Drag Reduction in Vascular System

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Abstract. Polymer drag reduction (DRP), or the Toms effect is an extraordinary hydrodynamic phenomenon that comes being applied in the last decades in bioengineering. It can decrease internal resistance in liquid flow so markedly under appropriate conditions that nanomolar concentrations of certain polymers increase flow as much as threefold or more under a constant driving pressure. Blood soluble DRPs were also demonstrated to have the ability to increase blood flow and tissue oxygenation with no direct effect on vessel tone when injected at nanomolar concentrations in various animal models of normal and especially pathological circulation. Thus, several polymers with high molecular weights and fairly linear structures but chemically dissimilar monomers were shown to facilitate blood flow in vitro in pipes and in vivo. Among this polymers each one can show more drag reduction effect and this effect can be related with average molecular weight and concentration. Then, research of the type of polymers blood soluble and your molecular weight will be made with purpose to identify which polymer and which average molecular weight can show more drag reduction effect.

Keywords. Drag reducing polymers, hemodynamic
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1. Introduction

Engineers have been interested in understanding the polymer solution effects on turbulent friction mechanisms since Toms (1949) first published his drag reduction data. The application of polymers are very ample. These polymers are constantly been used for transport of oil in pipelines (Trans-Alaska Pipeline), for sewer and drainage systems, in fire-fighting equipment, and, on blood flow, among others. In the blood flow, the polymers are been using for reducing atherosclerotics plaques, lethal hemorrhagic shock, reducing hemolysis, among others. The phenomenon of polymer drag reduction in the vascular system involves disciplines as diverse as hydrodynamics, rheology, physiology of vascular system and polymer chemistry among others. Because the interactions among several areas of knowledge, the phenomenon of drag reduction become interesting and challenging.

The substances that are added in the flow and that show drag reduction can be synthetics, biopolymers or surfactants. A detailed description of the drag reduction phenomenon is given by Gyr and Bewersdorff (1995). These substances act in the turbulent region of the fluid. These regions show different eddies scales which are related with the Reynolds numbers. As higher are the Reynolds numbers, greater will be the distance between the smallest scale and the biggest scale of the eddies. Although the polymers are primarily active on the smallest length scales, they are able to influence the macroscopic scales of the flow by which the drag is determined. Therefore, understanding the phenomenon of drag reduction is challenging from a fundamental point of view.

Generally, the application of polymers are in turbulent flow, but there is in the literature some works showing these effects in laminar flow. It was found that drag reducing polymers (DRP) additives reduced hydrodynamic resistance in systems with nonturbulent (disturbed laminar) flow, such as pulsatile flow in straight and spiral pipes or Couette flow with Taylor vortices at low Reynolds numbers (Driels and Ayyash, 1976; Keller and Mackley, 1975). The Toms effect has been intensively investigated for decades and has been applied for various industrial applications. However, the exact mechanism underlying the Toms phenomenon remain incompletely understood.

In the last decades, numerous studies have been developed in area of the drag reduction by polymer additives. The main works are described by Lumley (1969) who shows the different physical phenomena of drag reduction and Virk (1975) who presents experimental evidence that drag reduction is limited by an asymptotic value. And, the book of Gyr and Bewersdorff (1995) gives an extensive overview of the more recent material on the subject.

Over the years, the drag reduction by polymer additives blood soluble have been shown to produce positive hemodynamic effects in various acute and chronic models. The nanomolar concentrations of these polymers injected intravenously caused an increase in aortic and arterial blood flow and a decrease in both blood pressure and peripheral vascular resistance, with no effect on blood viscosity or blood vessel tone (Coleman *et al.*, 1987). Polymers when inserted intravenously show a significant increase in collateral blood flow in rabbits (Gannushkina *et al*, 1981) and the number of capillaries in normal and diabetic rats (Golub *et al*, 1987). Chronic intravenous injections of DRPs diminished the development of atherosclerosis in several atherogenic animal models (Faruqui et al, 1987; Sawchuk et al., 1999).

2.Drag Reduction

2.1 Definition

Drag reduction (DR) has been defined as "a modification in a turbulent fluid system which results in a decrease in the normal rate of fluid energy loss" (Sellin *et al*., 1982). Then, drag reduction is a decrease in resistance of turbulent flow in pipes and tubes. Of the same way, it can be reformulated that the polymers reduce the wall shear stress, τ_w, or the skin friction. Lumley (1969) proposed the following definition: "Drag reduction is the reduction of skin friction in turbulent flow below that of the solvent". Following the Lumley definition, the skin friction of solution is lower than that of the Newtonian fluid for the turbulent flow. Drag reduction, DR, is calculated through the friction factor ratio defined bellow, in equation 1. In this equation, the suffix s is related with polymer solution and the siffix N is related with the Newtonian solvent.

$$DR = \frac{\tau_{ws}}{\tau_{wN}} \tag{1}$$

The wall shear stress and Fanning's friction factor f, for pipes are related by

$$f = \frac{2\tau_w}{\rho \bar{u}^2} \tag{2}$$

where $\bar{u}$ is the mean velocity given by

$$\bar{u} = \frac{Q}{A} \tag{3}$$

with Q being the discharge or flow rate and A the cross-section of the pipe. The wall shear stress in equation (2) is isolated and substituted in equation (1), then result:

$$DR = \frac{f_{ws}}{f_{wN}} \tag{4}$$

for the same discharge condition and, $\rho_N \approx \rho_s$. It is more common, to report on drag reduction by giving the percent drag reduction

$$DR(\%) = 1 - \frac{f_s}{f_N}$$

or (5)

$$DR(\%) = 1 - \frac{\Delta p_s}{\Delta p_N} * 100$$

with Δp being the pressure drop over the considered pipe length. DR is often used for DR(%).

2.2 The friction factors

For laminar flow, the friction factor is given by:

$$f = \frac{16}{\text{Re}} \tag{6a}$$

or

$$f^{-\frac{1}{2}} = \frac{\text{Re}}{16} f^{\frac{1}{2}} \tag{6b}$$

This friction factor is showed in "fig. 1" in the laminar (1) and turbulent branch (2). The turbulent branch (2) is represented by the Prandtl-von Karman law

$$f^{-\frac{1}{2}} = 4\log_{10}\left(\text{Re}\, f^{\frac{1}{2}}\right) - 0.4 \tag{7}$$

The gap between the laminar and the turbulent branchs is know as transition from a laminar to a turbulent flow and occurs because the instability processes, which are not universally reproducible. In this gap is the region of DRP. Virk´s defined the maximum drag reduction asymptote (MDR) represented by the branch (3) and by equation (8):

$$f^{-\frac{1}{2}} = 19\log_{10}\left(\text{Re}\, f^{\frac{1}{2}}\right) - 32.4 \tag{8}$$

By definition the friction factor of a drag reducing flow must have values between Prandtl-von Karman law (smooth turbulent flow) and Virk's MDR asymptote for a given Reynolds number. However it is possible to consider drag reduction also for laminar flow subjected to small perturbations, like in vascular flow system.

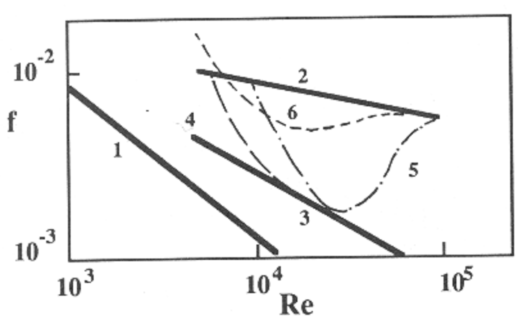

Figure 1. Schematic friction factor versus Reynolds number plot. 1: laminar pipe flow; 2: for turbulent pipe flows in smooth tubes; 3: Virk's maximum drag reduction asymptote; 4, 5 and 6: three typical drag reducing behaviours for flexible polymer, for surfactant and fibre solutions (Gyr & Bewersdorff, 1995).

2.3 The mean velocity profiles

In general, in the turbulent flow there are three regions or layer near the wall, and these layers are: viscous layer, buffer layer and logarithmic layer. In the viscous layer, the velocity profile is given by:

$$u^+ = y^+ \qquad \left(0 < y^+ < 5\right) \tag{9}$$

and the logathimic layer ($y^+ > 30$) is given by

$$u^+ = 2.5\ln y^+ + 5.5 \qquad \left(y^+ > 30\right) \tag{10}$$

It is not possible to define clearly an equation for the buffer layer region ($5 < y^+ < 30$). So that, by convenience, the three layers are reduced to a two layer model in which the buffer layer is eliminated by an extrapolation of the two other zones to their intersection.

The Virk's profile is represented in "fig 2", through the dashed thick line. The equation that show the MDR Virk's ultimate profile is given by

$$u^+ = 11.7 \ln y^+ - 17 \tag{11}$$

ΔB in figure 2 corresponds to a shift of the logarithmic velocity profile of a Newtonian fluid due to the presence of additives, like polymers. The Virk's general equation is reproduced bellow and ΔB is function of polymer type, molecular weight and concentration.

$$u^+ = 2.5 \ln y^+ + 5.5 + \Delta B \tag{12}$$

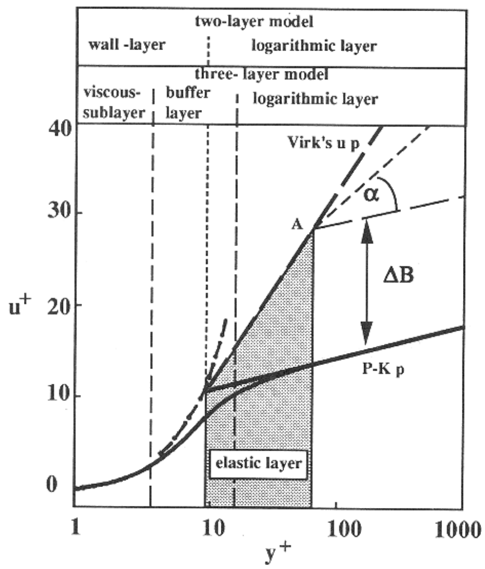

Figure 2. A schematic representation of the universal velocity profiles. Explanations are given in the text (Gyr & Bewersdorff, 1995).

3.Polymer Molecules in Solution and Suspension

3.1 Molecular weight and molecular configurations

A macromolecule (or polymer) is a large molecule composed of many small simple chemical units, generally called structural units. In some polymers each structural unit is connected to precisely two other structural units, and the resulting chain structure is called a linear macromolecule. In other polymers most structural units are connected to two other units, although some structural units connect three or more units, and it is called branched units.

It is sometimes useful to distinguish between synthetic and natural (biological) macromolecules. Many synthetic polymers are built from a single structural unit, and the polymer is then referred to as a homopolymer. Typical examples of synthetic homopolymers are polyethylene and polystyrene. In contrast copolymers are built from two or more different structural units.

In the polymer sample which the molecular weight of all macromolecules is the same, such a sample is called monodisperse. Indeed, some biological polymers are monodisperse. Synthetic monodisperse or almost monodisperse polymers may be prepared by special techniques, but are seldom used commercially. Most commercial polymers however are polydisperse, which means, that they contain molecules of many different molecular weights (Bird *et al.*, 1987).

It is a well known the fact that the efficiency of drag reduction strongly depends on the polymer molecular weight. Therefore the fraction with the highest molecular weight in a sample is the fraction which determines the drag reducing properties of the fluid (Gyr & Bewersdorff, 1995). In order to describe molecular weight distributions in sample quantitative terms, it is necessary to introduce the various molecular weight averages. Specifically, the fraction 1 contains

N_1 moles of molecular weight M_1, fraction 2 contains N_2 moles of molecular weight M_2 and so forth. Then, an average molecular weight is obtained by multiplying the molecular weight of each fraction by the number of moles in that fraction, adding and dividing by the total number of moles:

$$\overline{M_n} = \frac{\sum_{i=1}^{k} N_i M_i}{\sum_{i=1}^{k} N_i} \quad (13)$$

This is called the number-average molecular weight.

The molecular weight alone is not the only important parameter characterizing the polymers. Another important parameter is the configuration of the molecules, which can change due to the rotations of chemical bonds or by thermodynamic motions of the molecules.

3.2 The dilute solutions

The drag reducing long polymer molecules consist of linear sequences of n monomeric units. The number n of monomers in the molecule can be very large, usually of the order 10^5, which means that the molecular weight is also very large, usually of the order of 10^6 to 10^7 g/mol. Usually, these molecules are quite flexible, with exception when some groups, e.g. phenyl groups of sugar rings, are introduced in the polymer chain when the polymer becomes more rigid (Gyr & Bewersdorff, 1995).

With the drag reduction, mainly two kinds of water soluble polymers were investigated, the polyethylene oxide and the polyacrilamide, figure 3.

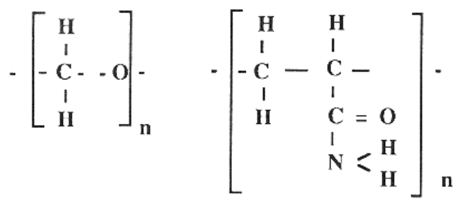

Figure 3. Chemical structure of polyethylene oxide and polyacrylamide (Gyr & Bewersdorff, 1995).

These molecules are suspended or dissolved in a Newtonian solvent. In dilute form one can study some of the properties of single molecules, de Gennes (1990). In this context dilute means

$$nL^3 << 1 \quad (14)$$

where n the number of molecules per unit volume, and L the length scale of the molecules. This quantity varies dramatically because the geometry of the polymer chain in solution is variable. The exact geometry of the dissolved molecules is unknown.

4.Polymer in the Vascular System

As comented before, the effect of the drag reduction occurs only in the presence of disturbed or turbulent flow, and it is associated with a laminarization of the flow. This phenomen has been observed in pipe blood flow. Application of polymer drag reduction principle to blood flow in vivo was first attempted in the mid 1970s. Mostardi *et al* (1976) demonstrated with hot-film anemometry that poststenotic flow disturbances in the dog were diminished subsequent to intravenous injection of Separan AP-30 (an anyonic polyacrylamide). Mostardi *et al* (1978) postulated that atherogenesis can be inhibit with the addition of the drag reducing polymers given the evidence that atheromas tend to form prefentially at vascular bifurcations, branches, and other regions of disturbed flow. Consequently, aortas of rabbits on a high cholesterol diet, some of which had been administered by Separan AP-30, were examined and compared. Based on a visual and photographic inspection of the inner walls of aortas slit lengthwise, it was concluded that the Separan treated animals were markedly less atherosclerotic than the untreated ones. Similar results were obtained in White Carneau pigeons fed an atherogenic diet (Green *et al*, 1980).

Faruqui *et al* (1987) studied the effect of the Separan AP-30 on atherosclerotic plaque formation in aortas of rabbits. This study was based in the Mostardi's hypothesis. For this study, the rabbits were submitted on a high (2%)

cholesterol diet over a period extending from 37 to 170 days. Atherogenesis was quantified morphometrically by appication of a computer-assisted image analysis of histologic cross sections of the aorta. The area of vessel wall-atheroma interface, fraction of lumem occluded, and other indexes of atherogenesis were measured in each of 26 serments of aorta excised from the animals, half of which were adiministered injections (intravenous) of Separan three times a week. Regression analysis of the morphometric data indicates that the polyelectrolyte exerts a powerful antiatherogenic effect in all regions of the aorta, inhibiting the formation of plaque mass to less than half in the aortic arch and about one-fitth in the descending aorta as compared with the aortic plaques masss in untreated rabbits. Therefore, these authors gave support for a new hemodynamic ideas based in polymer drag reduction applied to vascular system, and these ideas might be effectively applied against atherosclerosis.

Sokolova *et al* (1989) showed that the infusion of polyethylene oxide in anesthetised rats in a total dose of 10 g/ml caused a 10-16% reduction in blood pressure. This injection has been associated with 17% decreased pressure and 54% increased blood velocity in mesenteric arterioles, but no increase in their internal diameter was observed. Vasodilatation did not change the hemodynamic response of the arterioles to polymer infusion. In agreement with the description above, the microdisturbances of blood flow which can be diminished by drag reducing polymers are more important for the vascular resistance to blood.

In the literature, many articles show that regions of the low shear stress are associated with the development of the aterosclerotic plaque. Therefore, some researchers are working in these regions through addition drag reducing polymer. When the drag reducing polymer are added in blood flow, it increases the shear stress and reduces formation of the atherosclerotic plaque. Sawchuk *et al* (1999) studied concluded that drag reducing polymers increase shear stress in areas normally exposed to low shear stress. They used six dogs with half aorta arterie obstructed manually. The drag reducing polymer used in this study was a polyethylene oxide with an approximate molecular weight of $14*10^6$ (Polyox WSR 301, NF grade; Union Carbide, Danbury, Conn). The velocities were measured using Doppler ultrasound probe mounted at a 45 degree angle on a micromanipulator. Shear rates were calculated using linear regression and then compared using the t test. The drag reducing polymer solution was administered to the dogs for 30 minutes at a rate of 0.16 mg/kg/min using the syringe pump (model 22; Harvard Apparatus, South Natick, Mass). The blood viscosity remained constant at 0.04 poise during infusions of this amount of drag reducing polymer. The study showed an increase of the shear stress, when the polymer was added. Therefore, they concluded that one of the ways that drag reducing polymers inhibit the development of atherosclerosis appears to be the shear stress increasing in areas normally exposed to low shear stress. The drag reduction mechanism understanding may lead to the development of pharmaceutical agents that inhibit the development of atherosclerosis.

Generally, during extracorporeal circulation, blood cells are exposed to mechanical and enviromental factors, such as extremely high shear stress, prolonged contact with foreign materials and another factors. These factors can cause damage to red blood cells, which is manifested by hemolysis and alterations in red blood cells mechanical properties. These changes result in a decrease in red blood cells deformability that subsenquently blocked red blood cells passage through the microvasculature and that may contribute to many complications (Kameneva *et al*., 1999). Further, during routine hemodialysis, pumps and access systems may damage red blood cells because of the excessive shear forces and prolonged contact times associated with these biomaterial surfaces (Kameneva *et al*., 2002). For reduce this problem, the addition of 1 to 2 % PEG (polyethyle glycol), any molecular weight, reduce the hemolysis. Kameneva *et al* (2003) studied this problem and concluded that with utilization of the PEG, the erythrocytes were protected against mechanical hemolysis. This suggest that erythocytes in extracorporeal and circulatory assist devices would be more protected if a solution of PEG were used instead of current clinical use hemodilution products.

Kameneva et al. (2004) worked with three polymers with differents moleculars weights. One of this polymers was the polyethylene glycol (PEG, which is another name for polyethylene oxide) with average molecular weight of 3.5×10^6 Da (PEG-3500) (Aldrich Chemical Co), the other PEG with another molecular weight – 2×10^5 Da (PEG-200) and a new aloe vera-based drag reducing polymers discovered in your laboratory. The PEG are synthetic polymers and the aloe vera is a natural polymer. The molecule of the PEG-200 possesses the same chemistry and structure of the higher molecular weight PEG, but with no drag-reducing ability. According Kameneva *et al*. (2004), the PEG-200 was added in the solution until 75 ppm and it does not showed drag reducing, which means that no drag reduction occurs for PEG molecular weight bellow 10^5. The natural polymer, aloe vera, presented the phenomenon of the drag reduction. The extract of the aloe vera was purified and characterized as a complex of polysaccharides with average molecular weight of about 4×10^6 Da. For both polymers, the viscosity showed no variations independently of the polymer concentration. All polymer solutions demonstrated Newtonian flow behavior and the solution viscosity was constant at all applied shear rates (1.28-94.5 s^{-1}). It is important to continue this research for higher values of shear rates.

5.Drag Reduction versus Molecular Weight

Polymers with differents molecular weights showed differents values in drag reduction. However there are some contradictions in the literature related with drag reduction values and respectively polymer molecular weight. There is an agreement between researchers that polymers with high molecular weight showed more drag reduction than polymers with low molecular weight. However the low limit of drag reduction related with molecular weights is not clear. In the table bellow is presented differents results of drag reduction for different molecular weight, according some literature researches.

It is clear for Kameneva et al. (2004) that for molecular weight nearly 10^5 there is not drag reduction. The results of Vilalta & Ortiz (2002) however shows drag reduction for molecular weight nearly 10^3, however another polymers was used by these last authors. .

Table 1. Drag reduction polymers and molecular weight.

Researchers	Applications	Polymers	Concentration	Molecular Weight	Drag Reduction (%)
Vilalta & Ortiz (2000)	Channel	Iqapol PA	20 ppm	$1.7*10^6$	60
Vilalta & Ortiz (2000)	Channel	Iqapac	20 ppm	$4\text{-}6*10^3$	35
Scrivener *et al*. (1987)	Channel	PEO 301	1 - 2 ppm	$1*10^5 – 8*10^6$	40
Kameneva *et al*. (2004)	Pipe	PEG 200	75 ppm	$2*10^5$	0
Kameneva *et al*. (2004)	Pipe	PEG 3500	5 ppm	$3.5*10^6$	60
Kameneva *et al*. (2004)	Pipe	Aloe Vera (AVP)	5 ppm	$4*10^6$	60

6.Conclusions

This study is very complex and detailed studies are necessary regarding the drag reduction in the vascular system with the goal of understanding the mechanism of the drag reducing polymer. It is possible to concluded that not only molecular weight, but also concentration and molecular structure are responsible for the drag reduction. Further researches are necessary to understanding the weight of these parameter in the drag reduction (molecular weight, concentration, molecular structure), to give support for the development of new drugs, as auxiliary in pathologies like atherosclerosis and hemodyalisis.

7.Acknowledgement

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