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FINITE DEFORMATIONS AND INSTABILITIES OF STRESSED CYLINDRICAL TUBES UNDER HYDROSTATIC PRESSURE

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Abstract. *This paper investigates the large deformations of an extended cylindrical tube with internal hydrostatic pressure. The final scope of this research is to understand the behavior and formation of aneurisms. Aneurisms are one of the major causes of human mortality of the western world. A detailed numerical and experimental analysis was carried out involving different initial axial forces and the influence of the axial force was investigated. The thick-walled circular cylindrical tube was used throughout the research. The rubber-like material of the tube is considered isotropic, homogeneous, incompressible and hyperelastic, which is modeled as Mooney-Rivlin, A numerical analysis was done using the thick shell finite element S4R of ABAQUS. It was observed that when the extended tube was filled with water, the volume of the tube increased as the volume of water inside the tube increased. At a certain critical pressure a local buckling was responsible for the formation of a bulb in the tube. After this, the internal pressure decreased subtly with increasing internal volume. There was encouraging agreement between the numerical and experimental results.*

Keywords: *aneurysm, finite element, instability, finite deformation*

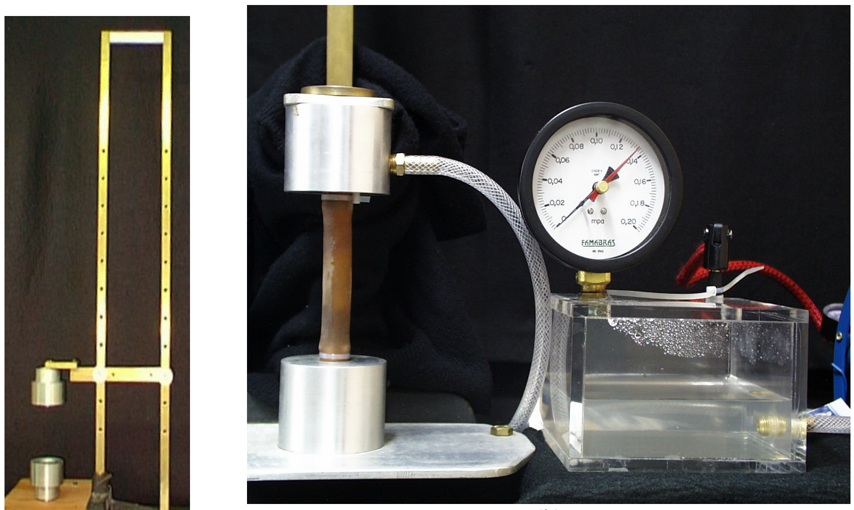
1. Introduction

The pioneering work of Green & Adkins (1960) on non-linear elasticity set up the basis for the analysis of membranes under large deformations. Since then many important papers have been published in this field, most of which deal with the equilibrium and stability of thin cylindrical membranes under uniform pressure loading or loads acting along the boundaries (Corneliussen & Shield, 1961; Alexander, 1971; Ratner, 1985; Li & Steigmann, 1993, Haseganu & Steigmann, 1994; Chen, 1995, Haughton, 1996). The number of experimental contributions to this class of problems is rather small compared with the theoretical and numerical ones. Among the experimental investigations in this field the publications of Green & Adkins (1970), Pamplona & Bevilacqua (1992) and Pamplona et al. (2001) should be mentioned. The analysis of large deformations of thick membranes is not so popular, nevertheless there are some important publications such as the ones by Kyriakides and Chang (1991), where the phenomenon of localization of the buckling mode was studied, and, more recently, the work by Tang et al (1999) and Haussy and Ganghoffer (2002) where the theory of thick hyperelastic shells was used for the modeling of carotid arteries and aneurysms, respectively.

Elastomeric membranes are load adaptive, as they change the geometry to accommodate external loads with the minimum variation in stress levels, and, therefore, may be an efficient engineering solution in many practical fields. In most of these applications the non-linearities of deformation and material response are very important. In this work the non-linear behavior of extended thick cylindrical membranes, both numerically and experimentally is investigated. In the numerical formulation of the problem, the membrane is considered to be incompressible, homogenous, and isotropic and it is modeled as a Mooney-Rivlin. This is in agreement with the physical characteristics of the rubber membranes used in the experimental investigation. In the experimental analysis several loading cases were investigated and these results were compared with the numerical results obtained by using the ABAQUS software (Hibbitt et al., 2001). This has been here used together with the Newton-Raphson algorithm and the Riks continuation method to obtain the pre- and post-bifurcational behavior of the membrane under traction and internal pressure.

2. Experimental Analysis

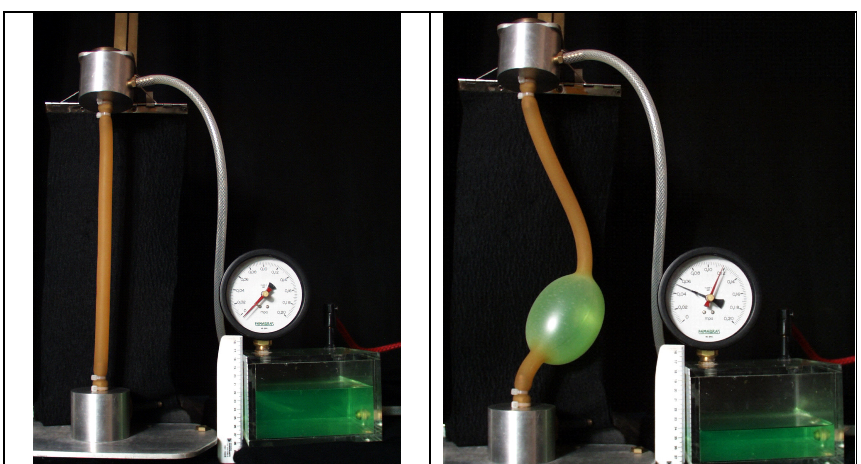
With the purpose to try to understand the mechanics of the formation of aneurysms, the whole experimental procedure was done using water under pressure. A thick latex membrane was used in the experimental analysis with a geometry that approaches the dimensions of an artery. The material of this membrane is considered to be, based on the experimental results, as homogeneous, isotropic, incompressible and hyperelastic rubber-like material of undeformed external radius, $R = 17.00\,mm$ and thickness, $H = 2.62\,mm$. An apparatus was developed to support vertically the extended cylindrical tube, Fig. 1(a), and a device to measure the internal pressure and pump the water under pressure to the interior of the tube, Fig. 1(b).



(a) (b)

Figure 1- Experimental apparatus

The experimental analysis of the membrane initially stressed and submitted to increasing pressure was carried out for a tube *317 mm* long. The results are shown in Figures 2 to 5, for increasing values of normalized axial extension $(\Delta L / L)$.

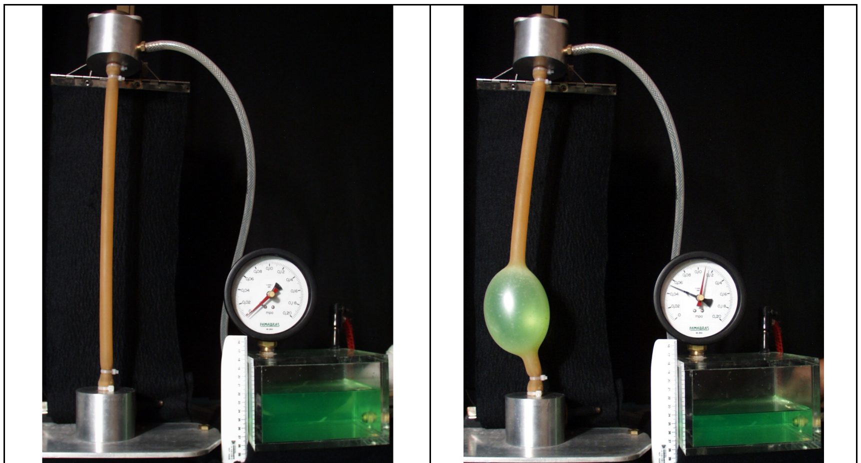


$P_i = 0\ MPa$	$P_{critical} = 0.102\ \ MPa$ $P_{final} = 0.052\ \ MPa$

Figure 2- Experimental analysis of an initially unstressed thick cylindrical tube $\Delta L / L = 0$.

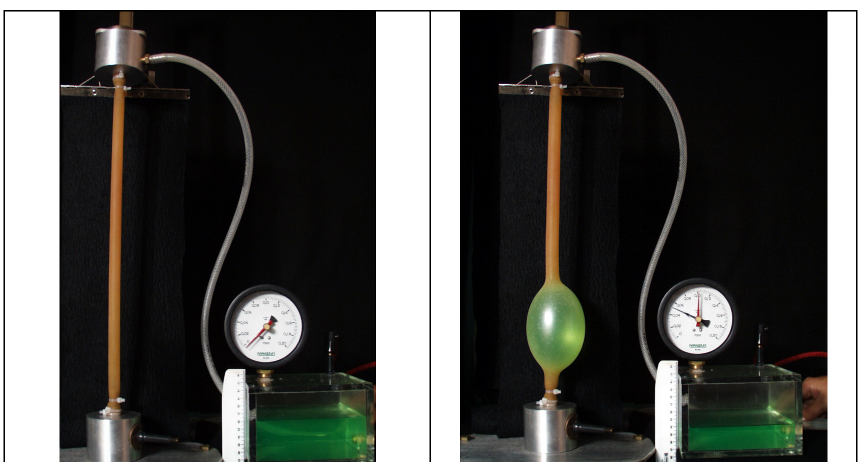
Figures 2, for $\Delta L/L = 0$ (membrane initially unstressed), as the pressure increases the membrane deflects laterally as a long column until a critical pressure is reached and a bubble is formed nearly in the middle of the specimen. At this critical point there is a sudden drop in pressure from *0.114 to 0.052 MPa* defining a critical pressure. However, the position of the bubble may differ from one specimen to another. This is probably due to the effects of initial material and geometric imperfections.

When the membrane is initially stressed, as can be seen in Figures 3-5 the tube remains in a vertical position until the critical point is reached. Again the shell reaches a limit point characterized by the formation of a bubble and a sudden drop in the internal pressure.



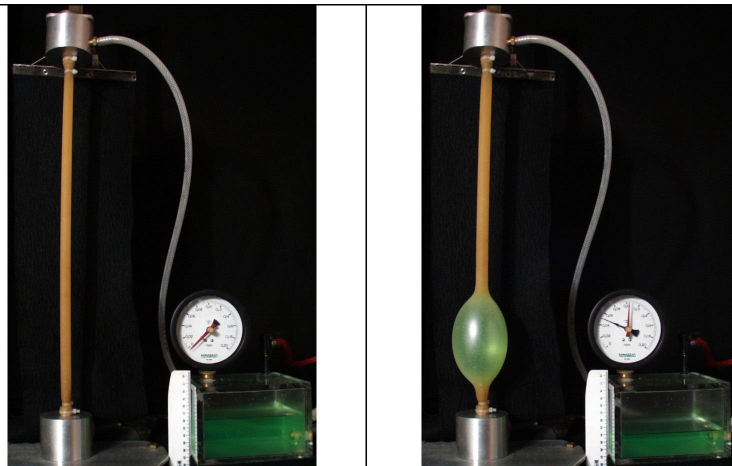
$P_i = 0\ MPa$	$P_{critical} = 0.096\ MPa$ $P_{final} = 0.052\ MPa$

Figure 3- Experimental analysis of an initially unstressed thick cylindrical tube $\Delta L/L = 0.158$.



$P_i = 0\ MPa$	$P_{critical} = 0.088\ MPa$ $P_{final} = 0.052\ MPa$

Figure 4- Experimental analysis of an initially unstressed thick cylindrical tube $\Delta L/L = 0.316$.



$P_i = 0\ MPa$	$P_{critical} = 0.082\ MPa$ $P_{final} = 0.052\ MPa$

Figure 5- Experimental analysis of an initially unstressed thick cylindrical tube $\Delta L / L = 0.474$.

Here again the thick membrane reaches a limit point characterized by the formation of a bubble and a sudden drop in the internal pressure is observed. Performing the same experiment in the same membrane several times it was possible to observe a drop in the critical pressure, probably, caused for the imperfections in the material done by the local buckling.

3. Numerical Analysis

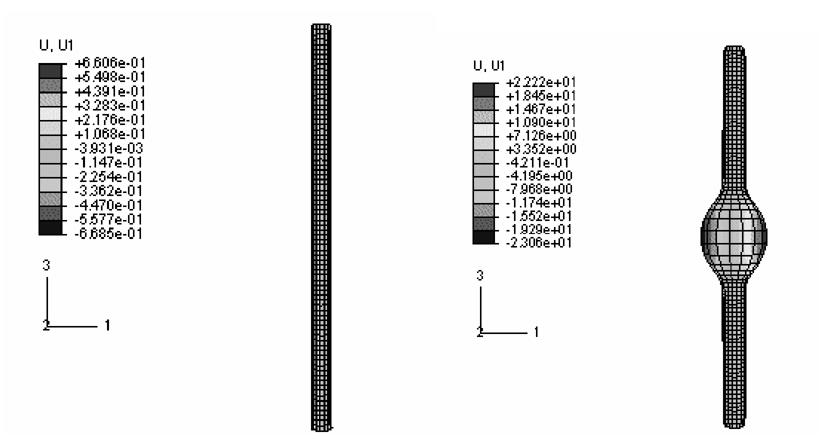
The numerical analysis for the thick membrane under hydrostatic pressure with the same geometry of the experimental analysis was performed with a mesh of *1230* nodes and *1215* shell elements *S4R* (quadrilateral with reduced integration).

The tube is considered to behave as made of Mooney-Rivlin material, *W*, and the constants of (1) are obtained comparing the experimental results for the cylindrical thick membrane under traction with the numerical solutions are: $C_1=0.0872MPa$ and $C_2=0.1508MPa$.

$$W^* = C_1 I_1 + C_2 I_2 \tag{1}$$

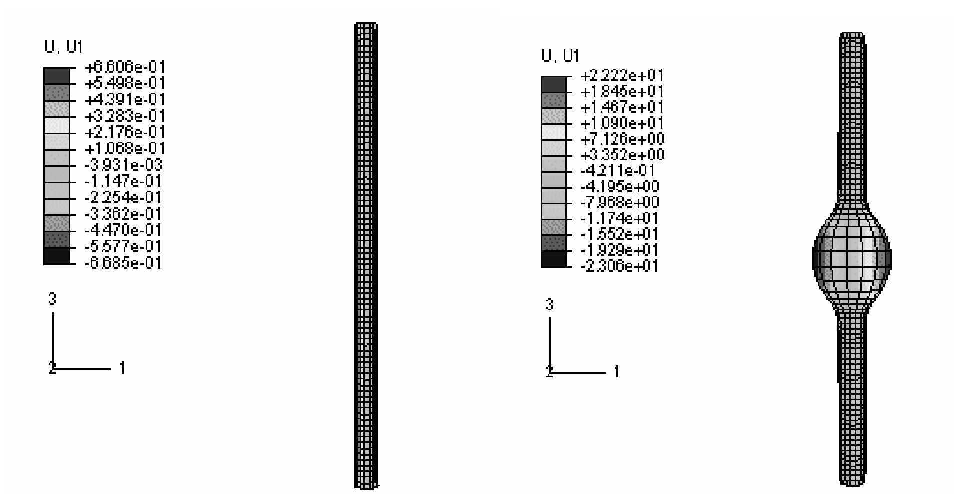
where I_1 and I_2 are the principal strain invariants.

The initial and final configurations are illustrated in Figures 6-8, where one can observe the good qualitative agreement between the numerical and experimental results.



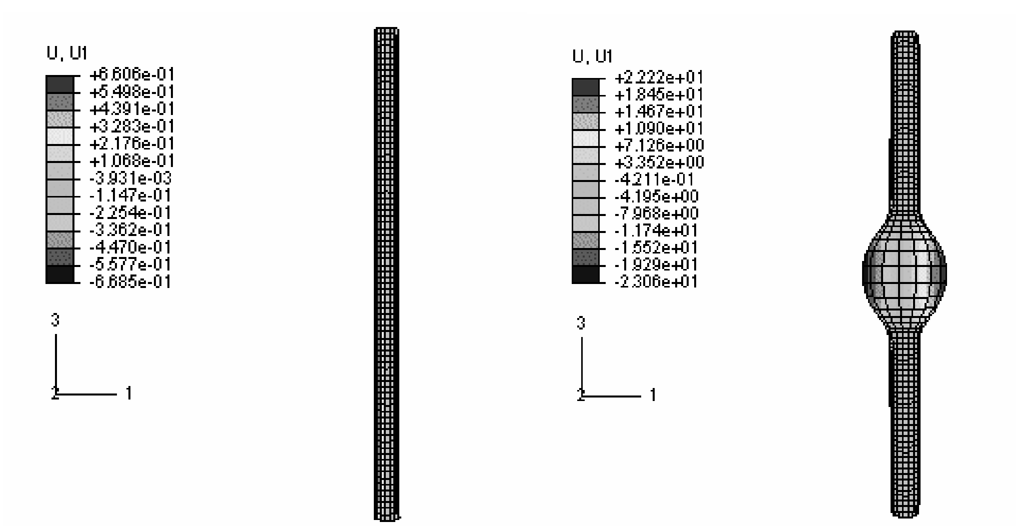
(a) (b)

Figure 6 – Shell deformation pattern. (a) Shell initially under traction; (b) post-buckled configuration after the critical pressure of *0.1033 MPa*, $\Delta L / L = 0.158$.



(a) (b)

Figure 7 – Shell deformation pattern. (a) Shell initially under traction; (b) post-buckled configuration after the critical pressure of *0.0922 MPa*, $\Delta L / L = 0.316$.



(a) (b)

Figure 8 – Shell deformation pattern. (a) Shell initially under traction; (b) post-buckled configuration after the critical pressure of *0.0834 MPa*, $\Delta L / L = 0.474$.

4. Discussion and Conclusions

In this work the finite deformations of an isotropic circular cylindrical tube subjected to a finite extension and gradually filled with water were investigated both theoretically and experimentally. Theoretical and, particularly, experimental investigations of thick membranes under variable pressure and traction are scarce in the literature. Nonetheless this is a problem of importance in many engineering fields including some relevant biomedical problems.

The agreement of the experimental and numerical results, as can be seen in Fig.9, is rather encouraging and indicates that the present formulation can satisfactorily model the deformation field under consideration.

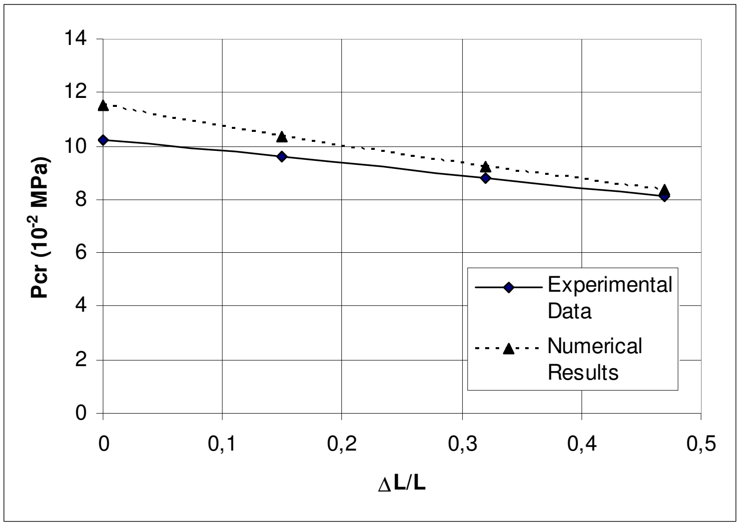


Figure 9 – Critical pressure x extension of the latex tube.

Also the experimental results presented here, covering a large collection of cases, can be used as a benchmark for future theoretical and numerical works in this area. An important instability phenomenon was observed during the experiments and numerical analysis with thick membranes, at a certain load level the membrane looses its stability forming a local bubble along its length. The critical pressure is a function of the membrane geometry and initial traction. This type of buckling mode is a characteristic of thick membranes under internal pressure and may explain the certain instability phenomena as localized buckling in tubes and aneurysms. As observed both experimentally and analytically, as the water is injected, the membrane deforms symmetrically and the internal pressure increases continuously until a maximum pressure is reached, after which the pressure decreases as more water is injected into the tube. This occurs because after this limit point the ratio of increase of the volume enclosed by the membrane is larger than the volume injected by the apparatus, leading to a decrease of the internal pressure. Also the numerical and experimental results presented here, is part of a large collection of cases, can be used as a benchmark for future experimental and theoretical works in this area.

5. Acknowledgments

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