MULTIPLE MOBILE ROBOTS IN A COOPERATING STRUCTURE

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Abstract. *This work relates computation intelligence algorithms (i.e. fuzzy logic, neural networks and genetic algorithms) with heuristics of path planning (i.e. artificial potential field, Voronoi diagram, graph of visibility, cells decomposition) in order to optimize the trajectory planning of autonomous mobile robots in an unpredictable or dynamic environment. In this work, the control system used (verification of the collision possibility and avoidance of obstacles, local planning) was based on the fuzzy logic, and the heuristic adopted was the artificial potential field (global planning).*

Keywords: *path planning, computational intelligence, potential field, mobile robots, cooperative robots, fuzzy logic.*

1. Introduction

This article approaches the path planning in robotic systems using path planning heuristic and computational intelligence-based control algorithms. Basically, the control algorithm considered will generate an established initial trajectory based on sensors information; and, next it will monitor the collision possibility of obstacles. If it occurs, an obstacle, the obstacles avoidance module will be initiated. In the case that this module does not have a solution, leader-following architecture will be used, or either, the leader robot that initiates and finishes the task of bar transport, exchange the leadership temporarily to the follower who will come back some steps after the leadership returns to the leader. With this, the leader will be able to take new decisions. The control algorithm uses movement sensors in the link bar and optic sensors for odometry and steering errors.

The studies carried out in the path planning area and cooperative work, had been divided in two forms: a) sweeping task of an unknown or dynamic environment, and b) the cooperation in the object transport in structured environments (static obstacles) and semi-structured (the obstacles position are not aware). In the works directed to recognition (sweeping) (Pereira, 2001; Rosa and Justel, 2004; Eder and Rosa, 2003), it exists two strategies: i) behavior-based, or either, the actions of robots are carried through by means of the election of a set supplied information depending on the situation, what it does not guarantee the path optimization; ii) model-based, where the actions are optimized had to the known model of the environment at the moment of the planning. However, the strategies have based on model have high computational cost, Trevai et al (2002).

2. Literature Revision

In studied articles for accomplishment of this work we concentrated in that combined heuristics, such as: graph of visibility, roadmap methods (Voronoi diagrams, cells decomposition, search random trees), artificial potential field with algorithms of computational intelligence (neural networks, fuzzy logic and genetic algorithms), in the search of systems of path planning and obstacles avoidance more efficient.

In the related works, the cooperation between robots, the algorithms of path planning have evolved of traditional heuristical roadmap, graphs, virtual potential field (Khatib, 1986; Miyazaki and Arimoto, 1985), which are strategies model-based - what it limits them how much to the power of processing. The recent concepts of computational intelligence had brought together with the increase of the power of processing and storage a greater autonomy of the mobile robots. However, the studies most recent had shown that combination of heuristical and computational intelligence improves the effectiveness of the algorithm of command and control, besides supplying the possibility of auto learning of the mobile robotic systems in unstructuralized or dynamic environments.

Another approach in cooperative systems is the leader-following architecture for Pereira et al. (2002), where two robots have the task to carry an object in a unstructuralized environment, being that a robot initiates and finishes the task as leader and the leadership exchange occurs when the sensors of the leader detect an concavous obstacle. In this work the authors left the command concentrated in the leader and the follower only has function to came back same steps if the leader find concavous obstacles that obstacles avoidance module does not find solution and to keep the object strong connected in order to prevent the bar fall, so keeping the implicit communication between them.

In our work it is used two robots link for a bar, which is monitored in both robots in order to implement an implicit communication between them and, thus to used leader-following architecture above described, however only when the leader it finds obstacles concavous (fig. 1), where the fuzzy logic can not decide which the best way, therefore the follower assumes the leadership when the leader stop, through of the bar sensoring, returning ten steps in straight line and, thus when the follower stop, it returns the leadership to the leader who will take another decision to avoid obstacles. Then the follower robot instead of storing the last steps of the leader, monitoring the bar to keep firmly connected and detects if the leader stopped or not. At first, the command and control algorithm use the sensors in its maximum reach of 6 (six) meters, to generate a partial mapping of the environment (global planning - artificial potential field (APF) and at long of trajectory to monitor and to avoid possible obstacles, with the sensors reach in 1 (one) meter (local planning - fuzzy logic). The fuzzy logic algorithm monitor collision possibility, when it detects the existence of an obstacle it selects the other module of the fuzzy controller to the avoidance obstacles and when reducing this collision possibility, the robot returns to the original way for the nearest coordinate. In the following section, the concepts related to the applied algorithms will be described.

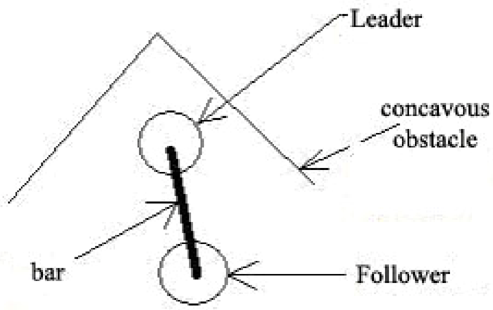


Figure 1. Leadership exchange when concavous obstacles found in the environment.

3. Theoretical support

In this section the concepts related to the problem in this article will be presented.

3.1. Kinematic constraints

In vehicles kinematics it is introduced holonomy concept used in the study of the position control ("Lie Algebra") of it. In a workspace T, a mobile composed of rigid bodies has its configuration specified for the position of all its points in relation to Cartesian coordinates system incorporated in T. The C space of configuration of the mobile have all its possible configurations, which has the structure of a soft distribution, or either, it represents any practical situation. The position interval you reached can be limited making with that C is a compact distribution.

In mathematical terms, get the C space a n-dimensional space, a configuration q is imagined as a list (q_1, ..., q_n) of n generalized coordinate with arithmetical module appropriate in angular coordinates.
It assumes that a scalar constraint of form:

$$G(q,t)=0 \tag{1}$$

with q Є *C* and t representing the time, it is applied to the robot movement. It assumes, moreover, that G is soft with derivate non null. Then, in theory, could use the equation for solution of one of the generalized coordinates in terms of the others coordinate and the time. Thus, the equation (1) defines a n-1 dimension submainfold of *C*. This submainfold is in fact the real space of the robot configuration and n – 1 remain coordinates are its generalized real coordinates. The constraint (1) is an equality holonomic constraint. More general, it can have k constraints of the form (1). If they will be independent, they determine one submainfold of dimension (n - k) of *C*, which is the real space of configuration of the robot.

A constraint of form G(q, t)< 0 (or ≤ 0) acts as an obstacle. Simply it determines one subgroup of *C* that it has same dimension of *C*.

A constraint of the form (1) is only a kinematic constraint. Another is a scalar constraint of the form:

$$H(q,\dot{q},t)=0 \tag{2}$$

with $\dot{q}\in T_q(C)$, the tangent space of *C* in q. The tangent space, which represents the space of the speeds of the robot, it is a space vector of n dimension. A constraint of the form (2) it is holonomic if it will be integrate, i.e. if $\dot{q}$ could be eliminated and the equation (2) it will be rewrite in the form (1). If not, the constraint is nonholonomic. As seen above, a nonholonomic constraint restricts the speed space that the robot reach in any q configuration in a linear subspace of (n - 1) dimension of $T_q(C)$ without to affect the configuration space dimension. If it will have k independent nonholonomics constraints of the form (1), the speed space reached is a subspace of $T_q(C)$ of (n – k) dimension. Then, a mobile subject to k independent constraints of the form (2) it is holonomic if the (n-r) codimension of the Lie Algebra control is equal to number k of constraints. In such case, the kinematic constraints are necessarily linear in terms of the speed parameters (Barraquand ad Latombe, 1991).

Therefore, in words, a definition more generality would be: "a vehicle is holonomic if the number of local degrees of freedom of movement are equal to number of global degrees of freedom."

3.2. Mathematical consideration - "Lie Algebra "

It is well-known that linear algebra concepts, as well as matrices theory are essential in the approach of control linearization problem in nonlinear systems, specifically when its relative degree is not clear. However, the relation between the control of nonlinear systems (that is, mobile robots) and the matrices theory is not always easy to understand. One of the inherent difficulties to the path planning of robots are the control algorithms. Another difficulty is the control to follow a trajectory. The keys topics of a nonholonomic dynamic of the robot will be approach and its space-state representation, the controlled linearization of input-output feedback, the kinematic model, the constraint equations of the mobile robot, the output equations that are essential for control algorithm and the determination or existence of relative degrees, beyond of the concepts of "Lie brackets" and position derivatives.

3.2.1. Dynamic equations and theoretician formularization

Consider a nonholonomic mobile robot with n generalized q coordinates, subjects to m constraints (assuming that m < n) whose dynamic equations of movement are described for

$$M(q)\ddot{q}+V(q,\dot{q})=B(q)\tau-A^{T}(q)\lambda \tag{3}$$

where $V(q,\dot{q})=C(q,\dot{q})\dot{q}, M(q)\in\Re^{mxn}$ it is inertia matrix, $C(q,\dot{q})\in\Re^{nxn}$ is centripetal forces matrix and of Coriolis, $B(q)\in\Re^{nx(n-m)}$ it is the matrix of transformation of entrance, T(q) is a Jacobi matrix, $\tau\in\Re^{(n-m)}$ is the input vector and $\Re^{m}$ is the confined forces vector. There are m equations of constraint of the mechanical system can be written in the form

$$w(q,\dot{q})=0 \tag{4}$$

If a constraint equation is in form $W_i(q) = 0$ are said holonomic; against case is a friction kinematic constraint non holonomic.

3.2.2. Classification of nonholonomic systems

It is important to know the type of system's motion constraints. Some concepts and mathematical formulations that allow to reach this purpose will be presented. Suppose that there are k holonomic and m-k nonholonomic constraints, all can be written in the form of

$$A(q)\dot{q}=0 \tag{5}$$

where A(q) Є $\Re^{mxn}$ is a full rank matrix. Let $s_1, \ldots, s_{n-m}$ be a set of smooth (continuously differentiable) and linearly independent vector fields in the null space of A(q), $\aleph(A)$, i.e.

$$A(q)s_i(q) = 0 \quad i = 1, \ldots\ldots, n-m \tag{6}$$

Let S(q) be the full rank matrix made up of these vectors

$$S(q) = [s_1(q) \ldots s_{n-m}(q)] \tag{7}$$

and Δ the distribution spanned by these vector fields

$$\Delta(q) = \mathrm{span}\{s_1(q) \ldots\ldots, s_{n-m}(q)\} \tag{8}$$

thus, it follows that dimΔ(q) = rank S(q) and any $\dot{q}$ satisfying equation (4) belongs to Δ .
Definition 1: For two vector fields f and g, the Lie bracket is a third vector field defined by:

$$[f,g](q)=\frac{\partial g}{\partial q}f(q)-\frac{\partial f}{\partial q}g(q) \tag{9}$$

It is obvious that [f,g] = - [g,f] and [f,g] = 0 for constant vector fields f and g. Also the Jacobi identity,

$$[h,[f,g]]+[f,[g,h]]+[g,[h,f]]=0 \tag{10}$$

The following notation is commonly used in Lie bracket representation:

$$\begin{aligned} &ad_f^0 g(q) = g(q) \\ &ad_f g(q) = [f,g](q) \\ &ad_f^k g(q) = \left[f, ad_f^{k-1} g\right](q), \qquad k > 1 \end{aligned} \tag{10}$$

Definition 2: A Δ distribution is involutive if it is closed under Lie bracket operation, that is, if $g_1 \in \Delta$ and $g_2 \in \Delta \Rightarrow [g_1,g_2] \in \Delta$. Then, analyzing whether Δ distribution is or not involutive. Let Δ* be the smallest involutive distribution containing Δ, in this case $\dim(\Delta) \le \dim(\Delta^*)$. According to Coelho and Nunes (2003), there are three possible cases: (1) for k = m, i.e. all the constraints are holonomic, Δ is involutive; (2) for k = 0, i.e. all the constraints are nonholonomic, Δ* spans the entire space; (3) for 0<k<m, the *k* constraints are integrable and *k* components of the generalized coordinates may be eliminated from the motion equations. In the last case dim(Δ*) = n-k.

However, we may be more precise, and distinguish among holonomic, and nonholonomic constraints. To verify the type of constraints it is necessary computing repeated Lie brackets of the vector fields $s_1, \ldots, s_{n-m}$ of Δ (or of the system $\dot{q}(t) = \sum_{i=1}^{n-m} s_i v_i(t) = S(q)v(t)$).

As observed for Luca, "the level of" bracketing " necessary to spread R^n it is related the complexity of the problem of planning of the movement. For this reason, gives a classification of nonholonomic systems based in the sequence and order of the "Lie brackets" in the corresponding algebra of accessibility."

Definition 3: The filtration generated by the distribution Δ (5) is defined as the sequence $\{\Delta_i\}$ with

$$\Delta_i = \Delta_{i-1} + [\Delta_1, \Delta_{i-1}], \qquad i \ge 2 \tag{11}$$

where

$\Delta_1 = \Delta$

and

$[\Delta_1, \Delta_{i-1}] = \text{span}\ \{[s_j, \gamma] \mid s_j \in \Delta_1, \gamma \in \Delta_{i-1}\}, j = 1, \ldots, n-m$

Note that $\Delta_i \subseteq \Delta_{i+1}$. Also, from the Jacobi identity follows that $\left[\Delta_i, \Delta_j\right] \subseteq \left[\Delta_1, \Delta_{i+j-1}\right] \subseteq \Delta_{i+j}$

A filtration is regular in a given neighborhood V of q_0 if $\dim\Delta_i(q) = \dim\Delta_i(q_0)$, $\forall q \in V$.

For a regular filtration, if $\dim\Delta_{i+1} = \dim\Delta_i$, then Δ_i is involutive and $\Delta_{i+j} = \Delta_i$ for all $j \ge 0$. Since $\dim\Delta_1$ = n-m and $\dim \Delta_i \le n$, the termination condition takes place after m steps, i.e. it agrees with the number of original kinematics constraints.

If the filtration generated by a distribution Δ is regular, it is possible to define the degree of nonholonomy of Δ as the smallest integer k that verifies the condition $\dim\Delta_{k-1} = \dim\Delta_k$. Note that the verification of this condition implies that $k \le m+1$.

The conditions previous for holonomy, partial nonholonomy and complete nonholonomy may be rewritten as follows: (1) for k = 1, i.e. $\dim\Delta_k$ = n-m, all the constraints are holonomic; (2) for $2 \le k \le m$ and if $\dim\Delta_k$ = n, all the constraints are nonholonomic; (3) for $2 \le k \le m$ and if $(n-m)+1 \le \dim \Delta_k \le n$, the constraints are partially nonholonomic (Coelho and Nunes, 2003).

3.3. Path planning - Artificial Potential Field

This approach was introduced by Khatib (1985), for arms of robotic manipulators and later suggested for platforms of mobile robots for J.C. Latombe and Barraquand (1991). Extracting of the concept of the field theory of the physics, this method shapes obstacles as emitting of a repulsive force and the goal as emitting of an attractive force in our robot. The navigation is played by the movement on way robot to minimize its potential energy. The main aspects of this approach are:

1. Environment: This approach assumes the knowledge of the type of obstacles in the environment whose are approximated by polygons or spheres. The environment for the original formulation of this idea was presumption to be static, however had some adaptations to use this approach for dynamic environments. The potentials are associates to objects in the environment such as it is found.
2. System model: The approach is almost independent of the system model and really it do not make intelligent planning of the knowledge vehicle constraints. For instance: in the case of a nonholonomic robot, this approach does not recommend the robot returns to reach the objective instead of to try to exert a lateral force in the robot that asymptotically would have to lead robot to the objective. This can lead to the behavior of the robot to be a spiral in objective if the robot orientation, when the approximation starts, will not be correct.

3. Objective: This robot is ideally projected to work with an only objective. If more of an objective exist it will be required a stack of objectives that is emptied to each time that the current objective is reached. Some does not have change in the behavior if a sequence of landmarks will be specified.

For this type of generation of control action, the entire knowledge of the environment and constraints need to be incorporated the project of the system. The several variants of the approach of artificial potential field had been developed since the time where the original thought was published, in the attempts to make more useful approach in dynamic or unstructured or semi-structured environments. An excellent tutorial in potential fields is presented by Goodrich. One classic approach of path planning using potential fields is presented in Beard et al. The approach most recent of the APF incorporates dynamic feedback of sensors in the robot control and, thus, it surpasses the limitations to react to unexpected obstacles in the environment than the approaches based on the optimization. The APF theory indicates that for any robot directed to the objective in an environment that contains stationary or mobile obstacles, a map of the APF can be formulated and be calculated, taking in account a attractive polar region in the position of objective and repulsive surfaces of the obstacles in the environment. The field potential used for the robot can be expressed as it follows:

$$U_{cpa}(x) = U_{objective}(x) + U_{obs}(x) \tag{12}$$

where $U_{cpa}(x)$,$U_{objective}(x)$ and $U_{obs}(x)$ denote artificial potential field, attractive potential of the objective, and the repulsive potential of the obstacles, respectively. The letter x indicates the operational coordinates that position and orientation of the robot describe.

Typically, obstacles are treated as exponentially repulsive bodies, that is are said to have repulsion when are close to the border of the obstacle, where becomes infinite. Therefore, for the cohesion of the obstacles the used potential field will be

$$U_{obs}(x) = log|r_j| + 1/r_j^2 \tag{13}$$

where r_j is the distance between robot and j-th obstacle. The objective is chosen typically to have an attractive force of parabolic form such as:

$$U_{objective}(x) = K|x - x_{objective}|^2 \tag{14}$$

The potential is calculated in each point to long of the trajectory and the robot it is moved in direction to the gradient descendant of the potential field until reaching a local or global minimum. The method was applied to mobile robots navigation specifically in (Rimon and Koditschek, 1992) . This approach is simple to extend to workspace of higher dimensions and new ideas allow dynamiclly to adapt the field to changeable environments.

3.4. Avoidance obstacles - Fuzzy Logic

In this section, will be presented basic concepts on the fuzzy logic, its methodology and application forms more usual of simple form, the fuzzy logic can be characterized as "one type of logic that recognizes more than simple values of true or false ". With the fuzzy logic, proposals can be represented with degrees of truth and falseness.

Or either, the fuzzy logic to provide a method to translate verbal expressions, inexact and qualitative or imprecise in numerical values, allowing that calculations are used as based of intelligent systems in the experience human being (Shaw and Simões, 1999).

The algorithm of the fuzzy controller was broken down in two parts: a) one that takes care of the collision possibility; b) another that deals with the avoidance of obstacles. The first module catches the sensors readings in accordance with the risk degree of collision or not, as seen in the figure below. The second module avoid of the obstacles do not detected in the global planning (APF).

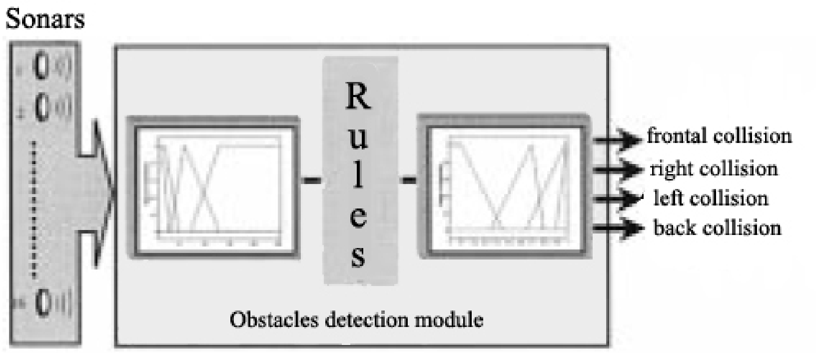


Figure 2. Collision possibility module.

3.4.1. Qualitative aspects of the Fuzzy Logic

Sight of qualitative form, the fuzzy logic can be presented of following form:

- Differently of the Aristotelian logic (bibrave - true or false), the fuzzy logic is multibrave, or either, the truth is graduated, usually receiving a value in interval [0,1], being 0 representing completely false and 1 completely true;
- Verbal expressions, inexact, qualitative, inherently to human beings, can be treated through the formalism of the fuzzy logic;
- The implications logic or logical inference, the inputs and outputs, or antecedents and consequences, the degrees of truth in the interval are associates [0,1];
- The use of the fuzzy logic facilitates to the interface calculating-man, for allowing the first one better understanding of the inherently inexact language of its operators.

In the next section the environment of simulation will be approached used in this article.

4. Simulation environment

The simulation environment used to validation of this work was mathematical software MatLab, in which was made modeling of the sensors and the control algorithms of the robots interfacing with a virtual environment in VRML(Virtual Reality Markup Language), fig. 3.

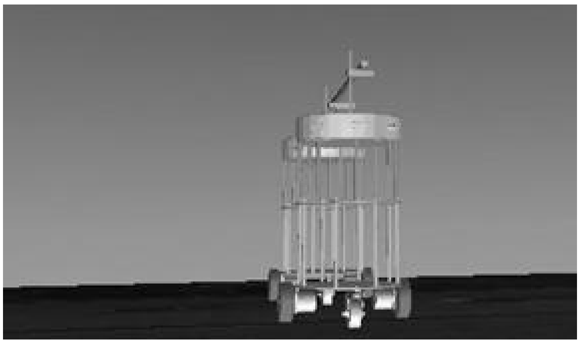


Figure 3. 3D environment of simulation

The ultrasonic sensors had been shaped in C, using the Bresenham algorithm for the generation of the smallest distance caught for the sensor which can modify its reach (fig 3). With the values of distance, a scene of detected obstacles is mounted, thus is created a best way across the computational potential field in which the robot will go to guide itself. When following this way, a collision possibility module monitoring in case of the sensor detects another not foreseen obstacle, the fuzzy controller will assume until the collision possibility to become a little value, after the robot will return to the point nearest to the original trajectory. For the construction of the sets fuzzy of the avoidance obstacles module, such as the set of rules was created empirically through the simulator.

In the simulator the following fields are defined for each sensor: i) name of the sensor; ii) relative position to the robot; iii) reach; iv) angle of the axle; v) angle of the sector - width of the beam; vi) Resolution - number of beams inside the sector, represented for the below figure. The prototype structure, fig. 4, was based on the Nomad architecture, for present little kinematic constraints and for the flexibility.

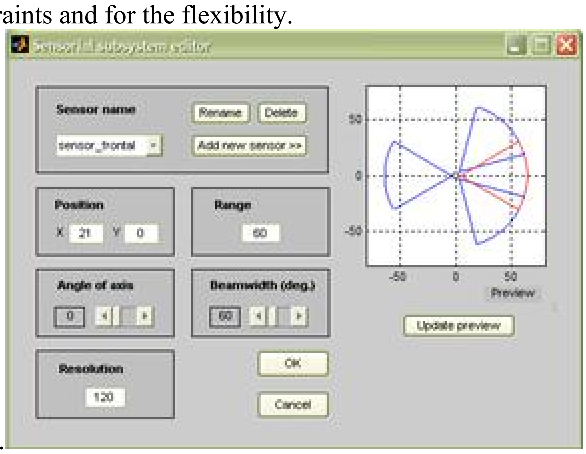


Figure 4. Sensors parameters.

The system of direction of the robots is carried through for differential speed, making possible the elimination of some kinematic constraints and allowing more positions reached for the robots, also reducing the number of variables used in the control algorithm (fig. 5).

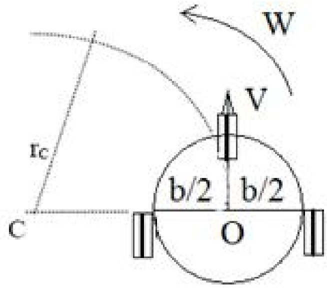


Figure 5. Direction for differential speed between the wheels.

Let r_c, the ray of the trajectory curve; b/2, the ratio of the used robot; v, the linear speed; W, the angular speed of the robot. The mapping of the degrees of relevancy of sensors, or either, a graduation of 0 to 1 used in the fuzzy logic to represent how much the value of a variable of the system belongs to a variable linguistic, was based on the figure of emission of the adopted ultrasonic sensors, as it shows in fig. 6.

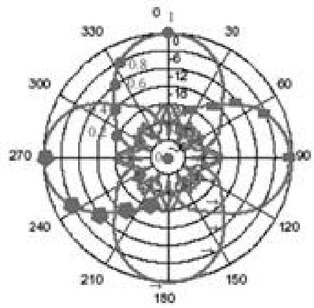


Figure 6. Map of abrangency and degrees of membership of the distance of the sensors (Devantech Ltd.).

It was adopted a distribution of the sensors priorizing the frontal part of the robots (fig. 7), therefore as both the robots are always near, the back sensor would be redundant, besides reducing the amount of variable in the fuzzy logic in order to not overload the computational system.

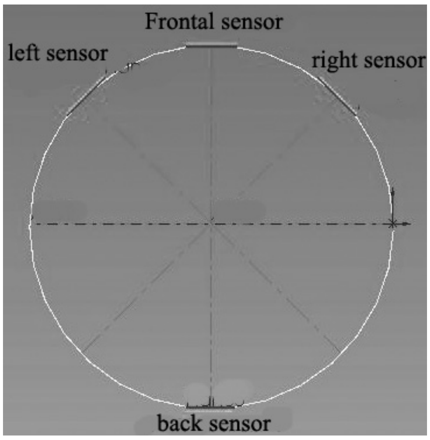


Figure 7. Distribution of the sensors for the structure of the robot.

The flowchart to follow represents the command algorithm and used control:

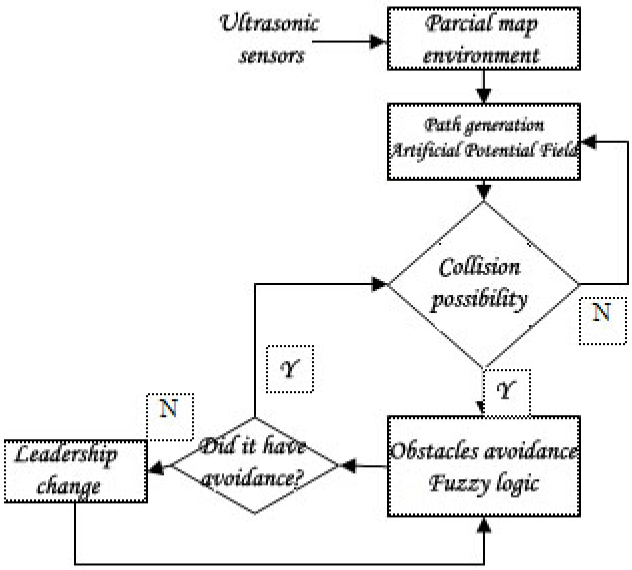


Figure 8. Flowchart of the control algorithm.

5. Results

The work consists of two robots carried out a bar, which is monitored in both robots in order to implement one implicit communication between them and, thus to use a leader-following architecture for a cooperative work to take a bar of an initial position to a known objective and that does not know "a priori" the obstacles localization. For this it is used ultrasonic sensors with the purpose of to supply data the heuristic of computational potential field (global planning) in order to get a environment map with the obstacles detected in its maximum reach, with this calculates the best way (minor distance trajectory) to the objective. As long of the way, a module of the algorithm in fuzzy logic monitoring the collision possibility and, in case that this occurs, selects another module of the fuzzy controller for the avoidance obstacles (local planning), shown in fig.10a in a 2D representation of the scene with the set leader-following represented for octagon, and when the collision possibility become smallest, the robot returns to the original way for the coordinate nearest to this way.

In case that it does not have attitude foreseen for fuzzy controller to the avoidance obstacles (concavous obstacle), the leader robot interrupts its movement and, through of the bar sensoring the leadership is exchange. The new leader returns same steps from the original leader robot and repass the leadership, stopping. Thus, the leader will try a new alternative of avoidance obstacles. The considered navigation system was implemented, as study of case, in mathematical software MatLab. The inference set used fuzzy logic blockset of this software in order to implement the system of collision possibility and the avoidance obstacles, beyond of the potential field heuristic, as seen in fig. 10b, where the circular regions represent obstacles seen for the sensors. The best way created of these sensors information in accordance with the potential fields generated by obstacles.

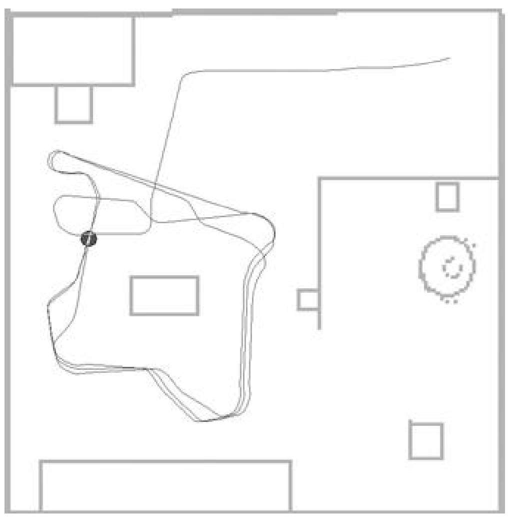
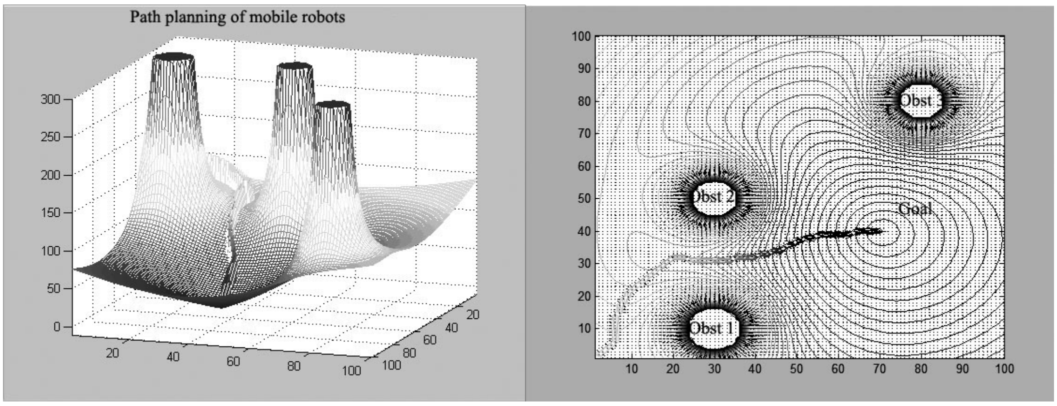


Figure 9. 2D representation of the fuzzy controller for avoidance obstacles of the leader-follower set.



a) b)

Figure 10. a) 3D representation of the best way; b) Creation of the best way for the virtual potential field

6. Future works and conclusion

The presented results justify the combination of path planning heuristic and the avoidance obstacles algorithm in terms of reduction in the execution time of the task. The prototype will serve for future implementations, besides have been of great value for validation of the considered algorithm. To improve more, can be used neuro-fuzzy control algorithm, that it supplies an auto-learning system to the robotic systems, the use of a robotic manipulator with the purpose to expand the tasks to carried out for this cooperative system.

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