta\dot{m}$ represents the mass flow rate in a elementary turbine, c_m is the meridian velocity in the rotor and r is the radius of the mean stream surface of the elementary turbine in the rotor.

2.3. Hydraulic loss power

According to Raabe (1985), only the hydraulic losses in the rotor ($P_{rotorloss}$) and in the draft tube ($P_{drafttubeloss}$) need to be considered in propeller turbines. In this work, however, one also considers the stator losses ($P_{statorloss}$). The sum of these losses represents the hydraulic loss power throughout the turbine, P_{hyloss}:

$$P_{hyloss} = P_{statorloss} + P_{rotorloss} + P_{drafttubeloss} \tag{4}$$

2.3.1. Stator loss power

The stator loss power is calculated by considering the contribution of each elementary turbine as follows:

$$P_{statorloss} = \sum \xi_S c_2^2 / 2\Delta\dot{m} \tag{5}$$

where c_2 is the absolute velocity in the stator cascade outlet and ξ_S is a friction loss coefficient, described in section 2.6.

2.3.2. Rotor loss power

Analogously to the stator, the rotor loss power is calculated by considering the contribution of each elementary turbine. Now, besides the friction loss, the shock loss in the rotor inlet must also be considered, due to the change of the relative flow direction with respect to nominal conditions. It follows that

$$P_{rotorloss} = \sum \left(\xi_R w_4^2 / 2 + \lambda w_{ch}^2 / 2\right)\Delta\dot{m} \tag{6}$$

where w_4 is the relative velocity in the rotor cascade outlet and w_{ch} is the shock component velocity in the rotor inlet, described in section 2.8. The friction loss coefficient ξ_R is calculated analogously to ξ_D. The shock loss coefficient, λ, is an empirical data. Pfleiderer and Petermann (1979) recommend to adopt it between 0.5 and 0.7.

2.3.3. Draft tube loss power

According to Raabe (1985), the draft tube loss is calculated by

$$P_{drafttubeloss} = \sum \left(X_{Dm} c_{m4}^2 / 2 + X_{Du} c_{u4}^2 / 2\right)\Delta\dot{m} \tag{7}$$

where c_{m4} and c_{u4} are respectively the meridian and circumferential components of the absolute velocity in the rotor cascade outlet. X_{Dm} is a coefficient that depends mainly on the geometry of the draft tube, being adopted as about 0.09 for straight draft tubes or 0.12 for elbow draft tubes. X_{Du} is a pressure recovery coefficient, adopted between 0.2 to 0.4.

2.4. Construction of the velocity triangles

2.4.1. Inlet of the turbine

A uniform distribution of pure axial velocity is admitted at the turbine inlet (no pre-rotation of the inlet flow):

$$c_0 = c_{m0} = Q / (\pi (R_e^2 - r_e^2) f_{bloc}) \tag{8}$$

where f_{bloc} is a blockade factor, which is adopted as constant between 0.95 to 1 for propeller turbines.

2.4.2. Stator vanes

The stator vanes are modeled as a set of linear cascades as shown by the simple geometrical arrangement of Figs. 2 and 3. By application of the mass conservation principle between the turbine inlet and a generic transversal section, it is possible to determine the axial component of the velocity at this section. Then, with the cone angle, θ, one calculates the corresponding meridian velocity. The flow deviation with respect to the geometrical camber direction of the vanes in the stator cascade outlet, δ, is calculated by the correlation of Carter and Hughes (Horlock, 1973), as shown in Fig. 3.

$$\delta = \alpha_{2vane} - \alpha_{2flow} = m\phi\sqrt{s/l} \tag{9}$$

where α_{2flow} is the flow angle, α_{2vane} is the vane geometrical angle, ϕ is the camber angle ($\phi = \alpha_{2vane} - \alpha_{1vane}$), l is the vane chord, s is the vane spacing and m is an empirical factor. In Horlock (1973), m is graphically provided as a function of the stagger angle, γ (in degrees), and the kind of camber line (circular or parabolic). This graph was approximated by the linear function $m(\gamma) = 0.12 - 0.05(\gamma/60°)$, for circular camber lines adopted in this work.

From Eq. (9), we can calculate $\alpha_{2flow} = \alpha_{2vane} - \delta$ (which will be adopted as equal to the rotor inlet flow angle, α_3).

2.4.3. Rotor blades

The geometrical model for the rotor is given in Fig.4. The assumption of c_m constant along the radius gives:

$$c_{m3} = c_{m4} = c_m = Q / (\pi (R_{re}^2 - r_r^2) f_{bloc}) \tag{10}$$

The absolute flow angle at rotor inlet, α_3, is considered equal to α_{2flow}. The flow deviation in a rotor cascade outlet is calculated analogously to the stator cascades, by applying the Carter and Hughes correlation with the appropriate

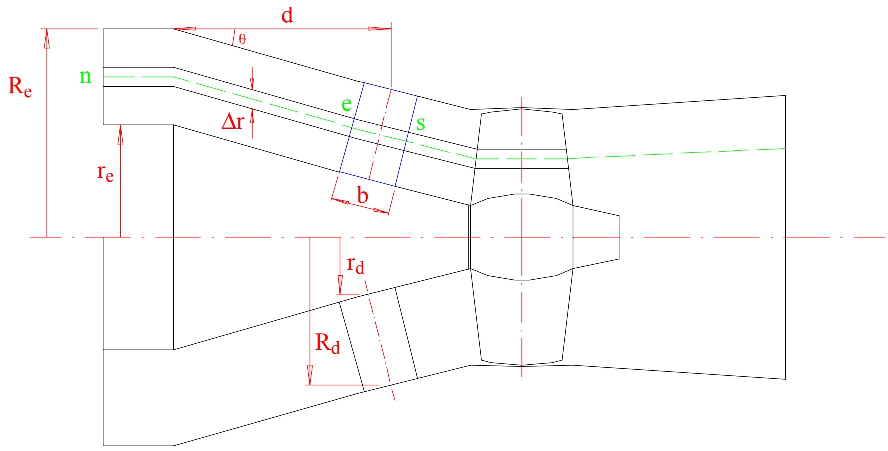

Figure 2. Geometrical model for the turbine stator

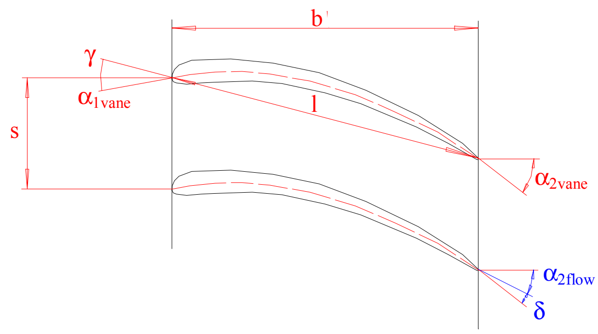

Figure 3. Vane cascade geometry

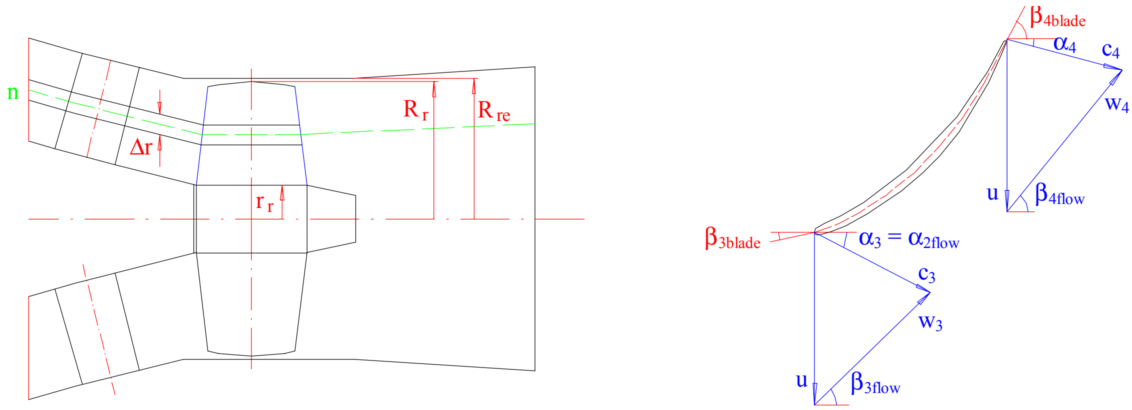

Figure 4. Rotor model and velocity triangles

geometrical relationships for the rotor cascades. It allows us to determine $\beta_{4flow}=\beta_{4blade}-\delta$ and hence all the other elements of the velocity triangles in the rotor cascade outlet (see also the next section).

2.5. Maximum deflection capacity of rotor cascade

The deflection of the relative flow, ε_{blade}, is defined as the difference between the outlet and inlet relative flow angles. Every cascade has a limit for deflecting the relative flow. The prediction of this maximum deflection capacity is not easy. The correlation of Carther and Hughes, for instance, can only be applied near nominal conditions, far away from cascade stall . For rotational speeds below the optimal speed, the deviation increases in relation to the values predicted by the Eq. (9). As a consequence, the specific work, Y_{blade}, is reduced. In this work, this effect is evaluated by a simple procedure, by avoiding the application of extensive airfoil databases (which are not always available in a straightforward manner). The procedure consists in changing the rotor outlet circumferential component, c_{u4}, adding to it a portion proportional to the difference between the nominal and actual rotational speeds, Eq. (11). If this correction is active, the velocity triangles on rotor outlet must be recalculated (the factor κ is adopted equal to 0.30.):

$$c_{u4} = \begin{cases} u - c_m \tan\beta_{4flow} & u \geq u_{nom} \\ (u - c_m \tan\beta^*) + \kappa(u_{nom} - u) & u < u_{nom} \end{cases} \quad ; \quad \beta^* = \beta_{4blade} - \delta \tag{11}$$

2.6. Profile loss coefficient

The Soderberg correlation (Horlock, 1973) is employed to evaluate the friction losses in the stator (ξ_S) and in the rotor (ξ_R). Although it had been developed for gas turbine applications (as the Carter and Hughes correlation for deviation), it is based on loading parameters of general cascades of finite aspect ratio without explicitly involving the Mach number. Thus it can also be applied to hydraulic turbines. The correlation uses a Reynolds number, R_e, defined below, and the cascade aspect ratio B/b (between radial and axial blade lengths). For the vanes, it has de form:

$$R_e = (\rho c_2 D_h)/\mu \quad ; \quad D_h = 2Bs\cos\alpha_2 /(s\cos\alpha_2 + B) \tag{12}$$

The Soderberg correlation is

$$\xi_S = \left(10^5/R_e\right)^{1/4}\left[(1+\xi_1)(0.975+0.075b/B)-1\right] \quad ; \quad \xi_1 = \xi_0 e^{0.01053\cdot\varepsilon} \tag{13}$$

where ε is the deflection (in degrees) and ξ_0 is a constant. The rotor loss coefficient, ξ_R, is computed in a similar way.

2.7. Tip clearance loss. Volumetric leakage

A reaction hydraulic turbine presents a tip clearance flow loss due to the static pressure difference across the rotor. This loss can be treated as a volumetric leakage loss. The leakage flow Q_e, is first computed and used to evaluate a volumetric efficiency, η_e. An empirical correlation typical of radial turbomachines is applied with a procedure similar to orifice flow calculation (Pfleiderer, 1960):

$$Q_e = \xi_e \pi (R_{re}^2 - R_r^2)(w_4^2 - w_3^2)^{1/2} \quad ; \quad \eta_e = (Q - Q_e)/Q \tag{14}$$

where ξ_e is a constant coefficient (Pfleiderer (1960) recommends $0.5 \leq \xi_e \leq 0.7$ for radial turbomachines).

2.8. Optimum rotational speed. Shock components

Manzanares Filho (1994) has developed a pure theoretical methodology for predicting the operating optimum parameters for driven axial-flow turbomachines of impeller/stator type. It can also be shown that the same methodology may be applied to the reverse case, that is, axial-flow turbines of stator / rotor type as considered in this work. The main result is a formula for the optimum flow coefficient, ϕ_{ot}, as a function of the spin coefficient of the flow between stator and rotor, f_e, the hub ratio, ν_i, and the global slip coefficients of stator, ε_S, and rotor, ε_R:

$$\phi_{ot} = \left[\sqrt{\nu^*(f_e p + 16\nu^*)} - 4\nu^*\right]4/p \tag{15}$$

$$p = f_e\left[(1+\varepsilon_S/\varepsilon_R)(16 + f_e^2/\nu_i) - 8\right] \quad ; \quad \nu^* = (1+\nu_i+\nu_i^2)/3 \quad ; \quad \nu_i = r_r/R_{re} \tag{16}$$

$$\phi = c_m/u_e \quad ; \quad f_e = 2c_{ue}/c_m \tag{17}$$

An energetic analysis of the flow in the space between stator and rotor (Manzanares Filho, 1994) suggests that $f_e = 2\nu_i$ is a good choice for nominal conditions. The relation $\varepsilon_S/\varepsilon_R$ must be ≤ 1. It allows the determination of a theoretical optimum circumferential velocity at each elementary turbine, u_{nom}, which is used to evaluate the shock components, w_{ch}:

$$w_{ch} = u - u_{nom} \tag{18}$$

2.9. Lateral friction loss

Although this loss is not well characterized in axial-flow turbines, it can be estimated by an empirical expression for lateral friction loss power of radial turbomachines, P_{lf}, (Mataix, 1975), and so the lateral friction efficiency, η_{lf}:

$$P_{lf} = \xi_{lf}\rho(\pi 2R_r n/60)^3(2R_r)^2 \quad ; \quad \eta_{lf} = (P_{blade} - P_{lf})/P_{blade} \qquad (n \text{ in rpm}) \tag{19}$$

where the coefficient ξ_{lf} is recommended to be about 0.0010 to 0.0012 for radial machines (Mataix, 1975).

2.10. Turbine efficiency and shaft power

Now it is possible to evaluate the turbine total efficiency, η, and hence its shaft power, P_s, as follows:

$$\eta = \eta_h \eta_e \eta_{lf} \eta_m \quad ; \qquad P_s = \eta (P_{blade} + P_{hyloss}) \tag{20}$$

where η_m is the mechanical efficiency, which is adopted in agreement to the turbine charge.

3. The POTHA code

The calculation sequence briefly described in section 2 was programmed in the POTHA code written in MATLAB™. Given the design flow rate and rotational speed and some basic machine dimensions (for instance, hub-to-tip ratio, vane/blade length and spacing), the code searches for the remaining dimensions (for instance, vane/blade angles) in order to maximize the efficiency, subjected to lateral bounds for those variables and to a maximum allowed available head. The code is formed by 3 iterative blocks, as illustrated in Fig. 5: the objective function block, which calculates the turbine efficiency; the block of constraints, which define lateral bounds for the design variables and a maximum limit for the available head; and the optimizer, which automatically varies the design variables and enforces the constraints in order to maximize the efficiency. The present version of the POTHA code uses the `fmincon` function from the MATLAB™ optimization toolbox. This function implements an algorithm to compute a local minimizer (vector $\boldsymbol{x}=(x_1,x_2,...,x_n)$) of a real differentiable nonlinear function (objective function) of the design variables x_i, subject to differentiable nonlinear constraints. It starts from an initial guess $\boldsymbol{x}_0$ and uses a sequential quadratic programming (SQP) technique and the BFGS formula for Hessian estimations. Derivatives can be computed by analytical expressions furnished by the user or by finite differences. This last option was adopted in the POTHA code.

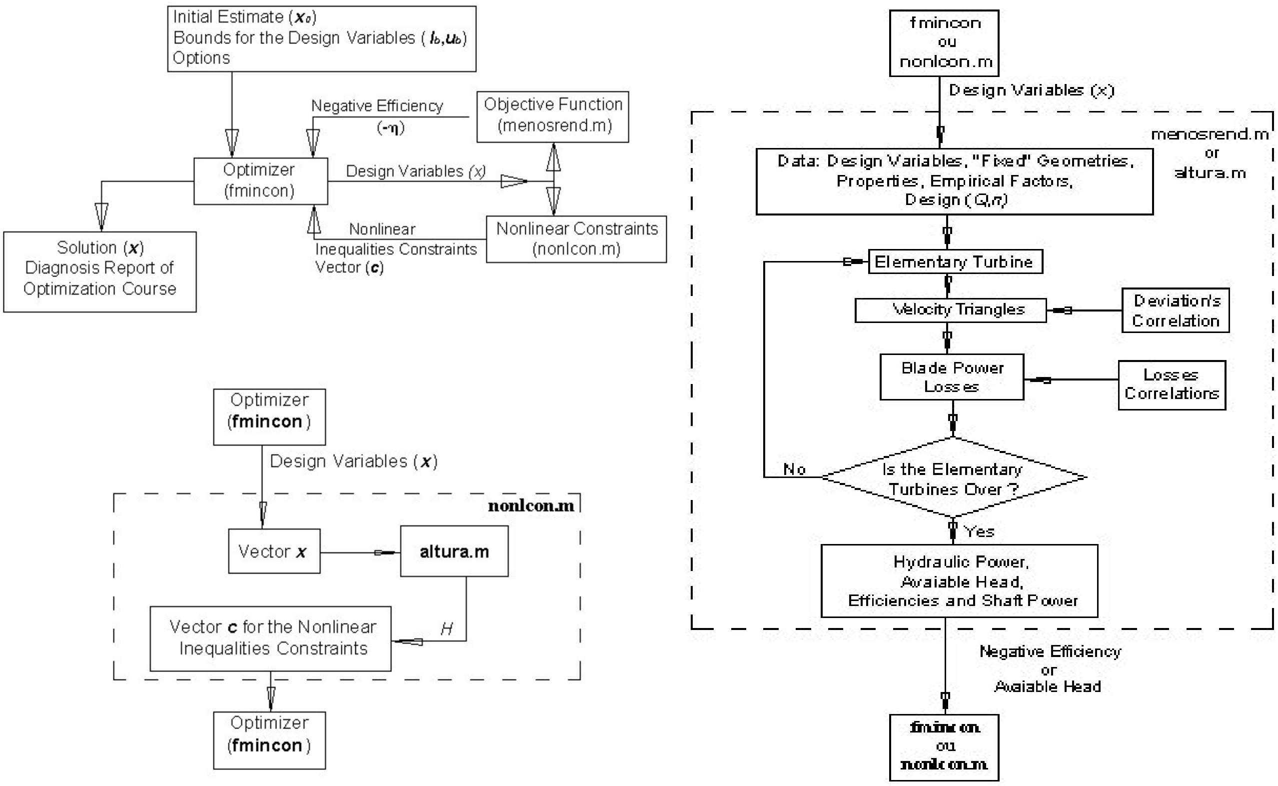

Figure 5. General algorithms of POTHA code (top left), objective function (right) and nonlinear constraints (bottom left)

4. Results and discussion

The POTHA code was applied to a propeller turbine with non-adjustable vanes designed and tested by Souza (1989) at the LHPCH-UNIFEI (Laboratory for Small Hydropower Plants at UNIFEI). It is a small propeller turbine, with internal diameter of 112 mm, external diameter of 280 mm, cylindrical stator ($\theta = 0°$), 8 vanes with 30° of outlet angle, rotor with 4 Göttinguen profile blades. The design point is Q = 0.286 m³/s, H = 4.0 m and n = 1145rpm (n_{qA} = 651). Three test cases are presented, with different sets of design variables. The available head was constrained to $H \leq 4.0$ m.

4.1. Test case 1 (β_{3blade}, $\beta_{4blade}=\beta_{3blade}$)

Design variables: geometric inlet blade angles, β_{3blade} (5 radial stations), and outlet blade angles, β_{4blade}, with $\beta_{4blade}=\beta_{3blade}$ (straight blades). The remaining geometrical data are the fixed (Souza, 1989, or Albuquerque, 2004). The

POTHA code correctly found a solution that reduces α_4 and c_{u4}. It does not produce high flow deflections for obtaining reasonable specific works (Y_{blade}) and also decreases the losses at the draft tube (by decreasing c_{u4}, according to Eq. 7). The radial variation of Y_{blade} obtained by POTHA code from tip to hub (2:1) is less accentuated than the original distribution (5:1). Thus, the code naturally tries to reach a circumferential velocity distribution at rotor inlet closer to the potential vortex distribution, where Y_{blade} is constant along the radius (1:1). A uniform distribution of Y_{blade} agrees with the hypothesis of uniform meridian velocity, but it could never be exactly achieved by a stator with constant outlet angle, as assumed in the present version of the code The principal effects verified in this test are the increase on blade power, the reduction on the draft tube loss and the increase on hydraulic and total efficiencies.

Table 1. Results of Test Case 1 (β_{3blade}, β_{4blade}=β_{3blade})

Design Variable	Original Data				POTHA			
β_{3blade}(1, tip) = β_{4blade}(1)	72.6°				70.3°			
β_{3blade}(2) = β_{4blade}(2)	68.7°				69.8°			
β_{3blade}(3) = β_{4blade}(3)	63.8°				63.1°			
β_{3blade}(4) = β_{4blade}(4)	56.3°				59.8°			
β_{3blade}(5, hub) = β_{4blade}(5)	40.7°				48.2°			
Flow Quantities	Elementary Turbine				Elementary Turbine			
	1	2	3	4	1	2	3	4
α_{2flow} (°)	26.9	27.2	27.5	27.8	26.9	27.2	27.5	27.8
c_{u3} (m/s)	2.92	2.95	2.99	3.03	2.92	2.95	2.99	3.03
c_{u4} (m/s)	-0.35	0.36	0.85	1.73	0.20	0.24	0.26	0.32
α_4 (°)	-3.5	3.6	8.4	16.7	2.0	2.4	2.6	3.2
Y_{blade} (J/kg)	50.7	33.7	22.5	10.4	42.3	35.3	28.6	21.6
η_h (%) ; η (%) ; H (m)	88.5 ; 80.3 ; 3.91				89.0 ; 81.2 ; 4.00			

4.2. Test case 2 (α_{2vane}, β_{3blade}, β_{4blade}=β_{3blade})

Design variables: geometric inlet blade angles, β_{3blade} (5 radial stations), outlet blade angles, β_{4blade}, with β_{4blade}=β_{3blade} (straight blades), and geometric outlet vane angle, α_{2vane}. The POTHA code has now produced a small reduction in the stator flow deflection (α_{2vane} was reduced in 1.4°). This allows the rotor to deflect the flow in order to decrease a little more the angle α_4 as compared with the POTHA solution of test case 1. However, the specific work distribution (Y_{blade}) is essentially the same as that of the previous test.

Table 2. Results of Test Case 2 (α_{2vane}, β_{3blade}, β_{4blade}=β_{3blade})

Design Variable	Original Data				POTHA			
β_{3blade}(1, tip) = β_{4blade}(1)	72.6°				70.4°			
β_{3blade}(2) = β_{4blade}(2)	68.7°				70.1°			
β_{3blade}(3) = β_{4blade}(3)	63.8°				63.3°			
β_{3blade}(4) = β_{4blade}(4)	56.3°				60.1°			
β_{3blade}(5, hub) = β_{4blade}(5)	40.7°				48.5°			
α_{2vane}	30.0°				28.6°			
Flow Quantities	Elementary Turbine				Elementary Turbine			
	1	2	3	4	1	2	3	4
α_{2flow} (°)	26.9	27.2	27.5	27.8	25.7	25.9	26.2	26.5
c_{u3} (m/s)	2.92	2.95	2.99	3.03	2.76	2.79	2.83	2.86
c_{u4} (m/s)	-0.35	0.36	0.85	1.73	0.01	0.08	0.14	0.22
α_4 (°)	-3.5	3.6	8.4	16.7	0.1	0.8	1.4	2.2
Y_{blade} (J/kg)	50.7	33.7	22.5	10.4	42.8	35.3	28.2	21.1
η_h (%) ; η (%) ; H (m)	88.5 ; 80.3 ; 3.91				89.0 ; 81.2 ; 4.00			

4.3. Test case 3 (α_{2vane}, β_{3blade}, β_{4blade})

Design variables: geometric inlet blade angles, β_{3blade} (5 radial stations), outlet blade angles, β_{4blade}, and geometric outlet vane angle, α_{2vane}. The POTHA code achieved a solution for the inlet blades angles very close to the original design. Once again, it has reduced the flow deflection at stator. Slightly cambered blades are now obtained, but the values of the outlet angle, β_{4blade}, are still fair close to the inlet ones, β_{3blade}, except at the hub radius. The small cambered Göttinguen profiles, used in the original design, now appear as a natural design solution.

Table 3. Results of Test Case 3 (α_{2vane}, β_{3blade}, β_{4blade})

Design Variable	Original Data				POTHA			
β_{3blade}(1, tip) ; β_{4blade}(1)	72.6° ; 72.6°				72.4° ; 70.3°			
β_{3blade}(2) ; β_{4blade}(2)	68.7° ; 68.7°				68.8° ; 70.2°			
β_{3blade}(3) ; β_{4blade}(3)	63.8° ; 63.8°				63.7° ; 63.3°			
β_{3blade}(4) ; β_{4blade}(4)	56.3° ; 56.3°				56.6° ; 60.4°			
β_{3blade}(5, hub) ; β_{4blade}(5)	40.7° ; 40.7°				41.3° ; 49.0°			
α_{2vane}	30.0°				28.6°			
Flow Quantities	Elementary Turbine				Elementary Turbine			
	1	2	3	4	1	2	3	4
α_{2flow} (°)	26.9	27.2	27.5	27.8	25.7	25.9	26.2	26.5
c_{u3} (m/s)	2.92	2.95	2.99	3.03	2.76	2.79	2.83	2.86
c_{u4} (m/s)	-0.35	0.36	0.85	1.73	0.01	0.08	0.14	0.22
α_4 (°)	-3.5	3.6	8.4	16.7	0.1	0.8	1.3	2.2
Y_{blade} (J/kg)	50.7	33.7	22.5	10.4	42.8	35.3	28.2	21.1
η_h (%) ; η (%) ; H (m)	88.5 ; 80.3 ; 3.91				89.0 ; 81.2 ; 4.00			

5. Conclusion

The results of the POTHA code have indicated potential improvements on the turbine efficiency in the order of 1%. Such relatively small improvement has occurred because the initial design was already close to the local optimum found by the code. Thus an accurate flow modeling is decisive for the success of the optimization methodology, since it is necessary to correctly predict the efficiency sensitivity in response to small changes in the design variables.

Besides the doubtful accuracy of the empirical correlations, certain limitations of the proposed model can be pointed out: (i) the hypothesis of uniform radial distribution of the meridian velocity c_m is not compatible with the radial flow equilibrium equation, ignored in the present work; (ii) detailed information about the vane/blade shapes is not accounted for; (iii) the adopted optimization algorithm searches for local minimizers only; (iv) it is necessary to furnish a reasonable guess for the initial design variables to achieve a successful optimization process.

Further work should to be carried out in order to overcome these limitations. However, the results already obtained by the POTHA code exhibited a trend that appears coherent with the design practice of axial-flow hydraulic turbines. Thus the developed code has a great potentiality in assisting the preliminary design studies of this kind of turbine.

6. References

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7. Responsibility notice