HOW TO PREDICT THE VIBRATION EFFECT DUE TO THE MACHINE OPERATION ON ITS VICINITY

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Abstract. *The satisfactory performance of a machine is concerned with its installation, i.e., with a well-designed foundation and soil investigation. The vibration amplitude of the machine-foundation system must be kept under permissible values generally specified by the machine's manufacturers and natural frequencies must be determined to avoid resonance. There may be situations when amplitudes of the machine may be within the acceptable limits for satisfactory machine operation but the vibrations may affect other machines around, be harmful to adjacent structures and precision equipment and annoying to persons. Here in will be shown how to predict in a simple way the effect of the vibration due to the machine operation on its vicinity considering the foundation as a vibration source on the surface of an elastic medium.*

Keywords*: structural vibrations, machine-foundation, wave propagation, vibration isolation*

1. Introduction

All machines generated unbalanced dynamic loads (forces and moments) that induce vibrations. These dynamic loads are caused by machine operation condition such as wear, imbalance, misalignment, etc, and must be kept under a safety limit by the maintenance personal for the satisfactory operation of the entire plant.

Another machine's vibration problem is caused by resonance. The resonance condition is reached when one or more frequencies of the exciting loads are near or equal to any of the natural frequencies of the machine-foundation-soil system. Unfortunately the resonance problem can't be readily solved because a structural modification will be necessary to bring the vibration level to accepted values.

The vibration generated by the machine operation condition is then transmits through the soil by the block foundation what may affect the vicinity in a bad way. A vibrating footing is then a source of wave generation in the soil.

Herein the response of a block foundation embedded in the soil and excited by a vertical harmonic load will be outlined and used to predict vibration response of the foundation. Also the problem of the vibration propagation through the soil due to concentrated dynamic load acting on the soil surface will be treated to estimated the decrease in amplitude of the vibration with distance from the source and to evaluated the effects of the vibration on the performance of other equipments in the vicinity and structures around.

2. Vibration Propagation Through the Soil Due to Periodic Concentrated Load

The problem of the surface wave propagation in an elastic medium (elastic half-space) was first studied by Lord Rayleigh (1885) and is known as the *Rayleigh* wave. Later Lamb (1904) studies the response of the medium as it was excited by oscillating vertical force or pulse loading acting at a point or along a line on the surface and within the body. See fig. (1).

Lamb's solutions for calculation of vertical surface displacement at a distance r from the concentrated dynamic load on the surface of the medium may be written as following,

$$w = \frac{Pe^{i\omega t}}{Gr}\left(F_1 + iF_2\right) \tag{1}$$

in which,

P = amplitude of the concentrated oscillating force applied to surface area,
ω = circular frequency of force application,
G = shear modulus of the ground,
r = radial distance from the load,
i = imaginary number,
F_1, F_2 = dimensionless Dynamic Boussinesq Displacement Functions and given by the following equations,

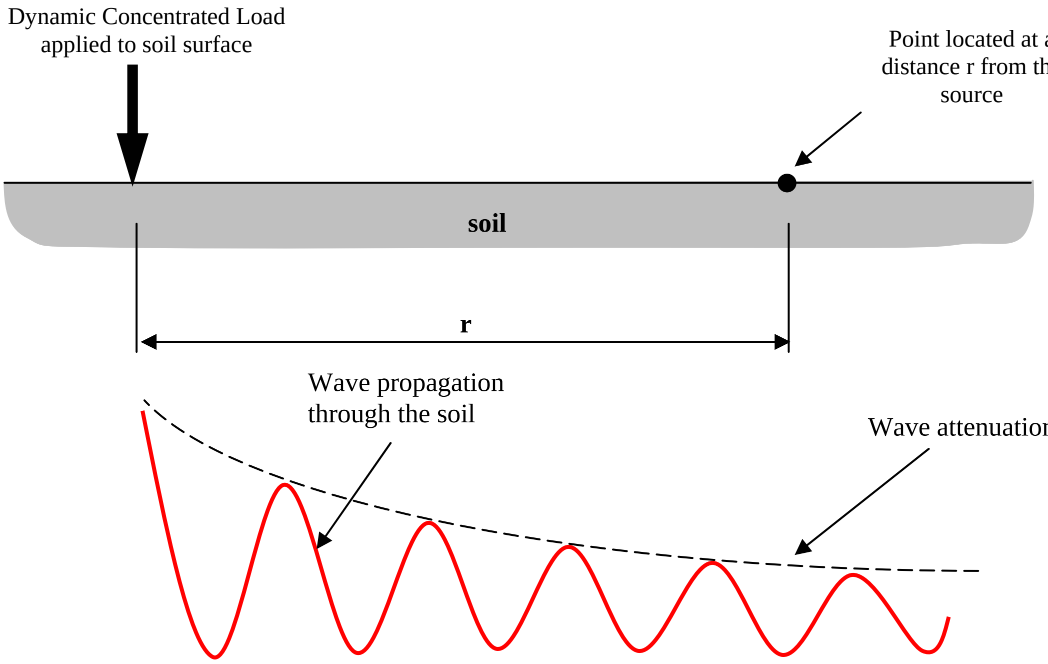

Figure 1. Dynamic concentrated load applied to soil surface and wave propagation through the soil.

$$F_1 = \frac{a}{2\pi}\left[\pi y K_d Y_O(ay) + \int_0^s \frac{x(s^2-x^2)^{0.5} Y_O(ax)}{(2x^2-1)+4x^2(s^2-x^2)^{0.5}(1-x^2)^{0.5}}dx \right.$$
$$+ \int_0^1 \frac{4x^3(x^2-s^2)(1-x^2)^{0.5} Y_O(ax)}{(2x^2-1)^4-16x^4(x^2-s^2)(x^2-1)}dx \qquad (2)$$
$$\left. + \frac{2}{\pi}\int_0^\infty \frac{x(s^2-x^2)^{0.5} K_O(ax)}{(2x^2-1)^2-4x^2(s^2-x^2)^{0.5}(1-x^2)^{0.5}}dx \right]$$

$$-F_2 = \frac{a}{2\pi}\left[\pi y K_d J_O(ay) + \int_0^s \frac{x(s^2-x^2)^{0.5} J_O(ax)}{(2x^2-1)+4x^2(s^2-x^2)^{0.5}(1-x^2)^{0.5}}dx \right.$$
$$\left. + \int_0^1 \frac{4x^3(x^2-s^2)(1-x^2)^{0.5} J_O(ax)}{(2x^2-1)^4-16x^4(x^2-s^2)(x^2-1)}dx \right] \qquad (3)$$

in which,

a is the dimensionless frequency and is defined by the following equation where ρ is the soil mass density,

$$a = \omega r\sqrt{\frac{\rho}{G}} \qquad (4)$$

ν is the Poisson's ratio of the soil and the s parameter is given by the equation below,

$$s = \sqrt{\frac{1-2\nu}{2(1-\nu)}}, \qquad (5)$$

J_O, is the Bessel function of the first kind and order zero;
Y_O, is the Bessel function of second kind and order zero;

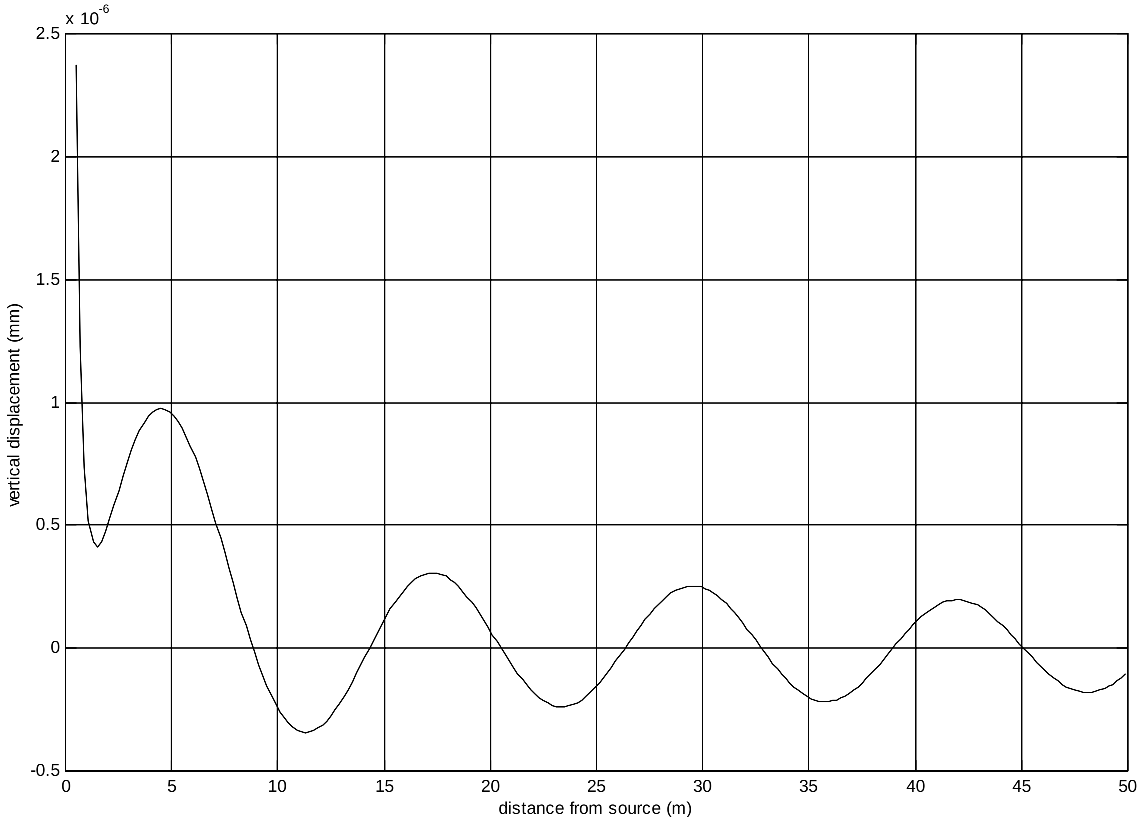

Figure 2: Surface displacement of an elastic medium due to an oscillating concentrated load with amplitude P=1(N) and a circular frequency ω = 62.83 (rad/s) or f=10(Hz). Soil data: G= 3.2x10^7 (N/m^2); mass density ρ=1800 (kg/m^3); Poisson's ratio ν=0.3. Rayleigh velocity V_R=123.7 (m/s) and wavelength λ=12.4 (m).

K_O, is the Modified Bessel function of order zero;
y is the positive real root of the equation below and is a function of Poisson's ratio ν only,

$$(2y^2-1)^2-4y^2(y^2-s^2)^{0.5}(y^2-1)^{0.5}=0 \tag{6}$$

K_d, is a function of the Poisson's ration ν and the value for y of eq. (6) and is expressed by eq. (7).

$$K_d=\frac{-(y^2-s^2)^{0.5}}{8y(2y^2-1)-\dfrac{4y(4y^4-3y^2(s^2+1)+2s^2)}{(y^2-s^2)^{0.5}(y^2-1)^{0.5}}} \tag{7}$$

A plot of the surface displacement caused by a unit-oscillating load with a circular frequency ω = 62,8 (rad/s) is shown in fig. (2). In this figure is possible to see how the amplitude of the surface wave decreases with the distance from source. The dimensionless functions F_1 and F_2 were numerically evaluated using the trapezoidal method. The surface wave, also known as Rayleigh wave, travels at a speed given by the equation below.

$$V_R=\frac{V_S}{y} \tag{8}$$

in which,

y is the value obtained using the eq. (6);
V_S is the shear velocity given by the eq. (9).

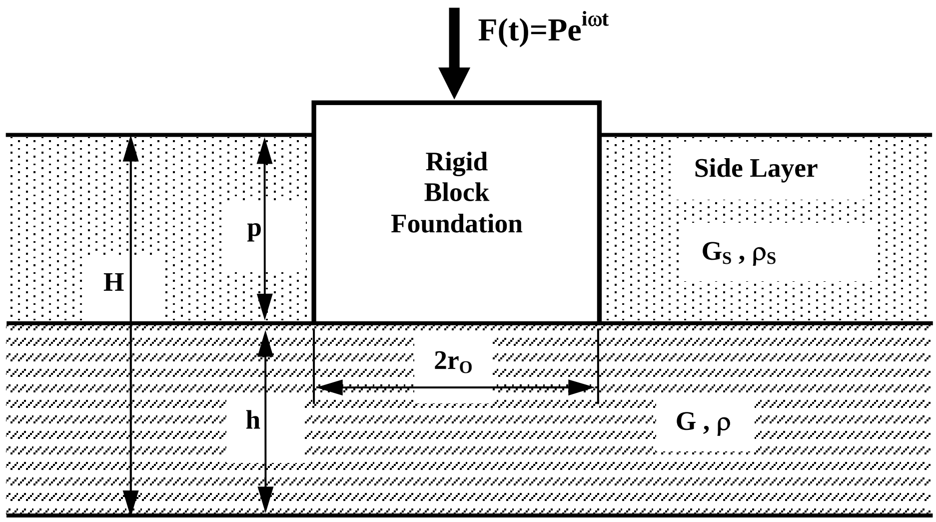

Figure 3: Schematic of a block foundation embedded in a soil and subjected to a periodic force F(t).

$$V_S = \sqrt{\frac{G}{\rho}} \tag{9}$$

3. Vibration of a rigid block foundation embedded on the Soil

The rigid block that supports machines may be of various kinds and is usually embedded on the soil as shown in fig. (3). The problem of a vibrating rigid circular footing on the surface of an elastic medium was examined first by Reissner (1936) and after by Sung (1953) and Quinlan (1956).

The solutions were modified for the purpose to obtain a solution for vertical vibrations similar to the equation for damped vibrations of a single degree of freedom system. See fig. (4). The vertical stiffness, k_{ZS} , and damping, c_{ZS} , for an embedded foundation are given by the following equations below (Prakash and Puri, 1988, Novak and Beredugo, 1972, Richart *et al.*, 1970).

$$k_{ZS} = Gr_O\left(C_1 + \frac{G_S}{G}\frac{p}{r_O}S_1\right) \tag{10}$$

$$c_{ZS} = \frac{Gr_O}{\omega}\left(C_2 + \frac{G_S}{G}\frac{p}{r_O}S_2\right) \tag{11}$$

Here, C_1 and C_2 are dimensionless parameters derived from the soil reaction at the base of the foundation, and S_1 and S_2 are parameters derived from the reactions acting on the sides of the block foundation. G and ρ are respectively the shear modulus and the mass density of the soil layer under the base of the foundation; G_S and ρ_S are the shear modulus and the mass density of the layer surrounding the sides of the block foundation as shown in fig. (3). The parameter p is the embedment depth. If the footing is circular, r_O is the radius of the footing. In the case of a rectangular, square, or other footing shape of area A_b the equivalent radius r_O is given by eq. (12).

$$r_O = \sqrt{\frac{A_b}{\pi}} \tag{12}$$

The parameters C_1 and C_2 depend on the depth H of the soil, Poisson's ratio ν, and the dimensionless frequency a_O given by eq. (13). The parameters S_1 and S_2 depend only on the dimensionless frequency a_O.

$$a_O = \frac{\omega r_O}{V_S} \tag{13}$$

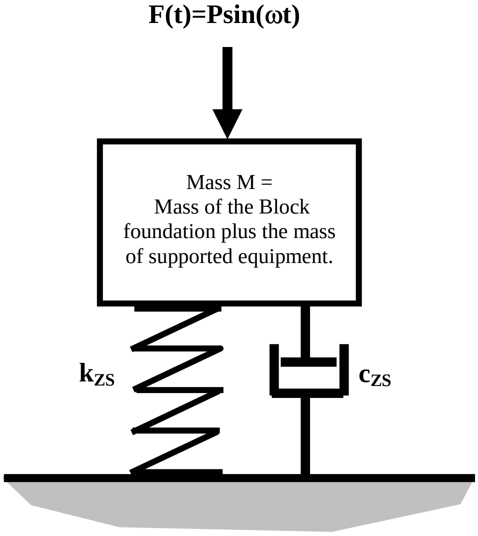

Figure 4: Schematic of a one degree-of-freedom model of a machine-foundation system.

The vertical amplitude w_O of the steady-state response of a single-degree-of-freedom system with respect to the harmonic forcing function F(t) is given by eq. 14.

$$w_O = \frac{P/k_{ZS}}{\sqrt{(1-\Omega^2)^2 + (2\xi\Omega)^2}} \tag{14}$$

in which, P is the amplitude of the harmonic forcing function F(t) as shown in fig. (3) and Ω is the frequency ratio defined by the following expression,

$$\Omega = \frac{\omega}{\omega_n} = \frac{\omega}{\sqrt{\frac{k_{ZS}}{M}}} \tag{15}$$

where ω_n is the natural frequency and M is the total mass, i. e., the mass of the inertial block and the equipment. ξ is the damping ratio defined by the expression below,

$$\xi = \frac{c_{ZS}}{2\omega_n M} \tag{16}$$

4. Dependence of the amplitude of Soil Vibration on Distance from the Source of Waves

A vibrating machine foundation generates waves in the soil. The energy of the oscillating footing is carried away by a combination of waves: the compression wave, shear wave and the Rayleigh wave. The energy density of each one of these waves decreases with distance from the footing. Figure 2 shows the decrease in amplitude of the wave with increased distance when a concentrated vertical load is applied at the surface of an elastic medium. The decrease in amplitude, as shown in fig. 2, can be expressed by the suggested expression below (Richart *et al.*, 1970, Barkan, 1962).

$$w = w_1\sqrt{\frac{d}{\chi}} \tag{17}$$

where,

w is the unknown amplitude at a distance χ from the source;

w_1 is the known amplitude at distance d from source; .

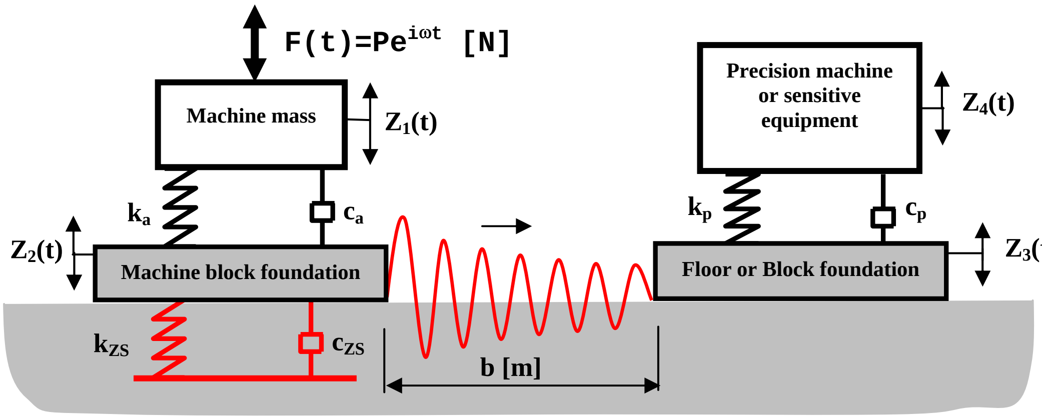

Figure 5: Schematic of active (left side) and passive (right side) isolation problem.

5. Effect on Surrounding of Waves from Industrial Sources

If a machine is directly attached to its block foundation, the vibration generated due to the normal operating condition of the machine is transmitted through the soil and may affect the performance of other machines in the vicinity or be harmful to adjacent structures even at large distance. On the other hand, it is sometimes possible to isolate the source of vibration from its surroundings (active isolation), or to isolate a device in such a way that the motion of its base caused by a disturbing source in the vicinity is reduced to acceptable limits (passive isolation). See fig. (5).

Herein, the problem of how to predict the effect due to the machine operation on its vicinity was analyzed considering three cases.

First, the problem was treated without the presence of isolation mounts, i.e., without active and passive isolators. In this case the machine was considered directly attached to the foundation and a single degree of freedom model was used to determine the amplitude of vibration of the machine and its foundation. The problem was solved applying eq. (14) with Z_1=Z_2= w_O. See fig. 6 for more details. Eq. 17 was modified to take in account the vibration attenuation with distance due to dispersion of the wave energy. The calculated vibration amplitude of the machine block foundation was considered as the known vibration amplitude at a distance d = r_0 . In this way, the eq. (17) becomes,

$$Z_3 = w_1 \sqrt{\frac{r_O}{r_O + b}} \tag{18}$$

In this case w_1=Z_1=Z_2 and Z_3=Z_4 due to the absence of passive and active isolators. A distance of b=1.5(m) was considered. See fig. 5 or fig. 6.

In the second case only active isolation was considered and a two-degree of freedom system was required to solve this problem. See fig 5 and fig 7. In this second case is necessary to consider w_1=Z_2 in eq. (18) and as in the first case Z_3=Z_4 due to the absence of passive isolators.

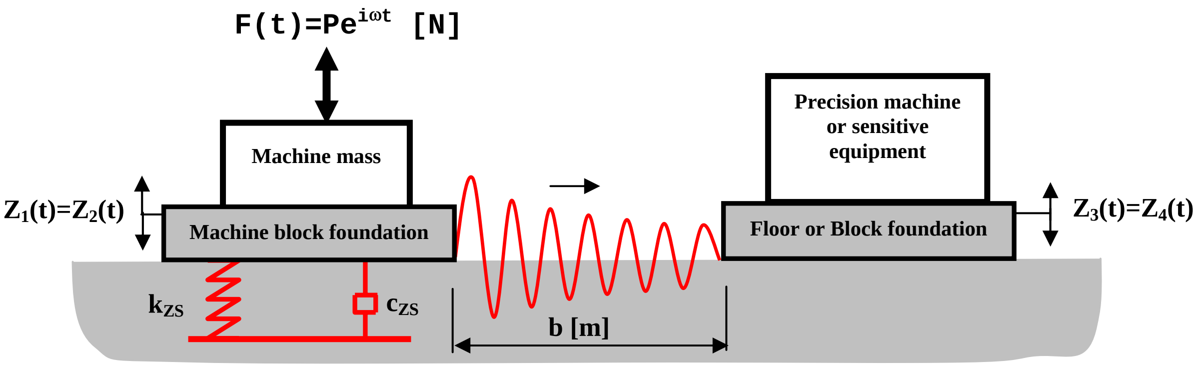

Figure 6: Schematic of the first case treated, i.e., without the presence of active and passive isolation mounts.

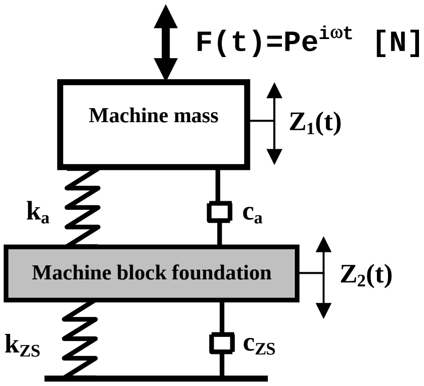

Figure 7: Schematic of the two degree of freedom of the system showing the machine with its active isolators and the block foundation with its equivalent soil stiffness and damping.

Finally both the active and passive isolators were applied. In this last case the transmissibility displacement ratio must be calculated using eq. (19) below (Inman, 1994).

$$\frac{Z_4}{Z_3} = \left[\frac{1+(2\xi_p \Omega_p)^2}{(1-\Omega_p^2)^2 + (2\xi_p \Omega_p)^2} \right]^{1/2} \tag{19}$$

In eq. (19), ξ_p is the damping ratio of the passive isolator and Ωp is the frequency ratio of the frequency of the oscillatory base motion ω_b to the natural frequency of the passive isolation system ω_{np}. The frequency of the base motion is equal to the frequency of the excitation, $\omega_b = \omega$.

Table 1 presents the data required to the computation. The results of the computation of each one of the cases is shown in table 2. The force amplitude used to excited the system was P=1500(N) at ω=188.5 (rad/s). The values to the natural frequency and damping ratio of the active and passive isolators were chosen to be equals to 5(Hz) and 0.05(/), respectively.

6. Conclusion

In the industry the problem of vibration is always of first concern and must be kept below permissible values for satisfactory operation of the entire plant. The goal of this paper was to predict the influence of the vibration in the vicinity due to operation condition of the machine in a simple way and aid the engineer to get some previous design requirements or remedial measures, in a low cost manner, by proper selection of the block size and contact area, active and passive isolators.

The problem of the vibration propagation through the soil due to concentrated dynamic load acting on the soil surface was treated and the results showed that the vibration attenuation with distance due to dispersion of the wave energy may be approximated by eq. (17) and used to estimated the vibration level that affect structures and delicate machines around as well cause discomfort to workers.

Table 1: Soil, Block Size and Mass, Machine data, dynamic soil parameters

Soil Data	Shear modulus	$G=3.14x10^7 (N/m^2)$
	Mass density	$\rho=1650\ (kg/m^3)$
	Poisson’s ratio	ν=0.25 (/)
Block Size and its mass	Lengthxwidthxthickness	1.50mx0.80mx0.60m
	Embedded depth	p=0.5m
	Block foundation Mass	M_B = 1728 (kg)*
Machine Data	Equipment Mass	M_E = 300 (kg)
	Speed	1800 rpm (30Hz)
Dynamic Soil Parameters	r_O	0.62m
	k_{ZS}	$1.34x10^8$ (N/m)
	c_{ZS}	$7.8x10^5$ (Ns/m)

***** The block foundation mass was calculated using the concrete mass density equal to 2400 kg/m^3

Table 2: Amplitudes Z_1, Z_2, Z_3, Z_4 of the vibratory movements of the system shown in fig. 5 to three specific cases.

First Case: without the presence of isolation mounts	
$Z_1=Z_2=w_O$	10(μm)
$Z_3=Z_4$	5.4(μm)
Second Case: active isolation	
Z_1	140(μm)
Z_2	0.3(μm)
$Z_3=Z_4$	0.16 (μm)
Third Case: active and passive isolation	
Z_1	140(μm)
Z_2	0.3(μm)
Z_3	0.16 (μm)
Z_4	0.0053 (μm)

The vibration problem of footings resting on or embedded on the soil was treated here considering the solutions given in literature to a circular footing with radius r_O. For the case of a square and rectangular contact area is acceptable to use solutions of a rigid circular contact area as was done herein for approximate response calculations (Prakash and Puri, 1988, Novak and Beredugo, 1972, Richart *et al.*, 1970). Solutions for irregular shapes of the foundation are available and must be found in the literature (Gazetas, 1991). Also Models with several degrees of freedom may be used for predicting the interaction of the machine with its foundation. The interaction changes the rotor-bearing critical speeds predicted by the manufacturer what may causes excessive vibration of the system (Chen, 1981).

The results obtained and presented in table 2 shows that when a machine is rigidly bolted to floor or its block foundation, the vibration of the machine itself may be reduced compared to the case when active isolators are used. The use of active isolators reduces the vibration transmitted through the soil what will reduce the bad effects of the vibration on its vicinity. So, when active mounts are provided the vibration transmitted to the soil will be reduced but this may cause significant vibration to the machine during its operation or even during its starting and stopping stages. The use of passive isolation helps to reduce the effect of the vibration and is also possible to provide isolation using wave barriers or trench barriers (Prakash and Puri, Richart *et al.*, 1970).

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