MODAL ANALYSIS OF HYDROGENERATORS USING CONDENSATION TECHNIQUES

Fernando Herzog
Centrais Elétricas do Norte do Brasil S.A – ELETRONORTE
SCN Quadra 6, Conjunto A, Bloco B, Sala 1102. DF – Brazil, CEP: 70718-900.
fernandoh@eln.gov.br

Alberto C. G. C. Diniz
Universidade de Brasília, Faculdade de Tecnologia, Departamento de Engenharia Mecânica. DF - Brazil, CEP: 70910 - 900.
adiniz@unb.br

Abstract: Currently, the discrete models constructed with the objective to estimate or to analyze the behavior of structures, employ very frequently the finite element method witch use a great number of degrees of freedom. In spite of the growth of computer power, the size of such models in the modal analysis or the calculation of the frequency response can be prohibitive and surpass the power of these computers, no matter how powerful they can be. With the aim at minimizing the size of the problem, approximate solutions were developed using the principle of cinematic reduction, which is to determine a satisfactory approximate solution in the complete space of the problem, using a subspace comprised of a reduced base of the space of solution. In this paper, the Guyan-Irons method is applied to the modal analysis of a hydrogenerator. The influence of the number of degrees of freedom used in the calculation of frequencies and mode shapes of a hydrogenerator is analyzed. Such procedure is used with the objective of determining an optimum methodology in modeling complex structures without loss of quality in the representation of their dynamic behavior.

Keywords: Reduction techniques, static condensation, modal analysis, rotordynamics, hydraulic turbine

1. Introduction

Model reduction is one of the important subjects in the dynamic analysis of flexible structures. A model with a large number of degrees of freedom, useful in a static analysis, cause numerical difficulties, uncertainties and high computational costs when used to study the dynamic behavior of a structure. In spite of the growth of computer power, the size of the models in the modal analysis or the calculation of the frequency response can be prohibitive and surpass the power of these computers, no matter how powerful they can be.

With the aim at minimizing the problem size, reduction techniques were developed reducing the order of the system, limiting the study to a restrict number of coordinates and/or to certain modes, important to solve the problem. In general way, theses techniques use the principle of cinematic reduction, which is to determine a satisfactory approximate solution in the complete space of the problem, using a subspace comprised of a reduced base of the space of solution. However, the reduction techniques usually employed, such as Guyan-Irons reduction (Guyan, 1965 and Irons, 1965), introduce some errors into the calculation of the reduced mass and stiffness matrices. The Improved Reduced System (IRS) (Callahan, 1989), makes use of Guyan-Irons method to obtain an estimation of the reduced system matrices compensating the inertia effect which are ignored in a Guyan-Iron process. Another process, the System Equivalent Reduction Expansion Process (SEREP) (Callahan, et al., 1989), was developed with the objective to preserve the frequencies and modes shape for a specific set of modes. Another technique is the dynamic condensation, which makes use of transformation matrix, which incorporates information about the dynamic stiffness matrix.

2. Techniques of Reduction of Models of Finite Elements

The Finite Elements Method is extensively used in the project and analysis of structures. However when it has great models with many degrees of freedom it becomes expensive. In order to reduce the number of degrees of freedom (DOF) sub-structuring and condensation techniques are used. These techniques reduce the models size and, consequently, the time and the cost of calculation, without loss of the quality of the results inside of the bands of industrial frequencies of interest.

2.1. Guyan-Irons Condensation Method

Developed simultaneously and independently by Guyan (1965) and Irons (1965) the static condensation is possibly the most popular and simple method of reduction. It is a truncation process of degrees of freedom (reduction) with the objective to obtain a saving in computational cost. The Guyan-Irons Method is a static condensation method where we use a procedure to eliminate incognito of a system of equations, substituting in other equations of the system.

The vectors of force f and displacement u and the matrices of mass M and stiffness K are separate in sub-vectors and sub-matrices related to the degrees of freedom "masters" r, that are restrained, and to the slave degrees of freedom c, that are condensed. If no force is applied to the master DOFs, the equation of the movement of the structure becomes:

$$\begin{bmatrix} M_{cc} & M_{cr} \\ M_{rc} & M_{rr} \end{bmatrix} . \begin{Bmatrix} \ddot{u}_c \\ \ddot{u}_r \end{Bmatrix} + \begin{bmatrix} K_{cc} & K_{cr} \\ K_{rc} & K_{rr} \end{bmatrix} . \begin{Bmatrix} u_c \\ u_r \end{Bmatrix} = \begin{Bmatrix} f_c \\ 0 \end{Bmatrix} \quad (1)$$

Ignoring the terms of inertia for the second set of equations (static condensation), we have:

$$K_{rc} u_c + K_{rr} u_r = 0 \quad (2)$$

This can be used to eliminate the slave degrees of freedom so that:

$$\begin{Bmatrix} u_c \\ u_r \end{Bmatrix} = \begin{bmatrix} I \\ -K_{rr}^{-1} \quad K_{rc} \end{bmatrix} . \{u_c\} = T_s . u_c \quad (3)$$

Where T_S is the static transformation between the vector of displacements and the master co-ordinates. The reduced mass and stiffness matrices are then given by:

$$M_R = T_s^T \quad M \quad T_s \qquad\qquad K_R = T_s^T \quad K \quad T_s \quad (4)$$

Where M_R and K_R are the reduced mass and stiffness matrices. Any response functions in frequency generated by the reduced matrices of the equation above are accurate only with frequency zero. The measure that excitement frequency increases, the ignored terms of inertia become more significant (Guyan, 1965).

2.2.2. Dynamic reduction

The static reduction method can be modified to reproduce the exact response of the structure to any frequency. This is an extension of the Guyan´s method although does not exist obvious choice for the frequency of the accurate reply. The equations above are modified to include the forces of inertia to a chosen frequency w_0. The transformation to generate the slave degrees of freedom from the master degrees of freedom is then given by:

$$\begin{Bmatrix} u_c \\ u_r \end{Bmatrix} = \begin{bmatrix} I \\ \left(K_{rr} - w_0^2 M_{rr}\right)\left(K_{rc} - w_0^2 M_{rc}\right) \end{bmatrix} . \{u_c\} = T_D . u_c \quad (5)$$

The dynamic transformation matrices then is used in the same way as the static transformation, Ts, to get reduced mass matrices and stiffness. Notice that if the frequency ω_0 is zero then this method is equivalent to the static reduction.

2.2.3. Improved Reduced System (IRS)

O'Callahan (1989) established this technique that became an advance in the method of static reduction. It supplies a perturbation to the transformation from static case, therefore includes the inertia terms as the pseudo-static forces. The transformation, T_i, used to generate the slave coordinates from the master coordinates are related to follow:

$$T_i = T_r + S \quad M \quad T_r \quad M_R^{-1} \quad K_R \qquad where \qquad S = \begin{bmatrix} 0 & 0 \\ 0 & K_{rr}^{-1} \end{bmatrix} \quad (6)$$

and M_R and K_R is the reduced mass and the stiffness matrices obtained for the static reduction.

2.2.4. System Equivalent of the Process of Reduction and Expansion (SEREP)

The SEREP uses the calculated eigenvectors to produce the transformation between the master coordinates and the slaves. The analytical eigenvectors are sprayed inside of the master and slave coordinates such that, $fi = [fi_c \quad fi_r]^T$ and the generalized inversion of $_c$ is used to give the transformation. The number of master DOF is greater that of slave, as:

$$T_u = \begin{bmatrix} fi_c \\ fi_r \end{bmatrix} \quad fi_c^{+} \qquad where \qquad fi_c^{+} = \left(fi_c^T \quad fi_c\right)^{-1} \quad fi_c^{I} \quad (7)$$

The reduced mass and stiffness matrices may be obtained in a similar way to equation (4). With this method, the reduced model will exactly reproduce the lower natural frequencies of the full model.

2.3. Methods of Direct Comparison in the Modes of Vibration

Aiming the comparison of mode shapes obtained using Guyan-Irons method and them obtained by a reference solution, without reduction, we used two techniques: the MAC and COMAC criterions, which are presented afterwards.

2.3.1 Modal Assurance Criterion (MAC)

Allemang and Brown (1982) had defined the Modal Assurance Criterion (MAC) to test if two modes of vibration obtained by different methods are similar.

When two equal modes are correlated through the MAC, the index of correlation between them will have to be a number next to 1. Already the distinct modes, present indices next to zero. The MAC is defined by the following expression:

$$MAC(x_k, y_k) = \frac{\left(\left| x_k . y_k \right| \right)^2}{\left(x^T{}_k . x_k . y^T{}_k . y_k \right)} \tag{8}$$

Where: x is the set of vibrations modes using the method “A” (experimental, for example) and y is the of vibrations modes using another method “B”.

2.3.2 Coordinate Modal Assurance Criterion (COMAC)

Wolff and Richardson (1989) had observed the utility of this index for localization of damages in structures. Lieven and Ewins (1988) defined the COMAC (Coordinate Modal Assurance Criterion) index. The COMAC, follows the same begins basic of the MAC, being that it is differentiated of this in the fact of being a prompt measure. This index already was conceived for localization of damages in structures for its authors.

Index COMAC measures the alteration between vibrations mode of a structure with and without damage in one same point. Its value is between limits 0 and 1, representing 0 the complete discord between the measured modes in the same point, and 1, the complete accord of modes. The COMAC is defined by the following expression:

$$MAC(i) = \frac{\sum_{k=1}^{N\ modes} \left(\left| x_{ki} . y_{ki} \right| \right)^2}{\sum_{k=1}^{N\ modes} x_{ki}^2 \sum_{k=1}^{N\ modes} y_{ki}^2} \tag{9}$$

3. Results

Diverse simulations, considering different numbers of restrained DOFs, had been made, with the objective of determine the influence of the number of restrained degrees of freedom in the Guyan-Irons method on the analyze modal of a complex structures. As application example a hydrogenerator of the Coaracy Nunes Hydropower Station of the ELETRONORTE was adopted. The turbine-generator was modeled using the finite elements code ANSYS.

The obtained results, using the Guyan-Irons method in the study of the Coaracy Nunes hydrogenerators, are presented and compared to a reference result, using the Subspace method in ANSYS. The model of the hydrogenerators (Figure 1) has 24,444 degrees of freedom and was constructed using solid elements with four nodes and six degrees of freedom for nodes.

Afterwards, the hidrogenerator frequencies and modes shapes are presented and the influence of the restrained DOF number in these results is evaluated.

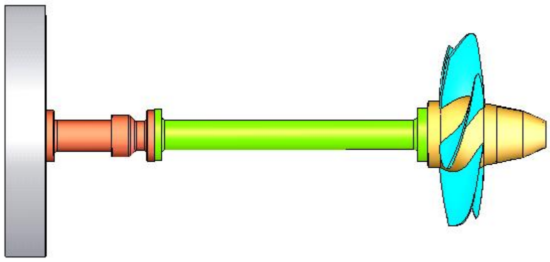

Figure1. Model of Hydrogenerator

The solutions using the method of Guyan-Irons had been determined for the following numbers of master DOFs: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 30, 40, 50, 100, 130, 150, 170, 200, 300, 400, 500, 1000, 1500, 2000.

The obtained results for the ten first modes, using the method of subspace, are shown in Figure 2. In this figure the used frequencies and modal forms can be seen as reference for the comparisons with the method of Guyan-Irons.

The Figure 3 shows the evolution of ten first natural frequencies of the Coaracy Nunes hydrogenerators considering different amounts of master DOFs. The finite elements method is based on the Rayleigh-Ritz method, which supplies the upper bonds eigenvalues calculated by a finite number of equations, which explaining the decreasing behavior of the curves in Figure 3.

Which internally uses a Jacobi interaction, the subspace method in Ansys uses all modes between –1.0 Hz and 10^8 Hz.

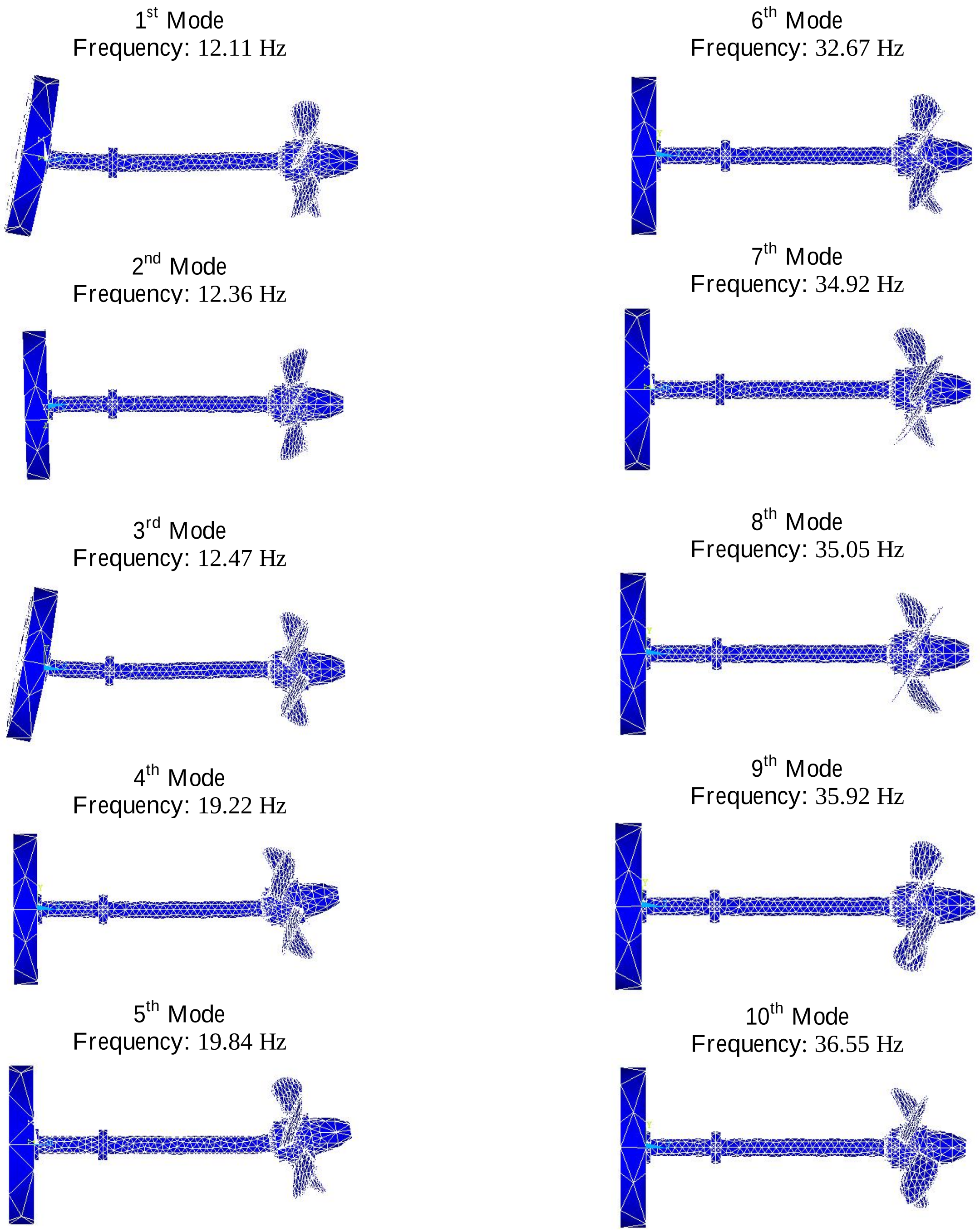

Figure 2. The obtained results for the ten first modes of the Coaracy Nunes hydrogenerators.

In Figure 4 and 5 are presented the “zoom" of the curves in Figure 3, together with the percentile difference between the obtained results, using the Guyan-Irons Condensation, and the Subspace Method, only for the first, fourth, sixth and tenth mode.

We can see in these figures, as for the other modes, that with a number of master DOFs higher than 100 modes, the solution using Guyan-Irons Condensation, presents a difference from the Subspace solution of approximately 0.1%.

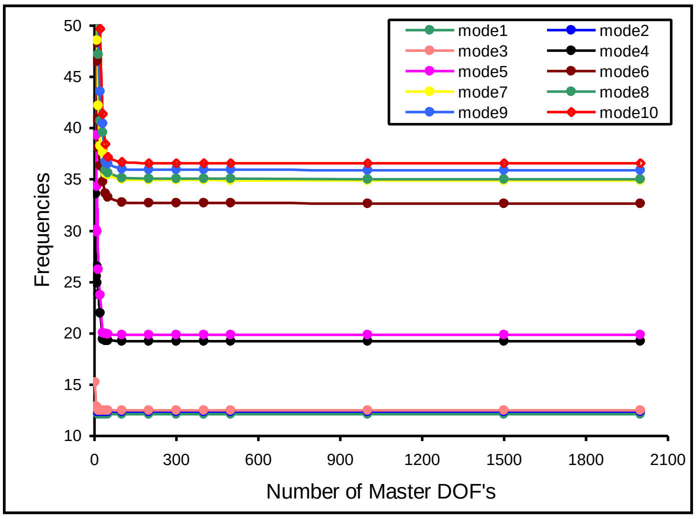

Figure 3. Frequency Evolution for all Modes

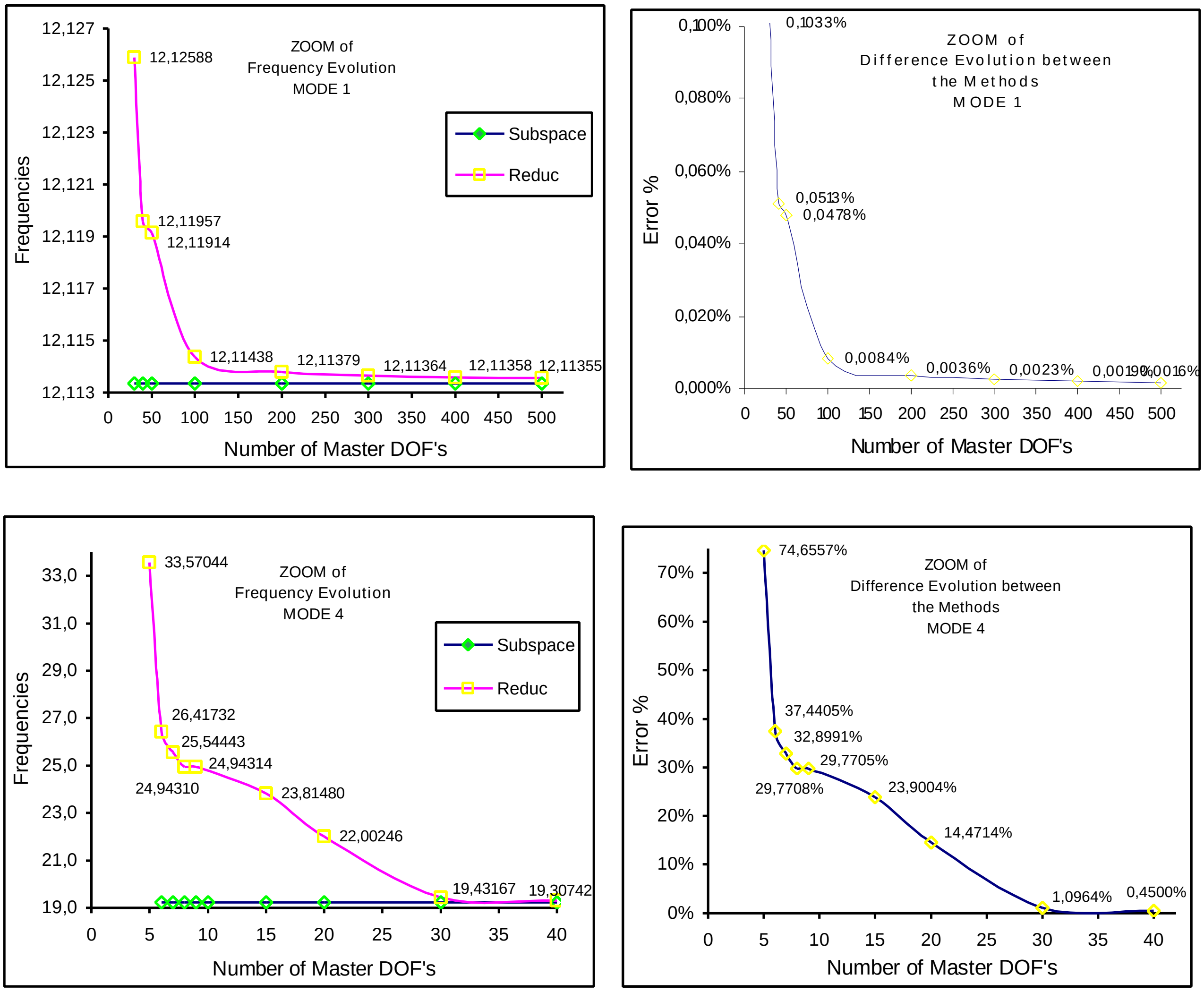

Figure 4. Evolution of the Frequency and of the Difference between the Methods for Modes 1 and 4.

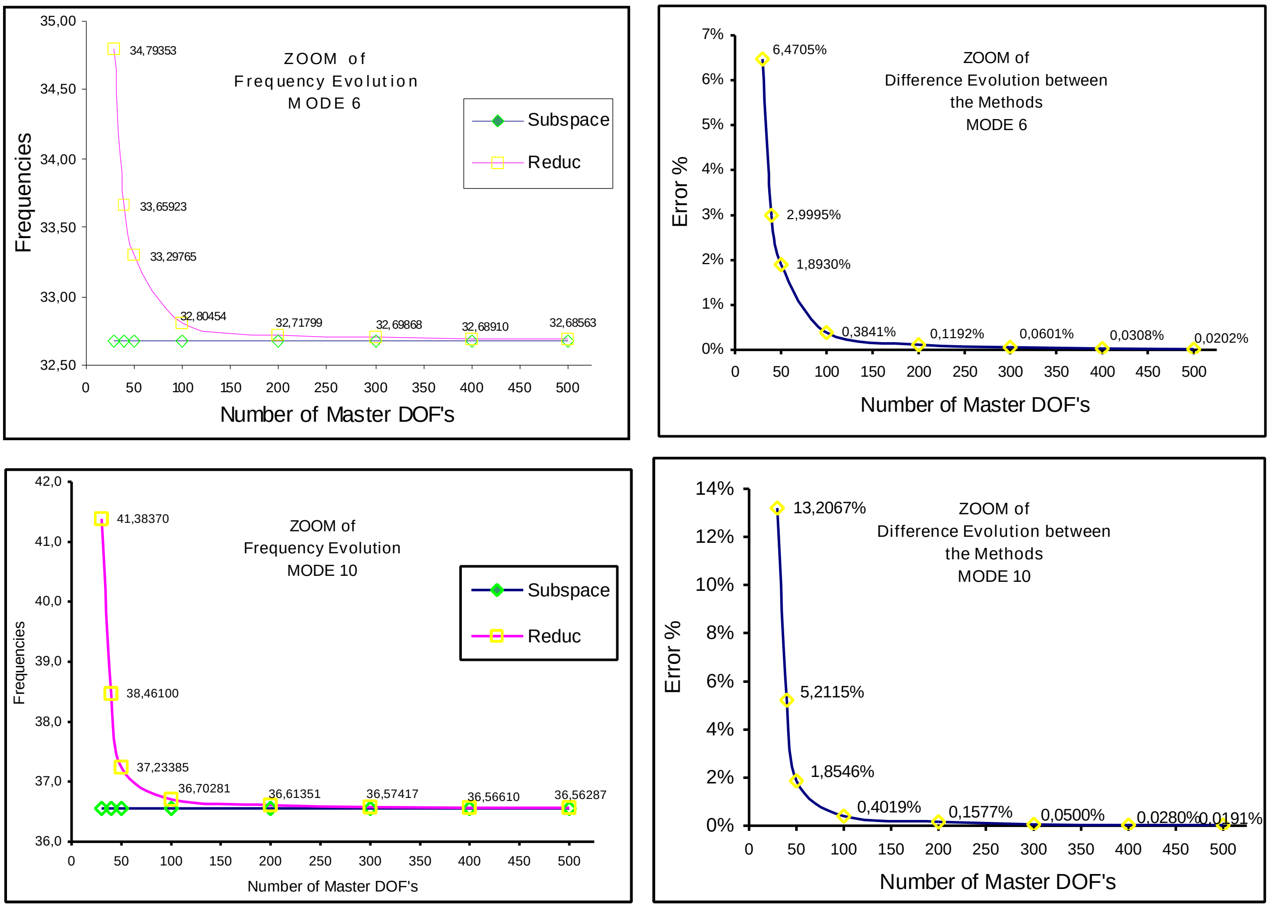

Figure 5. Evolution of the Frequency and of the Difference between the Methods for Modes 6 and 10.

The quality for the obtained modes, using a limited subgroup of equations, was compared with the full solution of the Subspace Method, calculating the matrix of MAC. This comparison is shown in Figure 6, 7, 8 and 9, which presents the matrix of MAC between the solution using the Guyan-Irons Condensation and the solution using Subspace Method for the 1st, 4th, 6th and 10th modes.

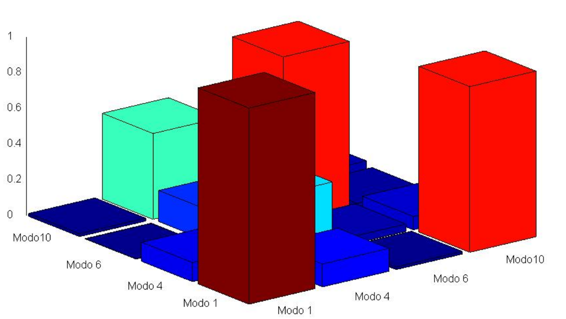

Figure 6. MAC Matrix for 10 Master DOFs

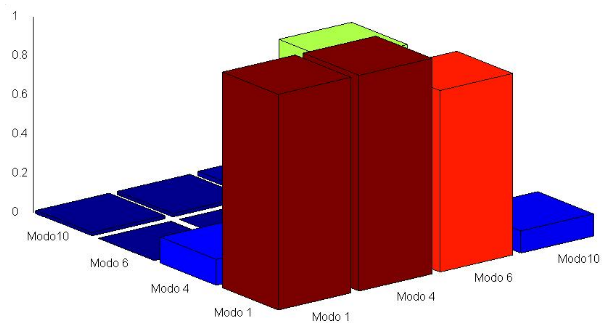

Figure 7. MAC Matrix for 20 Master DOFs

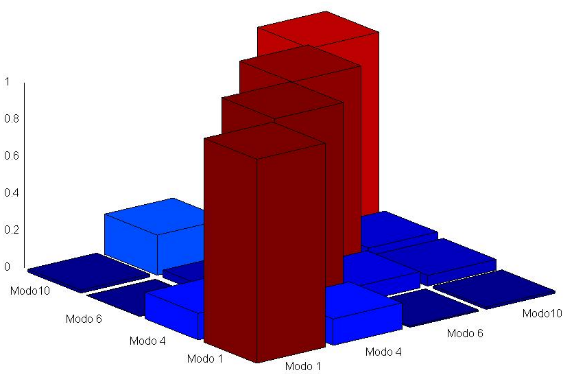

Figure 8. MAC Matrix for 30 Master DOFs

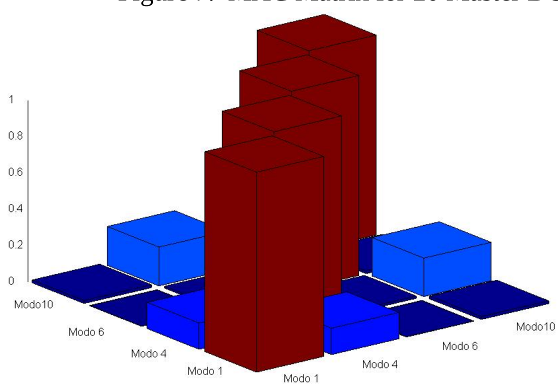

Figure 9. MAC Matrix for 130 Master DOFs

The Figure 9 shows that, to a number of restrained DOFs higher than 130, the results for both methods are identical. We can observe also that the Guyan-Irons Condensation, in Figure 6 and 7, using 10 or 20 master DOFs, doesn't allow a good representation of the hydrogenerator modes. The solution using 30 modes (Figure 8) allows a good representation for the 1st, 4th and 6th 1modes, but it's not enough for the representation of the higher frequency modes.

The Figure 10 shows the MAC coefficient evolution for the 1st, 4th, 6th and 10th for different quantities of master DOFs in the Guyan-Irons Condensation. We can observe, in this figure, that for 100 master DOFs the value of the MAC approaches to one, indicating that with this number of master DOFs we have already a representation of the vibration modes with same quality of the full solution using Subspace method.

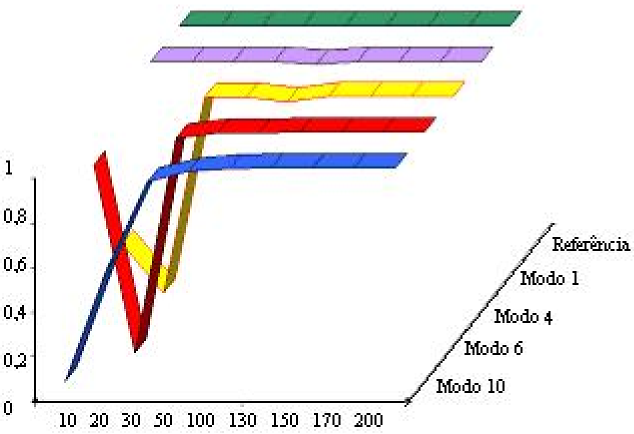

Figure 10. MAC coefficient evolution.

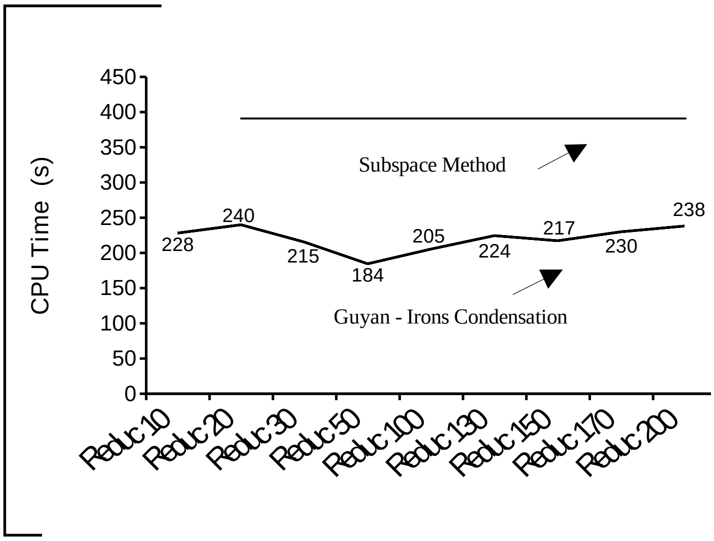

Figure 11. CPU time reduction.

With the objective to verify the representation quality of the local modes, using the Guyan-Irons Condensation, was calculated the COMAC coefficient between the two methodologies, also using different numbers of master DOFs for the Guyan-Irons Condensation. The COMAC coefficient allows identifying where the Guyan-Irons Condensation doesn't obtain the correct representation of the hydrogenerator local vibration shapes. As in the matrix MAC case, can we observe, in Figure 7, that with 100 master DOFs, we obtain already a representation with the same quality that for the solution using Subspace Method.

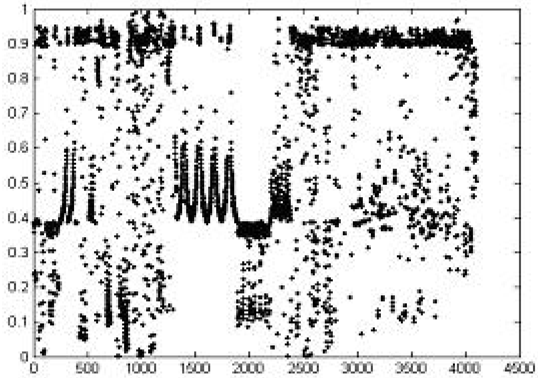

Figure 12. COMAC for 10 Master DOFs

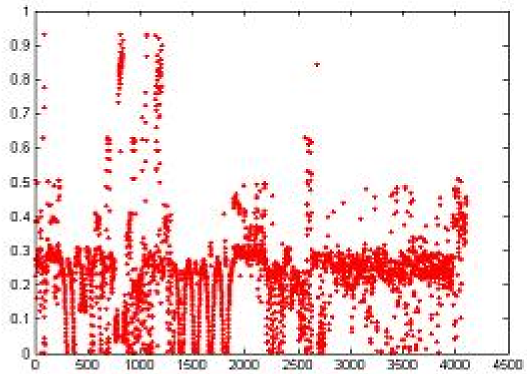

Figure 13. COMAC for 20 Master DOFs

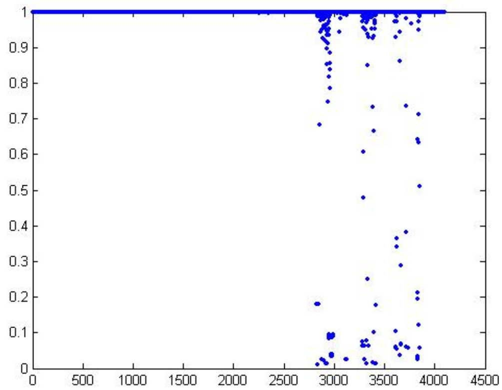

Figure 13. COMAC for 100 Master DOFs

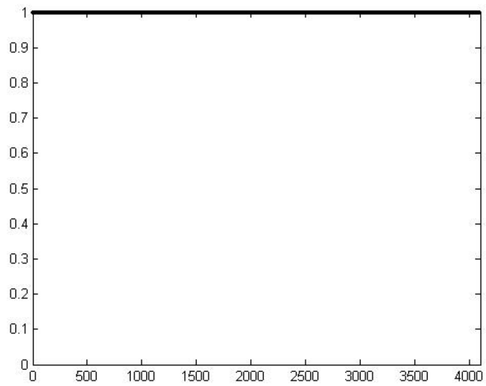

Figure 15. COMAC for 130 Master DOFs

Figure 11 shows the CPU time evolution for the different solutions, using Guyan-Irons Condensation. In this figure we can see that Subspace Method spend twice plus time that for the Guyan-Irons solution.

4. Conclusions

The most used method of reduction in the commercial computer codes of finite elements is the Guyan-Irons Condensation, however this method has limitations in the dynamical analysis.

Diverse simulations, considering different numbers of restrained DOFs, had been made, with the objective of determine the influence of DOFs restrained number in the Guyan-Irons method in the analyze modal of a complex structures.

The obtained results, using the Guyan-Irons method in the study of the Coaracy Nunes hydrogenerators are presented. These results show that 130 master DOFs are sufficient to obtain the good representation of the 10 first modes of the hydrogenation with the same quality of the full model of the Subspace Method but with the half of the time. For representation of the hydrgenerator vibration frequencies, only 100 master DOFs are sufficient in the Guyan-Irons Condensation.

The COMAC coefficient shows the Guyan-Irons method limitations in the representation of the vibrations local modes of the hydrogenerator. In this case the Guyan-Irons method isn't capable to represent the modes associated with the turbine blades vibrations.

5. Acknowledgments

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6. References

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