OPTIMAL DESIGN AND LOCATION FOR A VISCOELASTIC DYNAMIC NEUTRALIZER SYSTEM USING GENETIC ALGORITHM

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Abstract. *The use of viscoelastic materials in dynamic neutralizers (VDN) permits to construct devices with different forms and sizes, making them extremally usefull to control noise and vibrations in many types of structures. Considering the viscoelastic material caracteristics, this form of control device has proved to be very efficient in a wide frequency band. A general and robust method to design viscoelastic dynamic neutralizers (developed by the PISA-CNPq group) considers: non-linear optimization tecnique, equivalent generalized parameters, the structure's modal parameters and fractional derivatives based models for the viscoelastic material. Working in a modal subspace of the structure to be controled, it is possible to find the neutralizer's optimal physical parameters such the struture's response be minimized. In this process, the location of the neutralizers is predeterminated, once known the modal model for the structure. In certain applications, when the modal density of the structure is high or when the modes are coupled, the neutralizer's placement is not so clear for the designer. In these cases, to find the best localization for the control devices can be fundamental. An example that illustrates this fact is the best neutralizer's localization that are used in aerial cables of electric energy transmission. In this work, a general methodology is suggested allowing to determine, simultaneously, the localization and the optimal parameters of a neutralizer system. A numerical example on a steel plate is presented and its results commented.*

Keywords*: Dynamic Neutralizers; Viscoelastic Material; Optimal Design and Location; Optimization.*

1. Introduction

Vibration neutralizers, also called vibration absorbers, are devices connected to other mechanical systems or structures, called primary systems, with the purpose to reduce vibrations and sound radiation. Since they had been used to reduce the roll movement in ships (Den Hartog, 1956), many works and applications related with dynamic neutralizers had appeared. With the modern technology of the viscoelastic materials, the vibration neutralizers have become easy to produce and apply to almost any structure, despite its complexity. Recently, a great effort has been made in the direction to generalize the theory of the vibration absorbers, applied to the more complex structures that those with only one degree of freedom and undamped, studied by Ormondroyd & Den Hartog (1928).

In the work of Espíndola and Silva (1992), a general theory has been derived for the optimal design of a neutralizer system, when connected to a generic structure, with some distribution of damping. This theory has been applied for absorbers of several types (Espíndola and Silva, 1992; Freitas and Espíndola, 1993). The theory is based on the concept of the equivalent generalized quantities for neutralizers, introduced by Espíndola and Silva (1992). With this concept, it is possible to derive the equations of motion for the compound system (primary system + absorbers) in terms of generalized coordinates (degrees of freedom), previously chosen to describe the primary system, despite the fact of the compound system have degrees of freedom added. This fact allows the coordinates transformation using the primary system modal matrix, which is invariant during the optimization process of the neutralizers' physical parameters. In the modal space of the primary structure, it is possible to work with only certain equations, enclosing the frequency band of interest. If the coupling between equations is not considered, the neutralizer system can be designed to be optimal for a particular mode, as in the method of simple optimization of Den Hartog (1956).

Espíndola and Bavastri (1997) and Bavastri (1997), have introduced nonlinear optimization techniques (TONL). Bavastri, Espíndola and Teixeira (1998), using a hybrid algorithm (Genetic Algorithm and TONL), have developed a general technique for the optimal design of dynamic neutralizers in a frequency band. With this new approach, the control left of being projected mode to mode, like in Espíndola and Silva (1992). Now it is conceived in broad band of frequency, where one or more neutralizers can be designed simultaneously, to control one or more modes in the frequency band of interest. This methodology has been modified to ensure that the neutralizer's optimal parameters are a global optimum.

In those works, the localization of the neutralizer system in the structure is preset, once known the modal model of the structure. However, in some applications, when the modal density of the structure is increased or when the modes are sufficiently coupled, the localization of neutralizers is not so direct. In these cases, to find the best localization for the control devices can be fundamental.

In this work, a general methodology is considered that allows determining, simultaneously, the localization and the optimal parameters of a neutralizer system. For such, the optimization technique applied to find the optimal localization of the neutralizer system will be Genetic Algorithms (GA). To be able to compare the different localizations, a technique of internal optimization that allows determining the optimal parameters of the control system must be derived. Thus, when the program converges, it is possible to determine, simultaneously, the localization and the optimal physical parameters of neutralizers. In this work, the numerical code developed by Bavastri (1997) is used as a sub-routine.

2. Equivalent Generalized Quantities for the Simple Absorber

A simple neutralizer has a single lumped mass (m_a) connected to a rigid massless base through a resilient device, assumed as having a viscoelastic nature (figure 1), with complex stiffness $K_c(\Omega)$ equal to (Espíndola and Silva, 1992):

$$K_c(\Omega) = \ell G_c(\Omega) = \ell G(\Omega)[1 + i\eta(\Omega)]. \tag{1}$$

In the equation (1), $G_c(\Omega)$ is the complex shear modulus of the viscoelastic material, $G(\Omega)$ is the dynamic shear modulus, $\eta(\Omega)$ is the loss factor of the material, Ω is the circular frequency and ℓ is a geometric factor that depends on the assembly of the viscoelastic material in the neutralizer device. The complex shear modulus is defined in accordance with the fractional derivative model with four parameters, as defined in Espíndola, et. al. (2005).

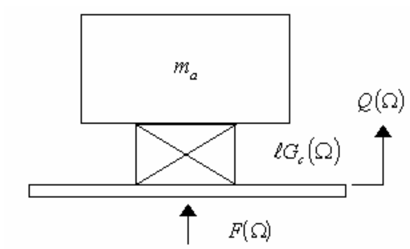


Figure 1: One simple viscoelastic neutralizer.

In figure 1, $Q(\Omega)$ e $F(\Omega)$ are the Fourier transform of the displacement of the base q(t) and the applied force f(t), respectively. This applied force results from the interaction between the neutralizer and the point of the structure where it is applied.

The impedance $Z_a(\Omega)$ and the dynamic mass matrix $M_a(\Omega)$ are given respectively by:

$$Z_a(\Omega) = \frac{F(\Omega)}{i\Omega Q(\Omega)} = \frac{-i m_a \Omega \ell G_c(\Omega)}{m_a \Omega^2 - \ell G_c(\Omega)} \tag{2}$$

$$M_a(\Omega) = \frac{F(\Omega)}{-\Omega^2 Q(\Omega)} = -m_a \frac{\ell G_c(\Omega)}{m_a \Omega^2 - \ell G_c(\Omega)}. \tag{3}$$

The anti-resonance frequency for a simple neutralizer is defined as:

$$\Omega_a{}^2(\Omega) = \frac{\ell G_c(\Omega_a)}{m_a}. \tag{4}$$

In equation 4, Ω_a presents the anti-resonance frequency of the neutralizer. Notice that, in the absence of damping, $G_c(\Omega)=G(\Omega)$. Thus, it can be said that:

$$\ell G_c(\Omega) = \ell G(\Omega_a) r_a(\Omega), \tag{5}$$

where $r_a(\Omega)=G(\Omega)/G(\Omega_a)$. Equations 2 and 3 can now be rewritten.

$$Z_a(\Omega) = -im_a\Omega_a \frac{\varepsilon_a r_a(\Omega)[1+i\eta(\Omega)]}{\varepsilon_a^{\,2} - r_a(\Omega)[1+i\eta(\Omega)]} \tag{6}$$

$$M_a(\Omega) = -m_a \frac{r_a(\Omega)[1+i\eta(\Omega)]}{\varepsilon_a^{\,2} - r_a(\Omega)[1+i\eta(\Omega)]} \tag{7}$$

where $\varepsilon_a=\Omega/\Omega_a$. The equivalent viscous damping is defined as the real part of the impedance (6) and for a simple neutralizer, it is:

$$c_e(\Omega) = m_a\Omega_a \frac{r_a(\Omega)\eta(\Omega)\varepsilon_a^{\,3}}{\left[\varepsilon_a^{\,2} - r_a(\Omega)\right]^2 + \left[r_a(\Omega)\eta(\Omega)\right]^2} \tag{8}$$

The equivalent mass, in the same way, is the real part of expression (7):

$$m_e(\Omega) = -m_a \frac{r_a(\Omega)\left\{\varepsilon_a^{\,2} - r_a(\Omega)\left[1+\eta^2(\Omega)\right]\right\}}{\left[\varepsilon_a^{\,2} - r_a(\Omega)\right]^2 + \left[r_a(\Omega)\eta(\Omega)\right]^2} \tag{9}$$

Thus, the two systems shown in figure 2 are dynamically equivalents (Espíndola and Silva, 1992).

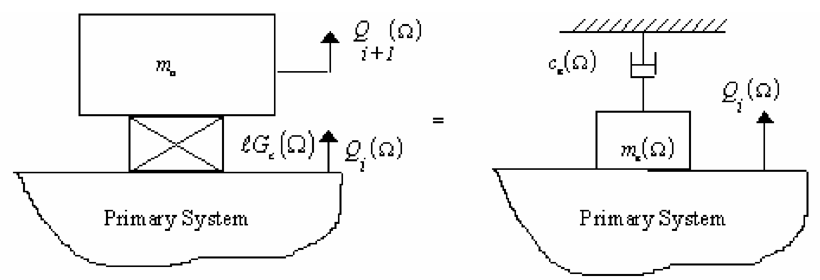


Figure 2: Equivalents systems.

The primary system "feels" the neutralizer as being a equivalent mass $m_e(\Omega)$, dependent on frequency, attached to the generalized coordinate $q_j(t)$ and a equivalent viscous damper with constant $c_e(\Omega)$, connected to the ground.

Therefore, the dynamics of the resultant system (primary + neutralizer) can be formulated in terms of the generalized coordinates of the primary system, where $Q(\Omega)$ is representative, despite the new system now having added degrees of freedom. This is the main advantage of the generalized equivalent quantities concept.

If many of these neutralizers are connected, the equation of motion can be rewritten as (see Bavastri, 1997 or Espíndola and Bavastri, 1995 and 1997):

$$\left[-\Omega^2\tilde{M} + i\Omega\tilde{C} + K\right]Q(\Omega) = F(\Omega). \tag{10}$$

Notice that the effect of coupling neutralizers falls again on modifications into the mass and damping matrices in the primary structure. The vector of the generalized coordinates of the primary system remains unchanged. To illustrate this idea, it is assumed that p neutralizers, with masses $m_a^{(1)}$, $m_a^{(2)}$,..., $m_a^{(p)}$, are connected to p physical generalized coordinates q_{k_1}, q_{k_2},..., q_{k_p}. The generalized equivalent masses and damping are $m_e^{(1)}(\Omega)$, $m_e^{(2)}(\Omega)$, ..., $m_e^{(p)}(\Omega)$ e $c_e^{(1)}(\Omega)$, $c_e^{(2)}(\Omega)$, ..., $c_e^{(p)}(\Omega)$. The resultant mass and damping matrices will be:

$$\tilde{M} = M + \begin{bmatrix} 0 & & & \\ & m_e^{(1)}(\Omega) & 0 & \\ & & \ddots & \\ & 0 & m_e^{(p)}(\Omega) & \\ & & & 0 \end{bmatrix} = M + M_A(\Omega) \quad \text{and} \quad \tilde{C} = C + \begin{bmatrix} 0 & & & \\ & c_e^{(1)}(\Omega) & 0 & \\ & & \ddots & \\ & 0 & c_e^{(p)}(\Omega) & \\ & & & 0 \end{bmatrix} = C + C_A(\Omega) \tag{11 and 12}$$

In equation (10), it is assumed the transformation,

$$Q(\Omega) = \Phi\tilde{P}(\Omega), \tag{13}$$

where Φ is the modal matrix of the primary system, obtained numerically or experimentally. Its order is nxñ, where n is the number of degrees of freedom and ñ is the number of computed or measured eigenvectors. Normally ñ<<n. The vector $\tilde{P}(\Omega)$ is called the generalized coordinate, in the sub modal space of the primary system (ñx1). If equation (13) is introduced in equation (10) and pre-multiplied by Φ^T, assuming proportional damping for the primary system, the result is:

$$\left\{-\Omega^2\left[I_{\tilde{n}} + \tilde{M}_A(\Omega)\right] + i\Omega\left[\Gamma_{\tilde{n}} + \tilde{C}_A(\Omega)\right] + \Lambda_{\tilde{n}}\right\}\tilde{P}(\Omega) = \tilde{N}(\Omega), \tag{14}$$

where

$\Gamma_{\tilde{n}} = diag(2\xi_j\Omega_j)$, $\Lambda_{\tilde{n}} = diag(\Omega_j^{\;2})$, $\Phi^T K\Phi = \Lambda_{\tilde{n}}$, $\tilde{M}_A(\Omega) = \Phi^T M_A(\Omega)\Phi$, $\tilde{C}_A(\Omega) = \Phi^T C_A(\Omega)\Phi$ and

$$\tilde{N}(\Omega) = \Phi^T F(\Omega) = \sum_{j=1}^{n}\psi_j^{\;T} F_j(\Omega) \tag{15}$$

In (15) ψ_j is j[th] line of the matrix Φ e $F_j(\Omega)$ is the j[th] component of the vector $F(\Omega)$. In the equations (15) it is assumed that the eigenvectors of the primary structure are orthonormalized in relation to the mass matrix M (see Espíndola and Bavastri, 1997). Ω_j are the natural frequencies of the primary structure and ξ_j is the corresponding modal damping. Equation (14) represents a set of ñ<<n equations and can be resolved directly for any frequency with the use of equations 8 and 9. Returning to equation (13), it is obtained the solution in physical coordinates. Through equations (15), it is simple to show that:

$$Q(\Omega) = \Phi\tilde{A}(\Omega)\Phi^T F(\Omega), \tag{16}$$

where

$$\tilde{A}(\Omega) = \left[\tilde{A}_0(\Omega) - \Omega^2\tilde{M}_A(\Omega) + i\Omega\tilde{C}_A(\Omega)\right]^{-1} \tag{17}$$

$$\tilde{A}_0(\Omega) = diag\left(\Omega_j^{\;2} - \Omega^2 + i2\xi_j\Omega_j\Omega\right) \tag{18}$$

in the case of orthonormalization. For a system with one degree of freedom, the reason of masses between the neutralizer and the primary system recommended by Den Hartog (1956) is $\mu = m_a/m_s = 0.1$ to 0.25. For a system of multiple degrees of freedom this relation is given by (Espíndola and Silva, 1992):

$$\mu_j = \frac{\left(\sum_{j=1}^{p} m_a^{(i)}\Phi^2{}_{k_i j}\right)}{m_j}, \tag{19}$$

where $m_a^{(i)}$ is the i[th] mass of neutralizer. The symbol m_j represents j[th] modal mass of the primary system that, in the case of orthonormalization of the eigenvectors, is equal to one. In this work, the masses of neutralizers are defined by an arithmetic mean of $m_a^{(i)}$ calculated for each mode, inside the frequency band of interest. Considering all the masses equal ($m_a^{(1)} = \ldots = m_a^{(p)}$),

$$m_a = \frac{\sum m_a^{(i)}}{n_m}, \tag{20}$$

where $i = 1, n_m$ and n_m is the number of modes in the frequency band of interest.

3. Constitutive Equations for Viscoelastic Materials in Fractional Derivatives

Equation (21) shows the definition of the called complex elasticity modulus, which is a function of the frequency. It is also function of the temperature, once its parameters are also dependents on the temperature. Being complex, $E_c(\Omega)$ can be written as:

$$E_c(\Omega) = E(\Omega) + iE'(\Omega) \quad \text{or} \quad E_c(\Omega) = E(\Omega)(1 + i\eta(\Omega)) \qquad \text{(21 and 22)}$$

where $\eta(\Omega)=E'(\Omega)/E(\Omega)$.

$E(\Omega)$ is known as storage modulus of the viscoelastic material, $E'(\Omega)$ is the loss modulus and is associated with the capacity of the material to dissipate vibration energy. $\eta(\Omega)$ is called loss factor. The mathematical formulation in terms of fractional derivatives has a close relation with the molecular theories for the behavior of the viscoelastic materials (Bagley and Torvik, 1983).

In this work, the model of four parameters, based on fractional derivatives is used for an optimal design of the physical parameters in viscoelastic neutralizers. The model can be written as (see Pritz, 1996):

$$E_c(\Omega) = \frac{E_0 + (i\Omega)^\alpha E_1}{1 + (i\Omega)^\alpha b_1}. \qquad (23)$$

Similarly, a model for the shear modulus will be:

$$G_c(\Omega) = \frac{G_0 + (i\Omega)^\alpha G_1}{1 + (i\Omega)^\alpha b_1}, \qquad (24)$$

or in an equivalent form:

$$G_c(\Omega) = \frac{G_0 + (ib\Omega)^\alpha G_\infty}{1 + (ib\Omega)^\alpha}, \qquad (25)$$

where $b=b_1^{1/\alpha}$ and $G_\infty=G_1/b_1$.

Equation (24) defines $G_c(\Omega)$ in terms of four parameters: G_0, G_∞, b e α. The time dimensional parameter b is called relaxation constant of the material.

4. Optimization of the Neutralizer's Parameters in a Frequency Band.

In the optimization process of the neutralizers' physical parameters, the anti-resonance frequencies are considered as design parameters, while the masses of neutralizers are pre-established. Thus in Bavastri (1997), the design vector is defined by:

$$x^T = (\Omega_{a1}, \ldots, \Omega_{ap}). \qquad (26)$$

being p the number of components of the design vector of equation (26), assuming the same viscoelastic material to all neutralizers of the system. The objective function used in the optimization methodology (Espíndola and Bavastri, 1995):

$$f_{obj}(x) = \left\| \left[\max_{\Omega_1 < \Omega < \Omega_2} \left| P_j(\Omega, x) \right| \right]^T \right\|_2, \; x_i^L < x_i < x_i^U, \; i = 1, p, \qquad (27)$$

where $P_j(\Omega, x)$ are the components of the main coordinates in the modal subspace of the primary system for each design vector *x,* and Ω_1 and Ω_2 are the inferior and superior limits of the frequency band of analysis. $\| \|_2$ is the Euclidian norm. As posed in Bavastri et al. (1998) this objective function is multimodal. After the optimization the stiffness of each neutralizer is given by:

$$k_{ai} = m_a \Omega_{ai}^{\ 2}, \text{ where } m_a \text{ are the mass of neutralizers.} \qquad (28)$$

5. Genetic Algorithm.

Briefly, a genetic algorithm (GA) is a search and optimization technique based on the natural selection. GA is equivalent to a numerical simulation of the Theory of the Evolution published by Charles Darwin in 1859. Holland (1975) was the first that considered the technique followed for Goldberg (1989) and others.

The user of the algorithm must parametrize the environment indicating the ways for which a population can evolve. This environment parametrization consists of the search space and the rules for the population evolution constitute the called objective function, or optimization function.

Inside of the search space, that in general is vast, some possible solutions are generated, in general of random form. These possible solutions are represented by points in the search space. This initial set of generated solutions represents the first population, or first generation.

The general structure of the GA can be summarized figure 3.

```
Begin
        k = 0
        Iniciate population P_k
        Avaliate P_k
        While stop criteria not satisfied
                k = k + 1
                Select parents in P_k-1
                Aply crossing in P_k
                Aply mutation in P_k
                Avaliate P_k
        Finish while
stop
```

Figure 3. Typical computational structure of a genetic algorithm.

The convergence of the method is guaranteed, even for multimodal problems, by the Theorem of the Schema or Basic Theorem of the Genetic Algorithms (Holland, 1975 and Goldberg, 1989). The schemata that possess superior aptitude to the average population will grow exponentially, in contrast of those with inferior aptitude to the average. These last ones will have increasing probabilities of extinguishing. The code of the GA used in this work has been developed by Carroll (2001) in FORTRAN 77 and updated for FORTRAN 90.

In the present work, the objective function is multimodal and multi objective as it shows the next equation.

$$F = w_1 f_1 + w_2 f_2 \tag{29}$$

where f_1 is the minimum value of the objective function given in eq. (27). f_2 is the value found for the mass of neutralizers added to the structure (all neutralizers of the system are designed with the same mass).

As function F must to be minimized, then w_1=-1 and w_2, proposed in this work, is:

$$|w_2| = \frac{-f_1}{0.02 M_{str}} \tag{30}$$

where M_{str} is the total mass of the structure that needs to be controlled and f_1=200 dB is adopted. In this way, to minimize F means to minimize f_1 with the minimal mass of the neutralizers.

6. Numerical Exemple.

The described approach has been tested in a numerical application. The object of study was a plain plate of dimensions 600x400x5 mm made of steel with a total mass of 9.42 kg. The plate was simply supported at its four vertices. The modal model of the plate was produced by finite elements using a mesh of 294 elements of shell type (Ansys shell63) with 4 nodes totalizing 330 nodes. Then the eight first natural frequencies and its modes have been computed. Of this set, the first five modes have been used for analysis. All the extracted frequencies can be seen in table 1.

The viscoelastic material used for this simulations carried through this work has been neoprene and its main properties can be seen in table 2.

Table 1. Natural frequencies of the plate.

Natural frequency	Hz	Natural frequency	Hz
1st	30.717	5th	181.90
2nd	74.070	6th	198.54
3rd	88.982	7th	240.73
4th	115.92	8th	309.93

Table 2. Principal properties of neoprene.

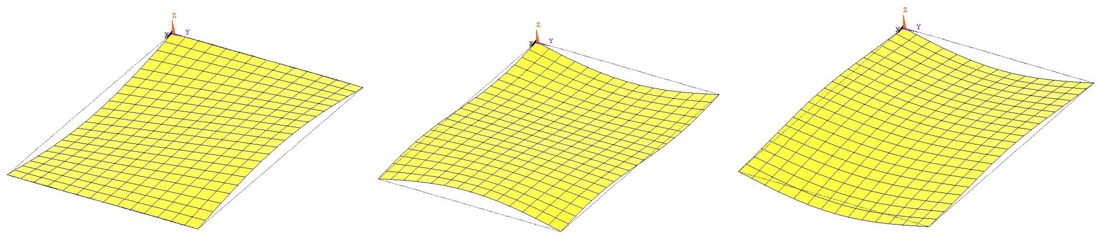
Viscoelastic material properties	Neoprene
G_0	1.53×10^6 MPa
G_∞	1.11×10^8 MPa
b	1.864×10^{-5} s
α	0.396

The genetic algorithm employed the following parameters: population size, 7; number of generations, 400; crossover probability, 0.95; and mutation probability, 0.03, suggested by Carroll (2001).

For the localization of the neutralizer system some nodes of the structure have been suppressed of the search space: the four nodes where the restrictions are applied and all nodes of the border of the plate. Therefore, only in the nodes inside the plate it is possible the place a neutralizer.

It has been used the micro-genetic algorithm with elitism and sharing. The code used in this work, developed by Carroll (2001), contemplates this condition and is capable to work with the micro-genetic algorithm.

With the objective to reduce the vibrations in the frequency band between 40 and 140 Hz, for the second, third and fourth modes (figure 5), a neutralizer system of such form has been design, aiming to reduce the response amplitude of vibration. Three different neutralizer systems have been proposed. The first system with just one neutralizer. Another with two neutralizers. And the last system with four neutralizers.



(a) (b) (c)

Figure 4. (a) Second mode (b) Third mode and (c) Fourth mode

Table 3 shows the results obtained for the three optimal designed systems.

Table 3. Neutralizers systems

Neutralizers System	Neutralizer mass [g]	Natural frequency [Hz]			
1 neutralizer	1128.9	43.3028			
2 neutralizers	212.26	58.4012		90.2262	
4 neutralizers	106.11	60.3491	69.3937	78.5895	82.3072

In figure 6 the frequency response functions of the primary structure with and without optimal neutralizers are presented. As it has been expected, the optimal localization of neutralizers determined with the proposed methodology, is a peak at modes of natural frequency, inside of the interest frequency band. However, when more than one mode is being subjected to consideration (e.g. a peak of a mode is close to a node of another mode), the proposed methodology is capable of determining, simultaneously: i) the optimal position to locate the neutralizer; ii) the minimal mass for the neutralizer and iii) the optimal neutralizer physical parameters (anti resonant natural frequency).

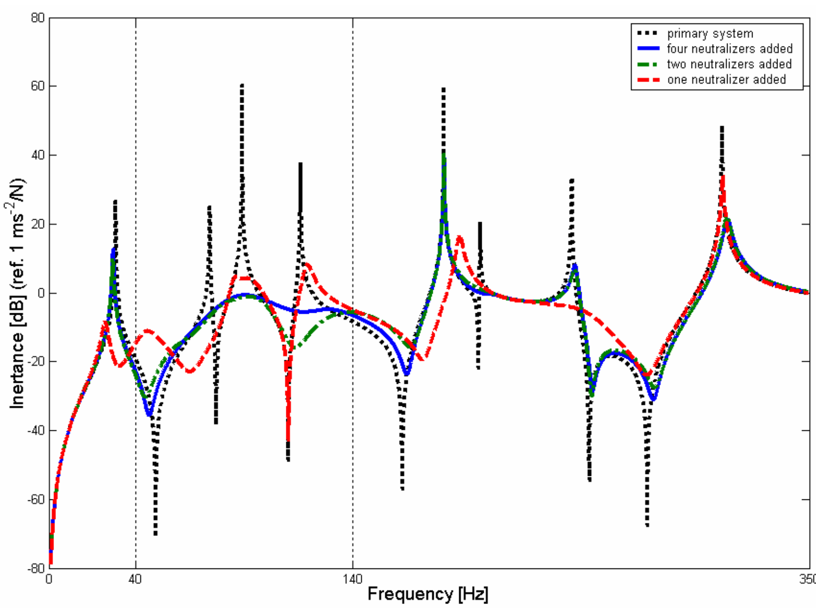
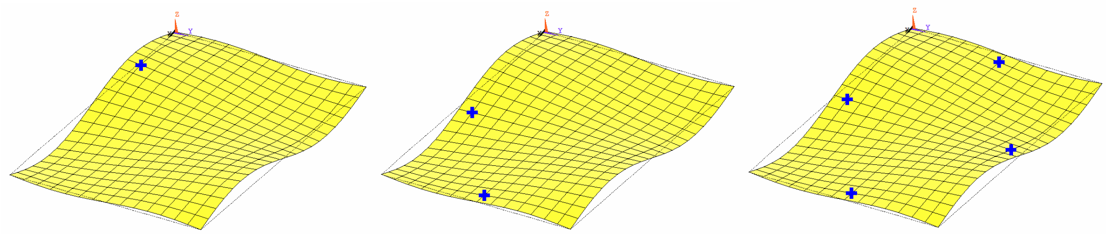


Figure 6. Reduction of vibration level accomplished by NDV's system.

The localization of the neutralizer system can be seen in figure 7. This figure illustrates the representation in the plate's fifth mode of vibration.



(a) (b) (c)

Figure 7. Vibration neutralizer system location. (a) One neutralizer, (b) Two neutralizers and (c) Four neutralizers

7. Conclusions.

- A revision of the concepts used in the general methodology of viscoelastic optimal dynamic vibration neutralizer design has been presented.
- A general methodology that allows optimizing, simultaneously, the localization and the optimal physical parameters of neutralizers has been proposed. This methodology is fundamental when there is a primary system with an increased modal density, inside the frequency band of interest.
- A numerical example showing the effectiveness of this methodology to control the vibration in a frequency band considering three natural frequencies has been introduced. Three different cases have been shown, indicating promising results.
- Despite being in its initial stages, this methodology revealed to be efficient for the optimal design of passive vibration control.

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