A NUMERICAL COMPARISON BETWEEN AXISYMMETRIC AND 2-D FINITE VOLUME FORMULATIONS USED IN THE THERMAL ANALYSIS OF OCULAR TISSUES

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Abstract. *In this paper, the analysis of heat transfer in human eyes is done using the Finite Volume Method that was developed for solving two-dimensional model problems with unstructured meshes and later extended to deal with axisymmetric applications. Both formulations use a vertex centered finite volume method implemented using an edge-based data structure. Results of the finite volume thermal analysis in ocular tissues are presented, without the presence of thermal loads, and compared with results obtained using an axisymmetric finite element formulation. Such analysis demonstrates the improvement obtained by using a more realistic axisymmetric formulation. Further, the axisymmetric model will be used in the analysis of the eye with retinal implants.*

Keywords: *finite volume method, axisymmetric, hyperthermia, unstructured meshes*

1. Introduction

Retinitis pigmentosa (RP) and age-related macular degeneration (AMD) are some of the leading causes of blindness in the population. Both involve a degeneration of the photoreceptor cells, rendering the visual system insensitive to light (Peachey & Chow, 1999). An alternative approach towards function restoration of the visual system involves the application of external electrical stimulus. The development of retinal prosthesis is based on this concept (Chow & Chow, 1997; Margalit et al., 2002).

The retinal prosthesis or implants consist of small chips composed by electrodes that create an electrical current which stimulate adjacent areas and so activate the visual system. Electrical stimulation of the retina through injection of current dissipates power and heart. In patients with degenerative retinal disorders, the choriocapillaris, wich normally provides heat dissipation in the retina, is damaged. The heat generated by the electronic sensors can damage the adjacent neuronal tissue and also the implant. Furthermore, the temperature increase can produce an environment suitable to proliferate bacteria that could cause infections (Schwiebert et al., 2002). Energy dissipation and temperatures must be carefully controlled to avoid damages in retina and adjacent tissues that could disturb retinal capillary blood flow. Blood flow perturbations in the retina is a feature of many ocular diseases, including diabetic retinopathy, age related maculapathy and glaucoma (Guan et al., 2003).

Actually, two kind of implants are being developed in United States, Germany and Japan: the epiretinal and the subretinal. These implants substitute different physiological functions. The subretinal substitutes the degenerated photoreceptors cells while the epiretinal stimulate directly the ganglion cells (Margalit et al., 2002). Consequently, the two kinds of chips are implanted in different places. The subretinal is located under the retina's surface, between the pigment epithelium and the photoreceptors cells, while the epiretinal one is fixed in the internal surface of the retina (Margalit et al., 2002; Schwiebert et al., 2002; Zrenner, 2002).

In this paper, the analysis of heat transfer in human eyes is done using an unstructured Finite Volume Method (FVM) that was developed for solving two-dimensional model problems and later extended to deal with axisymmetric applications. When studying complex problems using numerical analysis, bi-dimensional models may become quite limited. On the other hand, a fully three dimensional model may lead to heavy computational requirements which may sometimes turn the analysis unfeasible. In some situations the tri-dimensional modeling of a problem may be avoided due to its axisymmetric characteristic.

Here, the 2-D and axisymmetric FVM solutions are compared through the analysis of the thermal distribution in ocular tissues in the absence of thermal loads. Results of the finite volume thermal analysis in ocular tissues are presented, without the presence of thermal loads, and compared with results obtained using an axisymmetric finite element formulation.

2. Physical-Mathematical Model

The Bioheat Transfer Equation, Eq. (1), governs the physical process here analyzed:

$$\rho c \frac{\partial T}{\partial t} = k_t \nabla^2 T + Q_p + Q_m + Q_E \tag{1}$$

where ρ is the mass density; c is the specific heat; T is the temperature; k_t is the thermal conductivity; Q_E , Q_p and Q_m represents the source (or sink) terms.

In Equation (1), the source term Q_E represents external thermal sources, like lasers or heat dissipated by electronic devices such as retinal implants. The generation term of metabolic heat (Q_m) may be ignored because, generally, it is smaller than the external heat sources (Sturesson & Andersson-Engels, 1995). The term Q_p is a specific term of heat generation due to blood perfusion and represents the convective heat removal from the live tissues by the blood vasculature. The referred term is given by Eq. (2) (Diller, 1982; Charny, 1992):

$$Q_p = \omega \rho_s c_s (T_a - T) \tag{2}$$

in which, ω is the volumetric rate of blood perfusion [m^3 *of blood*/m^3 *of tissue.s*], ρ_s and c_s are, respectively, the mass density and the specific heat of the blood, T_a is the arterial blood temperature and T is the tissue temperature.

The eye is an organ with a few blood vasculatures. However, there is a high blood flow in the sclera/choroid/retina region and the retinal blood flow is mainly distributed within the inner retina (Scott, 1988; Guan et al., 2003). It will be considered only the blood perfusion in the retina. The blood flow in the choroid will be considered as convective heat transfer boundary condition between the retina and the choroid, by the use of an adequate heat transfer coefficient.

The problem described by the Eq. (1) is subjected to boundary and initial conditions. The boundary conditions of interest may be of three different types:

a) Dirichlet boundary condition: prescribed temperature $\overline{T}$ over a portion of the boundary Γ_D.

$$T = \overline{T}, \qquad \text{in } \Gamma_D \text{ X } \mathbf{T} \tag{3}$$

b) Neumann boundary condition: prescribed normal heat flux $\overline{q}_n$ over Γ_N.

$$-q_j n_j = \overline{q}_n, \qquad \text{in } \Gamma_N \text{ X } \mathbf{T} \tag{4}$$

in which n_j are the outward normal direction cosines.

c) Cauchy or Robin boundary condition: mixed boundary condition, in other words, prescribed flux and convection heat transfer over Γ_C.

$$-q_j n_j = \overline{q}_n + \alpha_S (T - T_\infty), \quad \text{in } \Gamma_C \text{ X } \mathbf{T} \tag{5}$$

with α_Γ representing the film coefficient and T_a being the bulk fluid temperature.

The initial distribution of temperature $\overline{T}^i$ is known for an initial time stage t^i, and the initial condition is expressed by:

$$T = \overline{T}^i \qquad \text{in } \Omega \text{ and } t = t^i \tag{6}$$

A detailed description of the mathematical model can be found in Lyra et al (2004a, 2004b).

3. Numerical Modelling

The temperature analysis in human eyes is done using an unstructured Finite Volume Method that was developed for solving two-dimensional model problems (Lyra et al, 2004a) and later extended to deal with axisymmetric applications (Lyra et al, 2004b). Both formulations use a vertex centered finite volume method implemented using an edge-based data structure.

For the 2-D formulation Eq. (1) must be re-written using a Cartesian coordinates system. For the axisymmetric one, it is convenient to re-write Eq. (1) using a cylindrical coordinates system. The Heat Conduction Equations for the 2-D and axisymmetric formulations are given by Eq. (6) and Eq. (7), respectively:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k_t \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_t \frac{\partial T}{\partial y}\right) + Q \tag{6}$$

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(k r \frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + Q \tag{7}$$

in which all the heat source terms are represented by Q.

Applying the Finite Volume Method into Eq. (6) and Eq. (7), integrating over a control volume, the semi-discretes equations obtained for 2-D and axisymmetric formulations are given by Eqs. (8) and (9), respectively:

$$\rho c \frac{d\hat{T}_I}{dt} V_I = -\left(\sum_L C_{IJ_L}^{j} q_{IJ_L}^{j(\Omega^*)} + \sum_L D_{IJ_L}^{j} q_{IJ_L}^{j(\Gamma)} + \sum_{k=1}^{2} \sum_L C_{IJ_L}^{j(R_k)} q_{IJ_L}^{j(\Gamma_I^*)} \right) + Q V_I \tag{8}$$

$$\rho c \frac{d\hat{T}_I}{dt} 2\pi r_C A_I = -\left(\sum_L C_{IJ_L}^{AX(j)} q_{IJ_L}^{j(\Omega^*)} + \sum_L D_{IJ_L}^{AX(j)} q_{IJ_L}^{j(\Gamma)} + \sum_{k=1}^{2} \sum_L C_{IJ_L}^{AX(j)(R_k)} q_{IJ_L}^{j(\Gamma_I^*)} \right) + Q\, 2\pi r_C A_I \tag{9}$$

where V_I denotes the volume around node I, the terms $C_{IJ_L}^{j}$ and $D_{IJ_L}^{j}$ are the 2-D weighting coefficients and the terms $C_{IJ_L}^{AX(j)}$ and $D_{IJ_L}^{AX(j)}$ are the axisymmetric ones, A_I is the cross-sectional area of the control volume associated to node I, and r_C is the centroid radius of the control volume.

The first term of the right hand side of Eq. (8) and Eq. (9) quantifies the flux over the interfaces of the control volume associated to node I. The second term quantifies the flux over the boundary edges, and the third one, the flux over the interfaces of different materials. The terms $q_{IJ_L}^{j(\Omega^*)}$, $q_{IJ_L}^{j(\Gamma)}$ and $q_{IJ_L}^{j(\Gamma_I^*)}$ represent, respectively, the fluxes over the domain, over the external boundary and over the interfaces of different materials.

In order to compute the edges fluxes, it is required to know the nodal fluxes values and, consequently, the values of the gradient of the nodal temperature. The nodal approximations of the gradient for 2-D and axisymmetric formulations are given by Eqs. (10) and (11), respectively:

$$\frac{\partial T_I}{\partial x_j} V_I \cong \sum_L C_{IJ_L}^{j} T_{IJ_L}^{(\Omega)} + \sum_L D_{IJ_L}^{j} T_{IJ_L}^{(\Gamma)} \tag{10}$$

$$\frac{\partial T_I}{\partial x_j} 2\pi r_C A_I \cong \sum_L C_{IJ_L}^{AX(j)} T_{IJ_L}^{(\Omega)} + \sum_L D_{IJ_L}^{AX(j)} T_{IJ_L}^{(\Gamma)} - 2\pi b T_I A_I \tag{11}$$

We can observe that the approximations are almost similar to each other. However, the axisymmetric one presents an additional term on the right hand side, where b is set one for the radial direction and zero for the axial direction.

The time discretization is done by a simple explicit formulation (Euler forward), where the nodes temperature are calculated in terms of the adjacent nodes temperature evaluated at the preview time level.

The time and domain discretization using triangular meshes is described in details by Lyra et al. (2004a) for the 2-D formulation; and by Lyra et al. (2004b) for the axisymmetric one. The discretization includes the treatment of boundary conditions, source terms, and domains with multiple materials.

4. Physical Problem

Some hypothesis adopted to simplify the model analyzed are presented in this section. Figure 1 shows a cross section of the eye.

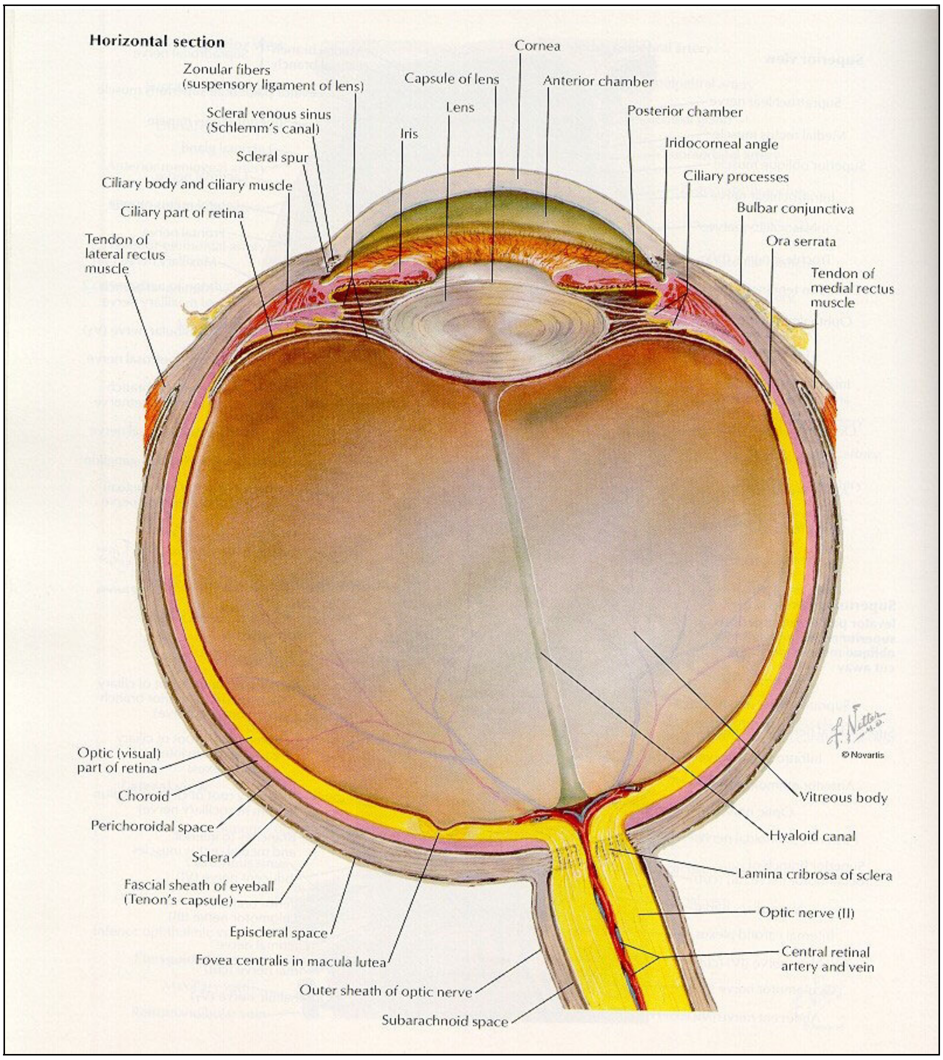

Figure 1 – Cross section of the human eye (Netter & Dalley, 1997).

I. The thermal properties of iris and ciliary body are assumed to be equal to those of the aqueous humor (Amara, 1995), so they are considered as an unique region;
II. Despite the foveae be anatomically different from retina, it will be consider as a constituent part of the retina;
III. Some structures, like nerves, blood vessels, etc, are negleted (Amara, 1995).
IV. The heat transfer between the cornea surface and the external environment at 293 K occurs by convection;
V. The heat transfer in the eye occurs by conduction;
VI. The heat transfer between retina and choroid occurs by convection;
VII. The blood temperature is considered at 310 K;
VIII. The eye is an organ with low blood perfusion, and the perfusion is concentrated at retina/choroid/sclera region. So, only blood perfusion at the retina will be considered

Five regions of different thermal-physical properties will represent the eye (see Figure 2): cornea, aqueous humor, lens, vitreous humor and retina.

The boundary conditions and initial condition are:

I. Heat transfer by convection between cornea surface and external environment;
II. Heat transfer by convection between retina and choroid;
III. Initial temperature of the eye is 309.5 K.

4.1 Thermal Properties and Eye Dimensions

The human eye dimensions are taken from Amara (1995) and Duane (1987). The pupillary axial diameter of a normal eye is 25.4 mm. The cornea thickness varies from 0.4 mm (at the middle) to 0.7 mm (at the boundary) and the horizontal diameter is 11.8 mm. The lens, located between humor aqueous and vitreous, has 4 mm thick and 9 mm of diameter. The retina is 0.2 mm thick near the equator, 0.1 mm deep at fovea and the maximum thickness is 0.5 mm, near the optic nerve. Its minimum thickness (0,05 mm) is located at "*ora serrata*" (vide Fig. 1). The humors are solutante with different NaCl concentrations, and their vertical lengths are 3 mm for aqueous humor and 15 mm for the vitreous humor.

The values of thermal conductivity (k), mass density (ρ) and specific heat (c) are assumed constant within each region of the ocular globe. The thermal properties of the analyzed tissues are described in Table (1).

Table 1 – Thermal properties of eye tissues

Tissue	*k (W/mK)* [(a)]	ρ *(kg/m³)* [(a)]	*c (J/kgK)* [(a)]	ω *(s^{-1})* [(b)]
Cornea	0.580	1050	4178	---
Aqueous humor	0.580	1000	3997	---
Lens	0.400	1050	3000	---
Vitreous humor	0.603	1000	4178	---
Retina	0.628	1000	4190	0.012

(a) Scott, 1988; Amara, 1995; [(b)] Welch et al., 1980.

Lagendijk (in Scott, 1988) estimated the heat transfer coefficient from cornea to environment considering the tear film evaporation and the heat transfer by radiation and convection to environment. The value obtained was $\tilde{h} = 20\ W/m^2K$. Lagendijk (in Scott, 1988) also estimated the heat transfer coefficient from retina to choroid as $h = 65\ W/m^2\,{}^\circ K$. This value includes the heat losses due to blood perfusion in the choroid region.

5. Results and Discussion

The domain of interest is obtained from Fig. 1 using a CAD program. For obtaining this domain, the geometry simplifications discussed in the previous item were considered. Due to the “quasi-symmetric” nature of the problem, a symmetric model was adopted where it is necessary to analyze only a half of the domain. Figure 2 shows the obtained domain and the regions of interest.

In order to discretize arbitrary two-dimensional domains, we used a computational system developed by Lyra & Carvalho (2000) and Carvalho (2001), which can generate triangular, quadrilateral and mixed consistent meshes. In the present work, we use triangular unstructured meshes. The adopted mesh generator, as any conventional unstructured mesh generator, provides the mesh data in an element-based data structure and the implementation of the finite volume solver described requires a pre-processing stage to convert the element-based data structure into an edge-based one. After the pre-processor stage is finished, the element-based data structure can be discarded. After this pre-processing stage, the edge-based data structure and the physical properties are provided to a FVM computational solver. The temperature profile and the damage function historic values are obtained from the FVM computational tool (Silva, 2004).

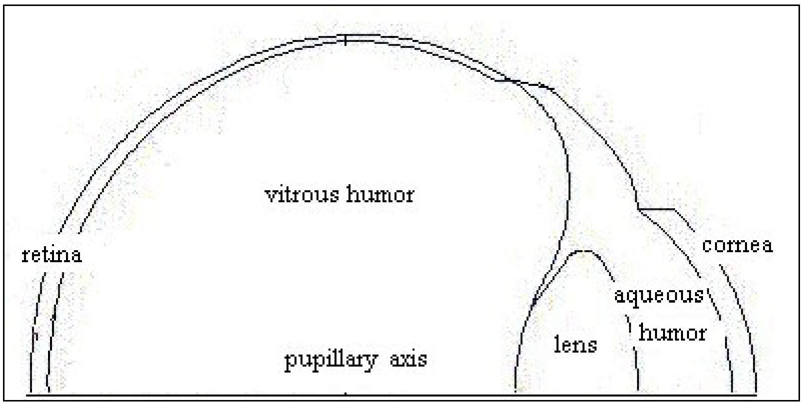

Figure 2 – Domain extracted from Fig. 1 using a CAD program.

5.1 Numerical analysis of the eye without external heat source

The heat transfer in the eye is analyzed by the two formulations: 2-D and axisymmetric. Both results are compared with results obtained using an axisymmetric finite element formulation (Amara, 1995). For the numerical solution, it was done a convergence study for both formulations. The 2-D formulation converged results were obtained using an unsctructured triangular mesh with 4,814 elements and 2,506 nodes. For the axisymmetric case, the converged solution was computed using an unstructured mesh with 1,736 elements and 929 nodes.

Figure 3 shows the temperature profiles along the pupillary axis including a comparison between the 2-D, axisymmetric formulation and the data obtained from literature (Amara, 1995). Considering the axisymmetric finite element formulation as a reference, the axisymmetric FV formulation gives better results when compared with the 2-D one as expected. The relative error in axisymmetric case varies from less than 0.01% to 0.05%, except at cornea’s surface where the error is 12%. For 2-D formulation, the relative error varies from 0.1% to 20.3%, and at cornea's surface the relative error is 16% (greater than the obtained in the axisymmetric case).

Figure 4 shows the temperature contours for the axisymmetric case.

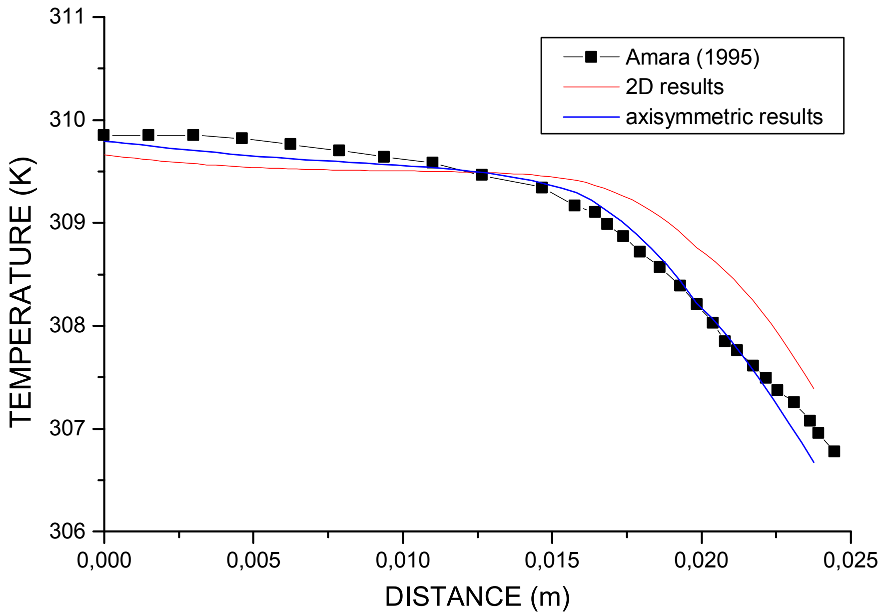

Figure 3 – Temperature profiles along the pupillary axe.

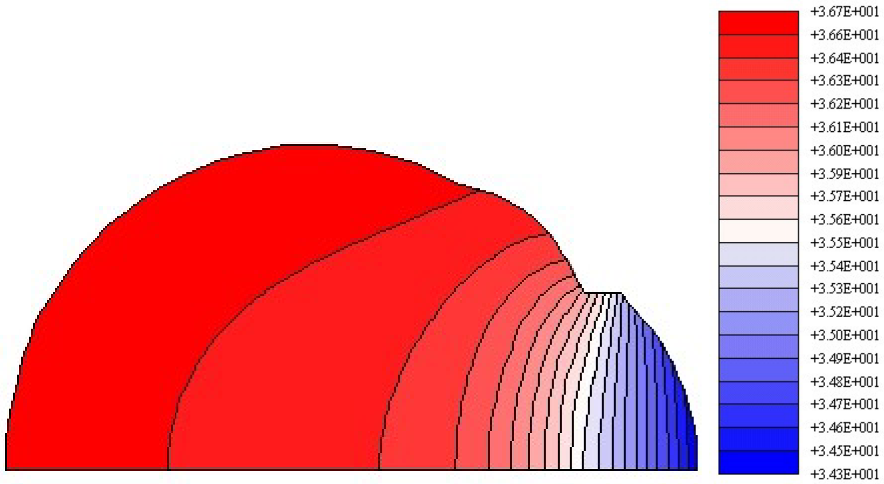

Figure 4 – Temperature contours for axisymmetric case.

6. Conclusions

The new technology of retinal implants has motivated the present work. Although, basic investigations in animals are in course presently, the implants are being used simultaneously in blind people and in animals. At first, it was necessary validate the computational tool and determine which formulation, axisymmetric or 2-D, presents more realistic results. The present paper made a comparison between the two finite volume formulations with an axisymmetric finite element formulation available in literature. As expected, we conclude that the axisymmetric formulation presented better results than 2-D one.

With the present results a continuation of this work is in course now. A thermal analysis of the eye when subjected to an internal heat source due to energy dissipated by retinal implants.

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