STATISTICAL ANALYSIS OF VIBRATION SIGNALS FOR CONDITION MONITORING OF DEFECTS IN ROLLING ELEMENT BEARINGS

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Abstract. *The article has as objective the use of vibration signals for defects detection in bearing components using statistical methods. Following parameters are studied in the statistical analysis: Defect Factor, K Factor, Crest Factor, RMS, Peak and the distribuition Kurtosis.For the data signal acquisition a vibration analyser was used, and for the calculations of the statistical parameters, the softwares MATLAB and MINITAB. Finally, the results obtained with the different types of parameters are compared to improve the reliability in the diagnosis and also to study the influence of the shaft speed (rotation) effect and defect size in the bearing component on the statistical parameters, mainly in machines with low speed. This method is a baseline for pattern recognition diagnostic method currently in research.*

Keywords*: Bearing, Vibration, Statistical Parameter*

1. Introduction

The article shows a study on the signals of vibrations for diagnoses of defects in rolling element bearing based on the techniques of pattern recognition using the statistical parameters.

We will study methods of pattern recognition and the mathematical theory for the transformation of the model measured space composed by several measures of vibrations in a rotative machine with defects in the components of the rolling element bearing for a pattern model with finite dimension, afterwards we will make a transformation in a feature space and later on another transformation that will result in a model of classified space. In the feature space we will use in the statistical analysis, the parameters as RMS, Peak, Factor of Crest, K Factor, Defect Factor and the moment distribution (Kurtosis), Heng and Nor (1998).

The statistical parameters are selected for the form of model feature space, which will be built in two dimensions and formulated by a nonlinear transformation.

The transformation Intraclass has the objective of clustering the data of different defects in rolling element bearing in different regions of the feature space and after to build the classification space by the function discriminant linear, using the method of Cluster (not supervised) or Fisher (supervised), with that, the classified space will be trained with the data of the faulty rolling element bearing.

Another studied aspect is the influence of the variation effect of the shaft speed and load in the results of the statistical methods and in the techniques of pattern recognition, due to the sensibility these components bearing rolling element.

To accomplish the calculations using statistical parameters of the vibrations signals of our test rig, with the accelerometer fixed in a strategiac point, close to the faulty rolling element bearing in the outer race, the softwares used were Matlab and Minitab.

The methods of the statistical parameters will be compared through graphs, with the objective of obtaining a good reliability in the location and severity related with defects in the rolling element bearing, mainly in machines rotating with low speed, and consequently this will give a base, so that in the next works the practical data may be used with the techniques of pattern recognition, and mainly the methods: Cluster and Fisher.

2. Theory and methodology of the pattern recognition

The pattern recognition can be classified as heuristic, linguistic and mathematical Andrews (1972), the mathematical pattern recognition can be described as that identifies classifiable patterns through the mathematical transformations in four spaces, the measurement space M, for the pattern space P, for the feature space F, and finally for the classification space C, as shown below:

$$M \rightarrow P \rightarrow F \rightarrow C$$

Note that this description is different from that given by Andrews (1972), in which we include the measurement space Xi, Sun and Krishnappa (2000).

2.1. Measurement space

The measurement space is constructed directly through the measured data, and the measures of vibrations accomplished in the machine with defect in the rolling element bearing and collected by the Bruel&Kjaer vibration analyzer.

Considering the finite dimension N, we have to follow the column of the vector data:

$$S = [S_1, S_2, ..., S_i, ..., S_N]^T$$

That represents a series of measurements with N data sampled and digitalized. Therefore, the state of measurements is formed by N axes, and vector S represents a point in the space.

We will use prototypes data of each class, with the objective of identifing and stablishing the classifiable patterns or model. For each class, we need a group of prototypes data sampled, for this reason, the vector of the measurement space represents the mth prototype of the class K, and it is expressed in the following way:

$$q_m^{(K)} = [q_{1m}^{(K)}, q_{2m}^{(K)}, ..., q_{Nm}^{(K)}]^T$$

Where m=1,…. ,M_k. M_k is defined as the number of the prototype vectors data for the class kth. For K classes, the total number of data groups, N_d, is given for:

$$N_d = \sum_{K=1}^{K} M_K$$

2.2. Pattern space

According to Andrews (1972), the pattern space is represented by the measurement data. The pattern space is essentially what dominates, selects and digitalizes the data collected by the sensor of vibration.

It could also be defined by means of statistical analysis methods. After this process, the vector data S becomes a new vector x_p.

$$x_p = [x_{p1}, x_{p2}, ..., x_{pi}, ..., x_{pR}]^T$$

Where the index p indicates the pattern space. The dimensions of the space pattern are defined as R, where $R \le N$.

A prototype is given by the following vector,

$$y_{pm}^{(K)} = [y_{p1m}^{(K)}, y_{p2m}^{(K)}, ..., y_{pRm}^{(K)}]^T$$

The problem classification is simply the discovery of the separation of the surfaces with dimension R, with the objective of classifying the correctly known prototype.

The solution is based on the hypotheses which the pattern space forms a metric space (relative) and satisfies the following conditions with relation to the points (a,b,c) of the analysed space Andrews (1972):

1. d(a,b)=d(b,a)   2. $d(a,b) \le d(a,c)+d(b,c)$
3. $d(a,b) \ge 0$;   4. d(a,b)=0 if a=b,

Where d is defined as the function distance, and a,b and c represent three vectors in the pattern space. In terms of the metric space (relative), the similarity of the point x_p for the class Kth, can be measured as:

$$S\left(x_p, \{y_{pm}^{(K)}\}\right) = \frac{1}{M_K} d^2\left(x_p, y_{pm}^{(K)}\right)$$

A similarity, is simply an average of the square of the distance between the point x and the group of prototypes $Y_{pm}^{(K)}$.

However, if the pattern space is a metric space (relative), in the sense that each dimension or axis is a quantitative measure of the same studied physical phenomenon, it is important not to make mistakes with the analyzed data.

Then, each dimension can be a measure of nonrelated parameters and this should be properly normalized before beginning the combinations with the other dimensions as in a calculated distance.

Comparing the vectors with the same unit and after normalizing the measured data, in which is referred for the square of the pattern space. The simplest way is to divide the data measured by its variances, knowing that the variances are normalized.The use of the normalization for the normalization variance with its dimension, can be less sensitive to values of extreme noises. Then, we have that:

$$x'_r = \frac{x_r}{\sigma_r}, \quad \text{Where} \quad \sigma_r^2 = \left(\sum_{k=1}^{k} M_k\right)^{-1} \sum_{k=1}^{k} \sum_{m=1}^{M_k} \left(y_{rm}^{(k)} - \bar{y}_r\right)^2 .$$

And for this example,

$$\bar{y}_r = \left(\sum_{k=1}^{k} M_k\right)^{-1} \sum_{k=1}^{k} \sum_{m=1}^{M_k} \left(y_{rm}^{(k)}\right)$$

Where $\bar{y}_r$ represents the type of variance of the rth dimension defined by the prototypes. Other techniques for normalization can also be applied, such as the normalization value extreme.

2.3. Feature space

The feature space is the intermediary space between the pattern space and the classified space, with the dimensions defined by L. The data vector x_f and the prototype vector $Y_{fm}^{(K)}$ in the feature space are defined as

$$x_f = \left[x_{f1}, x_{f2}, ..., x_{fi}, ..., x_{fL}\right]^T \text{ and } y_{fm}^{(K)} = \left[y_{f1m}^{(K)}, y_{f2m}^{(K)}, ..., y_{fLm}^{(K)}\right]^T .$$

Where the index f indicates the feature space. That space is to reduce the dimensions of the pattern space so that the classification algorithm can be computed with efficiency and the results presented correctly. The feature space also combines the dimensions of the pattern space with the extraction of the classified characteristics. The feature space studied is composed by six parameters.

The process of selection of the characteristic space is used to find a linear or nolinear transformation, with the objective of reducing the dimensions of the pattern space and maintaining. Its characteristics differentiated still for classification proposal, Donavan and Krishnappa (1999).

3. Classification space

The classification space is defined as the separation of the surfaces in the feature space.

The classification algorithm defines the division of N dimensions of the feature space in separate regions, each region is associated with a class.

For a certain class given K, its surfaces will be separate in K surfaces. The separate surfaces are plane, with dimension L-1. A classification algorithm is to define the space partition inside, in specific dimensional L of the separated regions, with each region associated with a class.

The separation is a point in the line for L=1, a line in the plane for L=2, and a plane in the volume for L=3. Since, the data is not linearly separated, the partition of the space generally results in nonlinear surfaces. For example, the boundary condition of the classified space L=2 can be formed by curves.

4. Pattern recognition of the statistical parameters

The recognition of the pattern space of the statistical parameters is based on the construction of the pattern space using the statistical analysis. In this section we will describe how we selected the statistical parameters to form the pattern space for diagnoses of defects in rolling element bearing. In the two following sections, we will show the construction of the feature space and the classification space, Sun and Chen (2004).

5. Selection of pattern space

The statistical parameters used for the diagnosis in rolling element bearing Almeida (2005) and the selection of the pattern space is: RMS, Peak, Factor of Crest, Kurtosis, Defect Factor e Factor K.

The application of the distribution of the moments as Kurtosis has had very stable results in the monitoring signals of vibrations for of the rolling element bearing in rotative machines.

The moments of first order, the value RMS is the squared root of the moment of second order, and the variance is the central moment of second order.

The moment can be written as:

$$M_r = \frac{1}{N}\sum_{K=1}^{N}(S_i - \bar{S})^r$$

Where N is the number of measured points and r it is the order of the moment. The following equations present the calculations for other statistical variables in discrete forms. It is noticed that the peak and RMS have units. For the normalization, we used the RMS value of the nondefects bearing data, defined as RMSu, as normalizing parameter.

$$RMS = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(S_i - \bar{S}\right)^2} \text{ and } RMS^* = \frac{RMS}{RMS_u}$$

$$Peak = \frac{1}{2}\cdot(max(S_i) - min(S_i)) \text{ and } Peak^* = \frac{Peak}{RMS_u}$$

$$CrestFactor = \frac{[Peak]}{RMS}$$

$$KFactor = RMS \cdot Pico \text{ and } KFactor^* = \frac{KFactor}{RMS_u^{\,2}}$$

$$DefectFactor = Peak - RMS \text{ and } DefectFactor^* = \frac{Peak - RMS}{RMS_u}$$

$$Kurtosis = \frac{\frac{1}{N}\cdot\sum_{i=1}^{N}(S_i - \bar{S})^4}{RMS^4}$$

The RMS_u are determined taking intoaccount the speed the shaft of the rolling element bearing. In terms of RMS_u, we can define the following normalized vector Z, Sun and Xi (1999).

$$Z = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{bmatrix} = \begin{bmatrix} RMS^* \\ Peak^* \\ CrestFactor \\ KFactor^* \\ DefectFactor^* \\ Kurtose \end{bmatrix}$$

6. Selection of pattern space

The statistical parameters of the signals of vibrations are affected by the operation conditions, for example, the rotating speed shaft out load .In this first work the load is an invariant parameter. The tendency analysis is used to investigate the effects of the conditions of operations of the rolling element bearing on the statistical parameters.

"Table1" shows the variation of the parameters with the shaft speed and the values obtained in our research.

Table 1. Measurement results from test

Vibration statistical parameter -Velocity vibration levels	**Race outer small defect**			**Race outer large defect**		
	29,3 Hz 1758 RPM	**25 Hz 1500 RPM**	**15,62 Hz 938 RPM**	**29,3 Hz 1758 RPM**	**25 Hz 1500 RPM**	**15,62 Hz 938 RPM**
RMS (mm/s)	3,20	2,84	0,99	10,91	6,44	1,97
Peak (mm/s)	6,13	5,54	1,91	23,18	17,26	4,04
CrestFactor	1,91	1,95	1,93	2,12	2,68	2,05
DefectFactor (mm/s)	2,93	2,70	0,92	12,27	10,87	2,06
Kfactor $(mm/s)^2$	19,63	15,72	1,88	252,88	111,23	7,97
Kurtose	1,65	1,87	1,45	1,71	2,23	1,74

Now we will define the following vector to form of space model

$$x_P = \begin{bmatrix} x_{p1} \\ x_{p2} \\ x_{p3} \\ x_{p4} \end{bmatrix} = \begin{bmatrix} RMS^* \\ Kurtosis \\ CrestFactor \\ DefectFactor^* \end{bmatrix} \quad (1)$$

The dimension of our pattern space is four.

7.Selection of feature space

7.1.Feature origin

The most important aspect of the pattern recognition is to select the feature space.
For this reason, the second feature space is determined as:

$$RMS^* \cdot \frac{CrestFactor}{Kurtosis} + DefectFactor^* \quad (2)$$

By substituting the equation (1) for the CrestFactor in the x_f in the second space, the vector defined as feature space is given as:

$$x_f = \begin{bmatrix} x_{1f} \\ x_{2f} \end{bmatrix} = \left[\left(\frac{Peak^{*Kurtosis}}{Kurtosis} + DefectFactor^* \right) \right] \quad (3)$$

The dimension of our feature space is two, L=2 and the prototype of the vector is defined as:

$$y_{fm}^{(K)} = \begin{bmatrix} y_{f1m}^{(K)} \\ y_{f2m}^{(K)} \end{bmatrix} \quad (4)$$

Note that the actual statistical parameters that can be used in our work for recognition pattern are: Peak*, RMS*, Kurtose and DefectFactor*.

The feature space can be built by using the equation (3) when considering both conditions of defects in the rolling pattern bearing incipient or large defects, obviously, the operation conditions will be considered in the speed and the load for several defects in the rolling element bearing.

7.2. Transformation intraclass

The transformation intraclass has the purpose of increasing the clustering of prototypes inside of the same class.

That is accomplished by minimizing a metric the points, defining the class. For the class Kth, there is a total of M_k prototypes $y_m^{(K)}$ being each a point in the two dimensions of the feature space.

The transformation intraclass of the feature space is defined as:

$$y_{fm}^{\prime(K)} = W^{(K)} \cdot y_{fm}^{(K)} \quad (5)$$

Where the apostrophe indicates the transformation intraclass, σ_1^2 and σ_2^2, as the variances of the two variables in the feature space, $W^{(K)}$ is defined by the following diagonal matrix, Xi and Sun (2000):

$$W^{(K)} = \left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} \right)^{-1} \cdot \begin{bmatrix} \frac{1}{\sigma_1^2} & 0 \\ 0 & \frac{1}{\sigma_2^2} \end{bmatrix} \tag{6}$$

The variances σ_1^2 and σ_2^2 are calculated**,** respectively, for the first and second row of the following data matrix Y^K formed by M_k prototypes.

$$Y(K) = \begin{bmatrix} y_{f11}^{(K)} & .... & y_{f1MK}^{(K)} \\ y_{f21}^{(K)} & .... & y_{f2MK}^{(K)} \end{bmatrix} \tag{7}$$

Through the intraclass transformation the mean square of the intrasets distance of the Kth class is minimized Andrew (1972). It can be seen in the equation (6) in which the two dimensional coordinates are inversely proportional to the variances of their own dimension.

The samples belonging to the same class are grouped or clustered.

For diagnoses in rolling element bearing, the vector x_f after the transformation intraclass, becomes:

$$x_f' = Wx_f \tag{8}$$

Where W is defined in the same way as the equation (6), except in this case, the variance will be determined with base on spanning x_f the number of samples.

8. Formation of the classification space

Remembering that the objective of the formation of the classification space is to partition the feature space, for in K classes, S1,....., Sk......, Sk, mathematically, this problem consists of finding a function that can measure each point in the feature space in terms of its degrees of membership for a given class.

This function is called discriminant function in the pattern recognition, and it is defined to all points in x_f feature space, inside the region described in S_k, where a function exists $g_k(X)$ Sun,Chen and Xi (2004), such as:

$$g_k(x_f) > g_j(x_f), \tag{9}$$

In other words, inside of the region S_k, the Kth discriminant function will have the largerst value.

The discriminant function linear piecewise is used to approach the conditions or boundary condition, separating the different regions (classes).These functions are defined by the minimum distance between the point x_f and the points prototypes in the class S_k.

$$d(x_f, S_k) = \min_{m=1,...M_k} \left\{ d(x_f, y_{fm}^{(k)}) \right\} \tag{10}$$

The point x_f belongs to the class S_k ,when the distance is minimum. The classification then comes to determine a minimum distance between all the prototypes of S_k and the unknown x_f, mathematically, this can be written as:

$$x_f \in S_k \quad \text{if} \quad d(x_f, S_k) = \min_k d(x_f, S_k) \tag{11}$$

Through the mathematical manipulations Andrews (1972), the discriminant function linear piecewise can be given as:

$$g_k(x_f) = \max_{m=1,\dots,M_k} \left\{ x_f^T y_{fm}^{(k)} - \frac{1}{2} y_{fm}^{(k)^T} y_{fm}^{(k)} \right\} \quad (12)$$

The boundary conditions that separate the different class regions are calculated by the following equation

$$g_k(x_f) - g_j(x_f) = 0 \quad (13)$$

The boundary conditions determined by the equation (13) show a avery different partition of the different class region for each one of the defects of the rolling element bearing.

9. Case study

9.1. Experiment

So asto develop the study we used the defects of the ball bearings in small electric motor**.** The motor bearing is SKF 6220 with an outer race defect. In this case only the speed rotation was varried and the load changed naturally for unbalance of the rotor (centrifugal force), which changes with the square speed. The figure (1) shows the simple test rig in UNIFEI Vibration Lab.

Figure 1 - Electric Motor with defect in the outer race of the rolling element bearing

The following figures show the comparison of all measured parameters, when the rolling element bearing was with small defect and large defect.

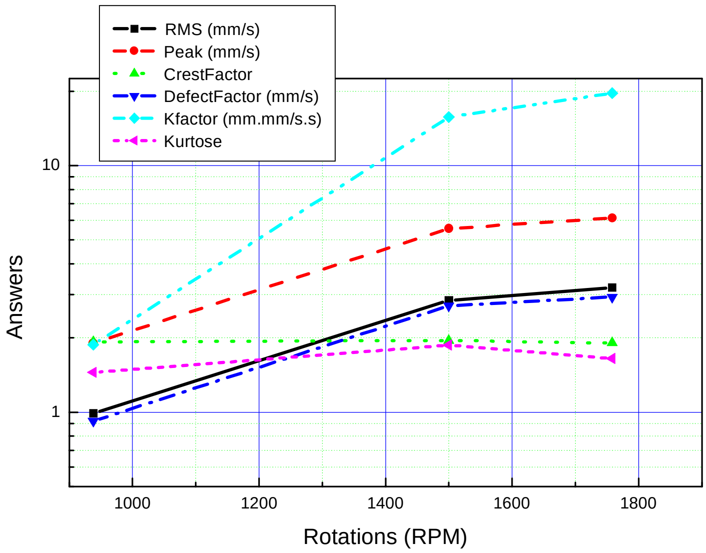

Figure 2 - Values of the parameters for different shaft rotations using the vibration analysis (rolling element bearing with small defect)

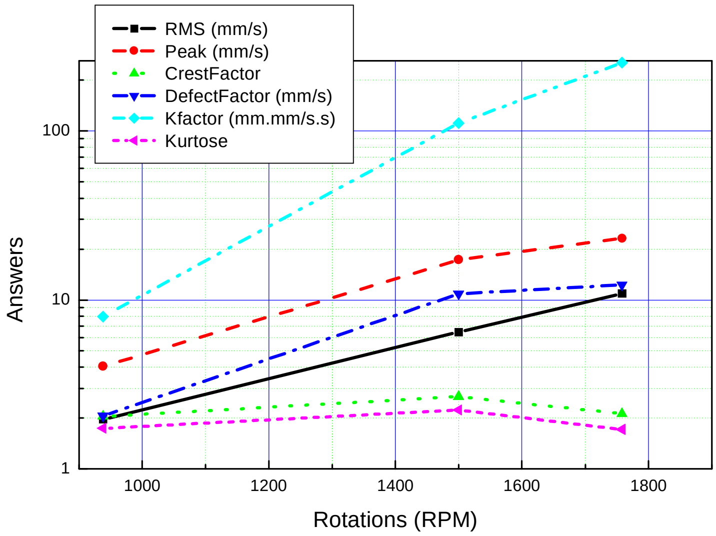

Figure 3 - Values of the parameters for different shaft rotations using the vibration analysis (rolling element bearing with large defect)

10. Conclusion

In this article a methodology is developed for diagnosis of defects in rolling element bearing through the statistical parameters and the technique of pattern recognition. We can observe in the previous graphs that when increasing the defect in the outer race of the rolling element bearing, the parameters as RMS, Peak, DefectFactor and Kfactor are more sensitive to the operational conditions and responded better to the others parameters as CrestFactor and Kurtose. This happens because when the defect is spread, the signal becomes random and consequently the levels of vibrations of the CrestFactor and Kurtose decrease.

Another important aspect is that when comparing all the parameters, it is concluded that the KFactor responds (larger sensibility) much better to the rolling element bearing both with a small or large defect than to other analyzed parameters.

Sometimes in the practice, we identify the problem, but we don't know how long that faulty element will last before breaking the equipment. The future of the development of the technique shown in this article will be of great importance to increase the capacity of predicting the remaining life of the components of the rolling element bearing.

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12. Responsibility notice