RHEOLOGICAL BEHAVIOR OF DENSE GRANULAR MEDIA

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Abstract. *The understanding of the physics of granular media is an old call. The study of deformation process in such material has lead to very valuable interpretations of powder mechanical behavior. The flow of granular material is an important issue in several industries. Granular medium is a set of distinct solid particles inside an interstitial fluid, which interact only in the contact points and move in a variable grade of independence, ones respect to others. Despite the apparent simplicity of such particulate systems, the actual behavior of these materials is complex, embracing hybrid properties of solid and gas. The problems related to grain flow are widely spread in some engineering field and are research subject in Physics. The most recent contributions to the granular media rheology have intended deal with phenomena linked to steady arc formation (Fayol´s dome), with the consequent interruption of the granular flow. This article treats on granular media behavior in bins and underground ore passes (or chutes) and displays results of assays in physical models. The following parameters were changed in the experimental campaign: chute inclination, grain size and material ltype, height of the material column, discharge section of chute and flow trajectory (straight chute or with dog-legged one). The widespread use of gravity flow, the cost of production losses, safety reasons and the opening of new underground mines strengthen the need of studies in this area.*

Keywords*: gravity flow, granular media, bins, ore pass, granular flow*

1. Introduction

Granular media is an important issue in many technical instances. The study of deformation process in such material has lead to very valuable interpretations of powder mechanical behavior. The flow of granular material is complex and dependent of several parameters.

The term rheology came from Greek linked to verb "to flow". After Doraiswamy (2002): "the name *rheology* was proposed to describe the study of the flow and deformation of all kind of matter by Bingham e Reiner, in 1929".

The understanding of the physics of dense granular media is an old call. The flow from confined reservoirs is very complex and depends on several parameters, related to the material and to the reservoirs. The study of deformation process in such material has lead to very valuable interpretations of powder mechanical behavior. The flow of granular material is an important issue in several industries. The problems related to grain flow are widely spread in some engineering fields and are research subject in Physics.

Granular medium is a set of distinct solid particles inside an interstitial fluid, which interact only in the contact points and move in a some grade of independence, ones respect to others. Despite the apparent simplicity of such particulate systems, the actual behavior of these materials is complex, embracing hybrid properties of solid and gas. System of non-stick solids have various properties of a fluid (McCabe e Smith, 1956). They exhibes pressure on side and bottom walls of the reservoirs and flow through excavations or through discharge chutes. The flow is usually dense, because dissipation occurs by friction and collisions. Under slow deformations, granular material behaves as a solid. If under strong vibration, it resembles a gas. In this paper, formal treatment is made in order to clarify the subjacent mechanisms and phenomena linked to rheology of granular media

2. Subject Review

The bulk density of a granular medium in rest usually differs from that density during the streaming or draining. Arteaga and Tüzün (apud Humby, 1998) have used acrylic and acrylo-butadiene-styrene beads and radish and turnip seeds to study flow. From those experimental data, one can see that dynamic density is about 99.8 % static density in mono-sized system. Although, Fickie (apud Verghese and Nedderman, 1995) reports flowing density values of up to 0.7 times the corresponding value in rest.

When flow in granular media starts, the bulk density migrates to a certain critical flowing value. This point is called critical *dilatancy* (also called Reynolds´ dilatancy according to Santomaso and Canu, 2001).

The friction is an important parameter to the behavior of material in flow. The gravity force and the friction control the behavior of large particles, with 20 cm or more (Kvapil, 1965). Due to particle interlocking, friction between the wall and the material, there is a tendence to decrease the effect of the weight and reduce the pressure on the recipient bottom. This fact may induce the arching in chutes. According McCabe and Smith (1956), for several materials, if the height of material column exceeds three times the bin diameter, additional material on top doesn't affect this pressure on the recipient bottom.

The most recent contributions to the granular media rheology have intended deal with phenomena linked to steady arc formation (Fayol´s dome), with the consequent interruption of the granular flow. This article treats on granular media behavior in bins and chutes and displays results of assays in physical models.

At discharge, the material flows, leaving a died zone, closed related to the internal friction angle. The discharge slope should be equal or larger than the effective internal friction angle for the occurrence of the flow. This angle varies with particle size, particle shape and moisture content. According Johanson (1968) the main limitation to mass outflow hoppers is the need of larger slopes, usually more than 65°. The friction between the wall and the material enlarges this need and contributes for the vessel construction cost.

2.1. Theoretical background

In an inviscid liquid there is no energy dissipation and in absence of a rotational component the draining from a reservoir can be described (in steady state) by Torricelli's law. So, one can calculate instant volumetric flowrate (***Q_v***) for conical or pyramidal reservoir using this equation in function of fluid head (***z(t)***) at instant ***t***:

$$Q_v = C_f \times A_o \times \sqrt{2 \times g \times z(t)} = C_f \times A_o \times \sqrt{2 \times g} \times \left\{ z_{\max} - \left[\frac{2.5 \times C_f \times A_o \times \sqrt{2 \times g} \times tg^2(\beta)}{k_{shape}} \right] \times t \right\}^{\frac{1}{5}} \quad (1)$$

Where:

- ***Q_v*** – Newtonian fluid volumetric throughput [m^3/s];
- ***A_o*** – outlet cross-sectional area [m^2] (***A_o*** = ***k_{shape}x D_{out}^2***);
- ***Cf*** – discharge coefficient [-];
- ***k_{shape}*** – outlet morphological coefficient [-] (for circle: ***k_{shape}*** = π);
- ***z_{max}*** – fluid surface height at ***t*** = 0 [m];
- ***t*** – leaking time in steady state [s];
- β – angle between the horizontal and the generator of inverted cone (or pyramid wall, see Fig 1) [°].

Part of the pressure in granular material is dissipated against the vessel wall and the exponent for z is less than 0.5, as in a true fluid. So, after a characteristic head (typically equal to vessel diameter, ***D***), the flow rate is practically constant (Khanam and Arunabha, 2005). So, the outflow equations ahead hold only for the early phase of draining when: ***$z(t) \geq D$***.

In a non-inviscid fluid, the energy dissipation mechanism can be ascribed to a property called viscosity, which encompasses momentum transfer and friction phenomena. On the other hand, in an expanded granular material such a feature can be expressed in terms of a corresponding "granular viscosity" (Dartevelle, 2003), which is defined as:

$$\eta_{gr} = \rho_s \times d_m \times \sqrt{\theta} = \frac{\rho_{ap} \times d_m \times \sqrt{\theta}}{c_v} = \frac{\rho_{ap} \times d_m \times \sqrt{\Theta}}{(1-\varepsilon)} \quad (2)$$

where:

- ρ_{ap} – bulk (or apparent, or poured) density [$kg.m^{-3}$];
- ρ_p – particle density [$kg.m^{-3}$];
- d_m – mean free path [m];
- c_v – solid volumetric concentration [-];
- ε – granular media porosity [-];
- Θ – granular temperature [$m^2.s^{-2}$]

The so-called granular temperature is proportional to the mean quadratic velocity of the random motion of the particles (v_{ran}):

$$\Theta = \frac{1}{3} \times v_{ran}^2 \quad (3)$$

The rheology becomes more complicate in high solid concentration. The dynamic of a granular flow depends on particle concentration and particle size as well. When particle concentration is high and particles are pressed ones against the others under an external force field, Eq. (2) and Eq. (3) do not hold anymore.

Parbery and Roberts (1986) studied the flow in duct leaving a reservoir. To keep the analysis in terms of fluid draining, they had incorporated the peculiarities of the flow of granular media to the concept of an *equivalent friction coefficient*. In the case of chute with rectangular cross section, the following expression for such an equivalent friction coefficient was recommended:

$$\mu_{eq} = \mu \times \left[1 + k_{pr} \frac{H}{B} \left(1 + C_p \times v^2 \right) \right] \tag{4}$$

Where:

- $\boldsymbol{v}$ – material point velocity with arc coordinate s [m/s];
- $\boldsymbol{\mu}$ – friction coefficient between granular material and pipe (trough) walls [-];
- $\boldsymbol{H}$ – height (depth) of the flowing film [m];
- $\boldsymbol{B}$ – effective width of the duct [m];
- $\boldsymbol{k_{pr}}$ – constant representing the pressure gradient;
- $\boldsymbol{C_p}$ – coefficient linked to the movement among particles.

Similarly, Santomaso and Canu (2001) have studied granular flow in inclined chutes and had established velocity profiles characterizing three regions (from bottom to top): stationary layer (with thickness about two particle diameters), frictional flow layer (with thickness about four particle diameters) and fast flow layer (the more dilated one). To keep the analogy with fluid streaming, they had used the concepts of pseudo-viscosity, given by the equation:

$$\eta_{gr}(\varphi_{din}) = \frac{k}{\frac{d\varphi}{dh}} \tag{5}$$

where: $\boldsymbol{k}$ is a constant factor, $\boldsymbol{h}$ is granular bed height (thickness) and φ_{din} is the dynamical friction angle, which varies with changing of $\boldsymbol{h}$ (because bed dilatancy is not constant). Silva and Luz (2004) has studied granular flow in tubes using simple experimental setup and have found that, normalizing flow velocity by multiplying it by slope angle cosine, area averaged velocity can be expressed by ($\boldsymbol{V_{ref}}$ and $\boldsymbol{C^*}$ are characteristic parameters):

$$v = v_{ref} \times \left[1 - e^{-\left(\frac{D/d_{max}}{C^*} \right)} \right] \tag{6}$$

. Stress in granular solids in small systems (mesoscopic on the scale of the grains) propagates by wave-like (hyperbolic) equations. However, in large systems (more common in engineering practice), the response is closer to that predicted by traditional isotropic elasticity models (elliptic equations). Static friction, often ignored in simple models, increases the elastic range and renders the response more isotropic (Goldberg and Goldhirsch, 2005).

In demand of fast way to foresee the flowability characteristics of powders, one can use some flow behavior indexes like the *Hausner ratio* and the *Carr's compressibility index*.

The Hausner ratio is given by (Conesa et alii, 2004):

$$HR = \frac{\rho_N}{\rho_{ap}} \tag{7}$$

In last equation, the numerator is the tapped density after $\boldsymbol{N}$ taps (since maximum density variation between ***N-th*** tap and the previous one is 2 %). The DIN ISO EN787 specification states tapping frequency of 250 times per minute (4.17 Hz) and stroke height of 2 mm. Usually material displaying Hausner ratios above 1.3 are supposed have poor flowability.

The Carr's compressibility index is calculated by:

$$C_{carr} = \frac{V_0 - V_N}{V_0} = \frac{\rho_N - \rho_{ap}}{\rho_{ap}} \tag{8}$$

Here, V_0 and V_N are the sample volumes before and after compaction (tapping), respectively and ρ_{ap} is the bulk density of unpacked powder ($N = 0$).

The flow patterns are defined by how granular material flows inside the silo or bin. Three patterns are common: the funnel flow, the mass flow and the hybrid flow. In the mass flow, the movement channel coincides with the walls of the silo and the hopper, and it is characteristic of bins where the height (h) is greater that 1.5 times the diameter (D). In funnel flow, the material flows through a narrow preferential vertical channel. The involving material outside to the channel remains stationary. The hybrid flow is a combination of the previous two regimes (Kelly and Spottiswood, 1982).

The minimum hopper angle (equal to the half of the conical vertex angle) for mass flow occurrence can be quantified by the following regression equation, which has been made from Raymus´ data (Raymus, 1984):

$$\theta_{cone} = -0.0072 \times \varphi^2 - 0.445 \times \varphi + 37.39 \quad (9)$$

For preliminary sizing purpose, McCabe and Smith (1956) have recommended the empirical equation (shown below) to calculate the gravity mass flowrate from a tall bin with conical hopper (modified for S.I. units):

$$Q_p = \frac{1{,}2 \times 10^{-5} \times \rho_p \times (39.37 \times D_{out})^n}{[6.288 \times tg(\varphi) + 23.16] \times (d_p + 0.048) - 1.1405} \quad (10)$$

Where:

- Q_p – granular media mass throughput [kg/s];
- D_{out} – outlet diameter [m];
- φ – angle of internal friction of the material [º];
- ρ_p – particles density [kg/m^3];
- d_p – average particle diameter [m].

The value of the exponent ***n*** depends on the of particles morphology. Thus, for irregularly shaped particles, one has n = 2.8, while for spheres, n = 3.1. However eq. (10) do not agree with experimental data from Luz and Silva (2002).

The classical work of Beverloo (Verghese and Nedderman, 1995) treats on free-flowing discharge material through a circular orifice in the base of a flat-bottomed cylindrical bin. The flow rate equation is:

$$Q_p = C' \times \rho_p \times c_{vdyn} \times \sqrt{g} \times (D_{out} - Z')^{\frac{5}{2}} \quad (11)$$

Where ***C'*** is opening discharge coefficient (***C'*** = 0.58), C_{vdin} is flowing particle concentration in volume (dynamical condition) and ***Z'*** is linked to the "empty annulus", that is a correction due particle collisions against walls, somewhat similar to the "vena contracta" in Newtonian fluids. The formula for width of empty annulus, ***Z'***, usually adopted is:

$$Z' = k' \times d_p \quad (12)$$

where ***k'*** is a constant. Verghese and Nedderman (1995) have attributed for ***k'*** the value of 1.5, but Ateaga and Tüzün (apud Humby et alii, 1998) propose, for binary mixtures (one coarse fraction and one fine fraction) that: ***k'*** = 1.85 when coarse phase is continuous and ***k'*** = 1.4 in the opposite case (fine continuous).

Alternatively, the proposed equation by Humby et alii (1998) for mass flowrate (which is a modification of Beverloo equation) is:

$$Q_p = C' \times \rho_p \times c_{vdyn} \times \sqrt{g} \times (D_{out} - Z')^{\frac{5}{2}} \quad (13)$$

In binary systems without segregation, Humby et alii (1998) have found this relationship between empty annulus and the effective diameter (d_{eff}):

$$Z' = 5.8573 \times d_{eff} - 0.0032 = 5.8573 \times \left(\frac{\sum X_i \times d_{pi}^3}{\sum X_i \times d_{pi}^2} \right) - 0.0032 \quad (14)$$

In last equation X_i is the volume proportion of component i.

For conical hopper Verghese and Nedderman (1995) suggest the following equation:

$$Q_p = \left[0.5 \times \tan^{-0.35} \alpha \times \sqrt{1 - \frac{1.46 \times 10^{-8}}{d_p^2}}\right] \times \rho_p \times c_{vdyn} \times \sqrt{g} \times \left(D_{out} - k \times d_p\right)^{\frac{5}{2}} \tag{15}$$

Where α is the half angle of the cone.

Formal and mathematical approach for granular material flow is important, but it is limited yet. Empirical relationships from experimental results by using physical models were reported by Rose and Tanaka (1959), McCabe and Smith (1956) for discharge rate from bins; Blight and Haak (1994) validated Janssen's equation for tensions in inclined chutes.

Molodtsof and Ould-Dris (1993) have reported a theoretical approach to foresee outflow rate from hoppers, examining the effect of several parameters on flow rate, including cohesion. The authors concluded that the square of the flow rate varies linearly with the outlet dimension, and that this rate is independent of material head.

As mines are concerned, among the gravity flow phenomena of bulk material in excavations, the main problem is the arching. Mining operation usually produces lumps with 40 cm or more. Lumpy material with negligible content of fines tends to mechanical arching (interlocking). These arches are caused also for changes on direction or pass geometry (as in fingers, knuckles, dog-legs) or changes in discharge section, in the draw points. The probability of the mechanical arch formation depends on the percentage of coarse particles in bulk material, the particle shape, the outlet dimension to particle size ratio (D_{out}/d_p) and velocity distribution. The D_{out}/d_p ratio is the main empirical rule used in mines. Silva and Luz (2002) has compiled project criteria to improve mine chute.

By the other hand, when the bulk material has too much fines, it may display cohesive arching, also depending on parameters such as moisture content and consolidation time.

3. Materials and Methods

3.1. Materials

The granular material used was glass beads, sand and graded aggregate (crushed and screened gneiss rock). Glass spheres in the following commercial specifications: 3R; AB; AC; AD; GAC and AH (Zirtec company) were used. Dolomite was used in three different sizes (table 1). Stone aggregates for concrete and sand were used in the tests of mine chute simulation (table 2). The physical models used were:

- Cylindrical hopper (bin/silo) with openings of diameter 6.9 mm and 21.2 mm (see Fig 1).
- Prismatic hopper (bin/silo) of square section with rectangular bottom openings of 11 mm by 5.5 mm and of 22 mm by 17 mm (see Fig 1).
- Wooden mine chute model in 1:20 scale in order to simulate a typical underground mine chute (see Fig 1).

Table 1 – Tested Material Properties (bins and hoppers)

Material	Size Range (µm)	Particle density [kg/m^3]	Bulk density [kg/m^3]	Characteristic angles	
				Dynamical repose	Material / wall friction (without rolling)
Dolomite	600 to 800	2820	1440	24.7	37.6
Dolomite	105 to 210	2820	1440	30.0	27.3
Dolomite	210 to 600	2820	1440	23.0	31.5
Glass beads 3R	600 to 800	2490	1460	19.5	19.7
Glass beads AB	177 to 297	2490	1420	16.0	22.0
Glass beads AC	149 to 250	2490	1420	21.8	18.8
Glass beads AD	105 to 210	2490	1420	20.8	20.9
Glass beads AG	53 to 105	2490	1420	21.7	19.3
Glass beads AH	53 to 105	2490	1420	22.8	19.3

Table 2 - Tested Material Properties (mine chute)

Material	Size Range (mm)	Particle density [kg/m^3]	Bulk density [kg/m^3]	Dynamical repose angle
Sand	1.2	2650	1420	45.7

Dolomite	8.7	2960	1680	32.3
Dolomite	25.2	2860	1720	34.5
Gneiss	12.0	2730	1420	32.9
Gneiss	18.9	2730	1500	32.0
Gneiss	47.8	2650	1360	33.3

3.2. Methods

The silo models aimed to test the draining of glass spheres and dolomite grits, both for two sizes of the discharge opening. The silo models had been fed completely with the material before the bottom discharge opening. Samples were collected at appropriate time intervals in order to assess the instant flow rate. The process was continued up to flow pattern abrupt changing or ending of material in hopper (crater formation and annular flow).

Experiments in the mine chute model were performed to measure gravity outflow rate, varying the following parameters in the experimental campaign: chute slope, particle size, and height of the material column, discharge section of chute and flow trajectory (straight chute or with doglegged one).

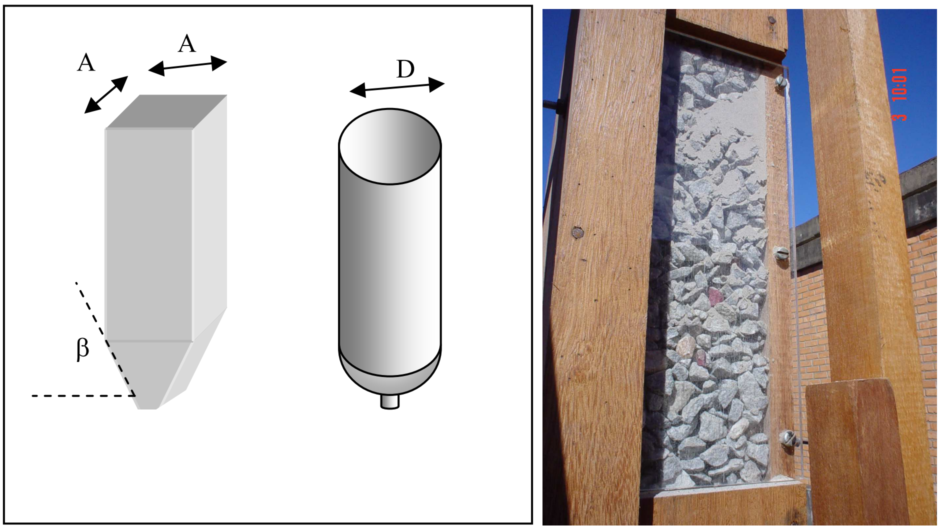

Figure 1. Prismatic and cylindrical bins used in confined gravity flow tests (A = 95 mm; D = 96.9 mm; edge slop: $\beta = 60\ ^{o}$) and mine chute model used in confined gravity flow tests (Fines segregation and red tracers can be seen across acrylic inspection window).

4. Results

4.1. Flow in bins and hoppers

Figure 2 displays flowrate evolution in draining for two opening size both cylindrical and prismatic hoppers or bins.

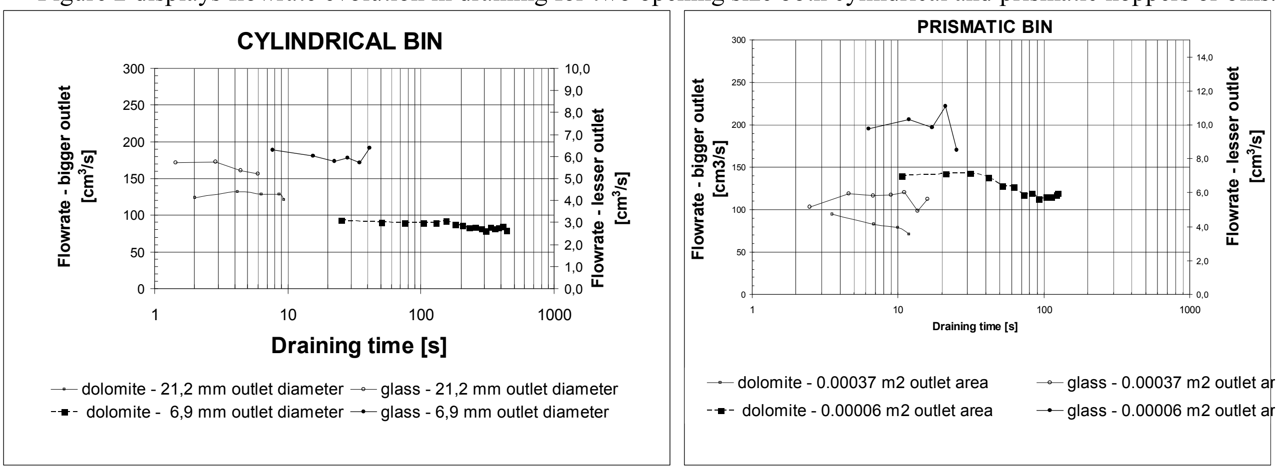

Figure 2. Effect of particle shape and density on discharge rates from bins (d_p = 0.7mm).

4.2. Flow in mine chute model

The most common phenomenon observed was mechanical arching that is supposed to occur, when the chute opening to maximum size of particles ratio is less than 3.0. Also was observed funnel flow (detected by tracers), and fine production (comminution). The arching for each outlet dimension and maximum particle size ratio (D_{out}/d_{max}) was respectively: D_{out}/d_{max} = 3: 44% of the tests; D_{out}/d_{max} = 5: 2%; D_{out}/d_{max} = 6: 4%; D_{out}/d_{max} = 7: 63%; D_{out}/d_{max} = 10: 12%; D_{out}/d_{max} = 14: 0%; D_{out}/d_{max} = 50: 100%; D_{out}/d_{max} = 100: 40%.

Tabel 3 summarizes tests results in strait chute (ore pass model without dogleg, rectangular section of 12 cm x 16 cm) varying material and chute slope

Table 3 – Discharge rates of sand and crushed dolomite and gneiss in chute model

Material head (Filling Level) [m]	Volumetric Flow Rate [$10^{-6}m^3/s$]								
	Sand (D_{out}/d_p =100), (d_{90}=1.2 mm) (ρ_{ap}=1.4 t/m^3)			Crushed dolomite (D_{out}/d_p =14), (d_{90}=8.7 mm) (ρ_{ap} =1.68 t/m^3)			Crushed gneiss (D_{out}/d_p =10), (d_{90}=12.0 mm) (ρ_{ap} =1.42 t/m^3)		
	α = 60 º	α =75 º	α =90 º	α =60 º	α =75 º	α =90 º	α =60 º	α =75 º	α =90 º
1.0	14,347	19,252	21,540	-	-	-	14,077	18,514	20,776
2.0	12,474	15,069	18,794	13,638	-	22,291	15,409	20,295	21,213
3.0	11,443	12,420	13,687	-	-	-	16,267	19,655	21,237

Material head (Filling Level) [m]	Volumetric Flow Rate [$10^{-6}m^3/s$]								
	Crushed gneiss (D_{out}/d =6), (d_{90}=18.9 mm) (ρ_{ap} =1.50 t/m^3)			Crushed dolomite (D_{out}/d =5), (d_{90}=25.2 mm) (ρ_{ap} =1.72 t/m^3)			Crushed gneiss (D_{out}/d =3), (d_{90}=47.8 mm) (ρ_{ap} =1.36 t/m^3)		
	α = 60 º	α =75 º	α =90 º	α =60 º	α =75 º	α =90 º	α =60 º	α =75 º	α =90 º
1.0	13,193	15,310	16,973	13,500	14,701	14,493	13,848	14,906	14,276
2.0	13,282	16,047	18,553	10,464	12,079	12,472	9,476	10,832	9,244
3.0	13,612	15,905	17,241	10,096	11,898	12,443	10,001	9,927	7,692

Table 4 displays outflow rates for two slopes and for several D_{out}/D_{max} in chute model. The previsions by McCabe and Smith equation and Beverloo equation (and its modified forms) do not agree with chute outflow obtained here. For the 4.0 scale-up factor for area would be expected flowrate increase of about 7.0 for Eq. (10) and 5.7 for Eq. (11). Comparing the results for sections, one can see that both previsions were very different to the actual rates for several materials (from table 4, the grand average value displays an outflow increase of 13.6 times, when the cross sectional area increases 4 times).

Table 4 – Discharge rates of sand and crushed materials in chute model without dogleg, for two rectangular cross sections.

Mine chute Outlet [mxm]	Volumetric Flow Rate [$10^{-6}m^3/s$] with 2.0 m of inicial material head											
	Sand (d_{90}=1.2 mm)			Crushed dolomite (d_{90}=8.7 mm)			Crushed gneiss (d_{90}=12.0 mm)			Crushed gneiss (d_{ap}=18.9 mm)		
	Chute slope			Chute slope			Chute slope			Chute slope		
	60 º		90 º	60 º		90 º	60 º		90 º	60 º		90 º
0.08 x 0.06	0		0	1,371		1,770	1,129		1,471	978		920
0.12 x 0.16	12,474		18,794	13,638		22,291	15,409		21,213	12,474		18,794

5. Conclusion

The widespread use of gravity flow, the cost of production losses and safety reasons strengthen the need of studies in this area. The minimum outlet dimension for free flowing occurrence in bins should be 8 to 10 times the maximum particle dimension, according to Reed (1991); for flow in underground excavations, this relation varies in the 4 to 6 range, as already registered in several works.

Considered the same head and same relation $\boldsymbol{D_{out}/d_{max}}$, it can be observed that the discharge rate increase according increasing of chute slope, except to $\boldsymbol{D_{out}/d_{max}}$ less than 3.

As a final remark, at least as authors´ knowledge is concerned, the equations listed in literature, did not hold in the situations studied here (like those presented in figure 2 and table 4), suggesting a strong need to continue research efforts. Furthermore, the usual equations do not include the effect of head variation as draining evolves.

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8. Responsibility notice

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